

# **TOWARDS COMPUTING THE PROPERTIES OF LARGE MOLECULES FROM ATOMS-IN-MOLECULES (AIM) DENSITIES**

by

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# Abstract

This thesis is divided into two main parts. In the first part, we proposed new three partition weights, namely; Fermi-Dirac, triangle, and Awad weights. In this part, we aim to develop new weights in which the core region close to a nucleus of an atom is assigned to that atom and not to the core or the bond of other atoms. We visually illustrate the molecular radial electron density (RDEN) and bond electron density (BDEN) using the proposed weights. We compared the molecular properties including the total number of electrons  $N_e$ , the electron-nuclear potential energy  $V_{ne}$ , and Coulomb potential energy  $V_{ee}$  which are calculated numerically using the Awad weight with those calculated by Hartree-Fock (HF) wavefunction (the exact values). Also, the computed results using the Awad weight were compared to the computed results obtained using Becke weight. Our findings show that the Awad weight gives better bonding-region representations for both RDEN and BDEN than those obtained using Becke weight. Also, the Awad weight gives better results in computing the molecular properties than those using the Becke weight.

In the second part, we proposed a new approach for calculating the total energy of molecules called atoms in molecules density (AIMD). In this part, we present the first version of AIMD. In AIMD, the molecular properties of the target molecules were computed in three steps: (i) generating small molecules (fragments) from molecules of interest, (ii) storing some properties of these fragments in a database, (iii) using the data stored in the database and by using the direct method to compute the molecular properties of the target molecule.



The electronic molecular energy ( $E^{ele}$ ) is obtained using AIMD as follows,

$$E^{ele} = T^{sum} + V_{ne}[\rho] + J[\rho] - K_{HF}^{sum} \quad (1)$$

where,  $T^{sum}$  is the molecular kinetic energy density calculated by summing over the atomic kinetic energy densities,  $K_{HF}^{sum}$  is the molecular Hartree-Fock (HF) exchange energy calculated by summing over the atomic HF exchange energy densities,  $V_{ne}[\rho]$  is the molecular potential energy calculated numerically using its atomic electron densities, and  $J[\rho]$  is the molecular Coulomb energy calculated numerically using its atomic electron densities. The mean absolute percentage errors (MAPEs) are relatively small. The MAPE of  $E_{database}$  are about 0.165%, 0.017%, and 0.008% for  $\mathbf{F}^1$ ,  $\mathbf{F}^2$ ,  $\mathbf{F}^3$ , respectively, and the MAPE of  $E_{direct}$  are about 0.164%, 0.022%, and 0.006% for  $\mathbf{F}^1$ ,  $\mathbf{F}^2$ ,  $\mathbf{F}^3$ , respectively.

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# Abbreviations

**2D** two dimensional. 104, 105, 108, 110

**3D** three dimensional. 12–14, 20, 27, 65, 104, 105, 108–110, 112, 164

**AIM** atoms in molecules. 11–13, 19, 207, 208

**AIMD** atoms in molecules density. ii, iii, ix, xi, 163, 165, 206–208, 236, 277, 341

**ALMO-EDA** the absolutely localized molecular orbital energy decomposition analysis.

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**BDEN** bond electron density. ii, xviii, xix, xxv–xxvii, 67, 72–74, 79–81, 83–85, 87, 88, 91–93, 95, 205, 215–220, 228–235

**DFT** density functional theory. 4, 20, 96

**EDA** energy decomposition analysis. 10

**GGA** Generalized Gradient Approximation. 9, 10

**HF** Hartree-Fock. ii, iii, xiv, 8, 10, 11, 96, 97, 146, 147, 165, 181, 205

**InChI** International Chemical Identifier. 128

**IUPAC** International Union of Pure and Applied Chemistry. 128

**KM** Kitauro-Morokuma. 10

**KS** Kohn-Sham. 6–8

**LDA** Local Density Approximation. 8–10

**MAE** mean absolute error. xii, 96–98, 100, 101, 205

**MAPE** mean absolute percentage error. iii, 165, 174, 177, 179, 185, 186, 192, 194, 198,  
200, 201, 206

**mGGA** meta-GGA. 9

**NNA** non-nuclear attractors. 14

**QTAIM** quantum theory of atoms in molecules. 14

**RDEN** molecular radial electron density. ii, xii, xvii–xix, xxiii–xxvi, 66, 67, 72–79, 81,  
82, 84–86, 88–90, 93, 94, 186, 188, 189, 205, 209–214, 220–227

**RHF** restricted Hartree-Fock. 10, 145, 146

**SDF** Structure-Data File. 128

**SIE** self-interaction error. 6

**SLN** SYBYL Line Notation. 128

**SMILES** Simplified Molecular-Input Line-Entry System. 127

**SVD** singular value decomposition. 112, 113

# Chapter 1

## Introduction

*“Small shifts in your thinking, and small changes in your energy, can lead to massive alterations of your end result.”*

— Kevin Michel

## 1.1 Wavefunction Electronic Structure Theory

Chemistry is the science of matter that deals with construction, transformation and properties of molecules. Theoretical chemistry is the subfield where mathematical methods are combined with fundamental laws of physics to explain the structures and dynamics of chemical systems and to correlate, understand, and predict their thermodynamic and kinetic properties [1].

Due to the rapid development of theoretical techniques carrying out electronic structure calculations for many complicated systems is becoming straightforward. High level wavefunction based methods together with a large basis set have proved to be capable of achieving chemical accuracy for small molecules [2].

In wavefunction electronic structure theory, molecular structures are calculated using only the electronic Schrödinger equation (Equation 1.1),

$$\hat{H}_{elec} \Psi_{elec}^{\mathbf{R}}(\{\mathbf{r}_i\}) = E_{elec}(\mathbf{R}) \Psi_{elec}^{\mathbf{R}}(\{\mathbf{r}_i\}) \quad (1.1)$$

where,  $E_{elec}(\mathbf{R})$  represents the electronic energy of the molecular system,  $\hat{H}_{elec}$  is the electronic Hamiltonian, which is given in atomic units as

$$\hat{H}_{elec} = -\frac{1}{2} \sum_{i=1}^N \nabla_i^2 - \sum_{i=1}^N \sum_{A=1}^M \frac{Z_A}{r_{iA}} + \sum_{i=1}^N \sum_{j<1}^N \frac{1}{r_{ij}} \quad (1.2)$$

where  $N$  is the total number of electrons,  $M$  is the total number of nuclei,  $Z_A$  is the atomic number of nucleus  $A$ ,  $\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}$  is the Laplacian operator, and  $r_{ij}$  is the distance between electrons  $i$  and  $j$ .

The Born-Oppenheimer approximation [3] is applied to the non-relativistic time-independent

Schrödinger equation (Equation 1.3) to derive the electronic Schrödinger equation (Equation 1.1).

$$\hat{H}\Psi(\{\mathbf{r}_i\};\{\mathbf{R}_A\}) = E\Psi(\{\mathbf{r}_i\};\{\mathbf{R}_A\}) \quad (1.3)$$

where,  $\Psi(\{\mathbf{r}_i\};\{\mathbf{R}_A\})$  is the molecular wavefunction which is a function of electronic ( $\mathbf{r}_i$ ) and nuclear ( $\mathbf{R}_A$ ) coordinates,  $E$  is the total non-relativistic energy of the system; and  $\hat{H}$  is the total non-relativistic Hamiltonian of the molecular system that can be written as

$$\begin{aligned} \hat{H} = & -\frac{1}{2} \sum_{i=1}^N \nabla_i^2 - \sum_{i=1}^N \sum_{A=1}^M \frac{Z_A}{r_{iA}} + \sum_{i=1}^N \sum_{j<1}^N \frac{1}{r_{ij}} \\ & - \frac{1}{2} \sum_{A=1}^M \frac{1}{M_A} \nabla_A^2 + \sum_{A=1}^M \sum_{B>A}^M \frac{Z_A Z_B}{R_{AB}} \end{aligned} \quad (1.4)$$

where  $M_A$  is the ratio of the mass of nucleus  $A$  to the mass of the electron and  $R_{AB}$  is the distance between nuclei  $A$  and  $B$ .

The Born-Oppenheimer approximation allows us to separate the wavefunction of a molecule into the electronic and nuclear (vibrational, rotational) components. This approximation is based on the fact that nuclei are much heavier than electrons or the electrons move much faster than the nuclei, therefore, the motion of electrons and nuclei in atomic and molecular systems can be separated. In the Born-Oppenheimer approximation, the molecular wavefunction can be express as product of the electronic and nuclear wavefunctions.

$$\Psi(\mathbf{r}, \mathbf{R}) = \Psi_{elec}^{\mathbf{R}}(\mathbf{r}) \Psi_{nucl}(\mathbf{R}) \quad (1.5)$$

Where,  $\Psi_{elec}^{\mathbf{R}}(\mathbf{r})$  is the electronic wavefunction with fixed nuclei that depends on electronic coordinates and parametrically on nuclear coordinates, and  $\Psi_{nucl}(\mathbf{R})$  is the nuclear wave-

function which depends only on nuclear coordinates.

In molecular orbital theory, each molecule contains a set of orbitals called molecular orbitals,  $\psi_a$ . Each molecular orbital can be described as a linear combination of a finite set of basis functions,  $\phi_\mu$ .

$$\psi_a = \sum_{\mu=1}^K C_{\mu a} \phi_\mu \quad (1.6)$$

where  $\psi_a$  is a molecular orbital represented as the sum of  $K$  basis functions  $\phi_\mu$ , each multiplied by a corresponding expansion coefficient  $C_{\mu a}$ . The number of molecular orbitals is equal to the number of basis functions included in the linear expansion.

## 1.2 Density Functional Theory

Density functional theory (DFT) is an alternative method to investigate the molecular electronic structure of  $N$ -electron systems. Modern DFT was rooted as a promising method to study many-electron systems. Many density functional (i.e. functions of another function) approximations has become more mature for practical applications. DFT applications have increased enormously in many areas of chemistry due to the excellent performance of the functionals. Moreover, DFT's parameters are so successful because they are not system dependent. This approach is computationally very different from the direct solution of the Schrödinger equation in which the time is spent in a search over the whole of Hilbert-space to find the wavefunction. Conversely, in DFT the search is to find the three dimensional electron density [4].

In DFT, the properties of  $N$ -electron systems can be determined using functionals of electron density  $\rho(\mathbf{r})$  [5]. The theory was initially developed by Hohenburg, Kohn, and Sham in



the early 1960s [6, 7]. The Hohenburg and Kohn theorem states that the exact ground-state energy  $E_0$  is a functional of the one particle density  $\rho_0(\mathbf{r})$ .

$$E_0 = E_0[\rho_0] \quad (1.7)$$

However, the theorem does not tell how to construct this functional exactly. For this reason much effort has been devoted to the task of obtaining approximate functionals for the description of the ground-state properties of many particle systems [8]. The energy functional can be written as:

$$E[\rho(\mathbf{r})] = V_{ex}[\rho(\mathbf{r})] + V_{ee}[\rho(\mathbf{r})] + T[\rho(\mathbf{r})] \quad (1.8)$$

where,  $V_{ex}[\rho(\mathbf{r})]$  is the external potential functional,  $V_{ee}[\rho(\mathbf{r})]$  is the electron-electron potential functional, and  $T[\rho(\mathbf{r})]$  is the kinetic energy functional. For molecules in the absence of any external effects, the external potential  $V_{ex}[\rho(\mathbf{r})]$  is

$$V_{ex}[\rho(\mathbf{r})] = V_{ne}[\rho(\mathbf{r})] \quad (1.9)$$

where  $V_{ne}[\rho(\mathbf{r})]$  is the Coulomb attraction between electrons and nuclei.

$$V_{ne}[\rho(\mathbf{r})] = - \sum_{A=1}^M \int \frac{Z_A}{|\mathbf{r} - \mathbf{r}_A|} \rho(\mathbf{r}) d\mathbf{r} \quad (1.10)$$

The  $V_{ee}[\rho(\mathbf{r})]$  is given by,

$$\begin{aligned} V_{ee}[\rho(\mathbf{r})] &= J_{ee}[\rho(\mathbf{r})] + \Delta V_{ee}[\rho(\mathbf{r})] \\ &= \frac{1}{2} \int \int \frac{\rho(\mathbf{r}_1)\rho(\mathbf{r}_2)}{|\mathbf{r}_1 - \mathbf{r}_2|} d\mathbf{r}_1 d\mathbf{r}_2 + \Delta V_{ee}[\rho(\mathbf{r})] \end{aligned} \quad (1.11)$$

where the first part ( $J_{ee}[\rho(\mathbf{r})]$ ) in the above equation is the classical Coulomb repulsion, and the second part ( $\Delta V_{ee}[\rho(\mathbf{r})]$ ) is the non-classical corrections to the electron-electron interaction energy, such as the exchange energy ( $K$ ), the correlation energy ( $E_{corr}$ ), and the self-interaction error (SIE). In 1965 [7], Kohn and Sham proposed a way to compute the kinetic energy of the system  $T[\rho(\mathbf{r})]$ . The concept of their approach is to separate  $T[\rho(\mathbf{r})]$  into the kinetic energy of the non-interacting electrons  $T_{ni}[\rho(\mathbf{r})]$  and the unknown part  $\Delta T[\rho(\mathbf{r})]$ ,

$$T[\rho(\mathbf{r})] = T_{ni}[\rho(\mathbf{r})] + \Delta T[\rho(\mathbf{r})] \quad (1.12)$$

The unknown part  $\Delta T[\rho(\mathbf{r})]$  is the correction to the kinetic energy in the real system due to the interaction between the electrons. The  $T_{ni}[\rho(\mathbf{r})]$  term can be computed exactly as follows,

$$T_{ni}[\rho(\mathbf{r})] = -\frac{1}{2} \sum_{i=1}^N \langle \psi_i^{KS}(\mathbf{r}) | \nabla_1^2 | \psi_i^{KS}(\mathbf{r}) \rangle \quad (1.13)$$

where,  $\psi_i^{KS}$  are called Kohn-Sham (KS) orbitals. The sum of the square of KS orbitals gives the electron density of the system,  $\rho(\mathbf{r})$ .

$$\rho(\mathbf{r}) = \sum_{i=1}^N |\psi_i^{KS}(\mathbf{r})|^2 \quad (1.14)$$

By substituting Equation 1.13 into Equation 1.12,  $T[\rho(\mathbf{r})]$  can be written as,

$$T[\rho(\mathbf{r})] = -\frac{1}{2} \sum_{i=1}^N \langle \psi_i^{KS}(\mathbf{r}) | \nabla_1^2 | \psi_i^{KS}(\mathbf{r}) \rangle + \Delta T[\rho(\mathbf{r})] \quad (1.15)$$

and by substituting Equations 1.10, 1.11, and 1.15 in Equation 1.8, one can write the energy functional as follows,

$$\begin{aligned}
E[\rho(\mathbf{r})] = & - \sum_{A=1}^M \int \frac{Z_A}{|\mathbf{r} - \mathbf{r}_A|} \rho(\mathbf{r}) d\mathbf{r} \\
& + \frac{1}{2} \int \int \frac{\rho(\mathbf{r}_1)\rho(\mathbf{r}_2)}{|\mathbf{r}_1 - \mathbf{r}_2|} d\mathbf{r}_1 d\mathbf{r}_2 + \Delta V_{ee}[\rho(\mathbf{r})] \\
& - \frac{1}{2} \sum_{i=1}^N \langle \psi_i^{KS}(1) | \nabla_1^2 | \psi_i^{KS}(1) \rangle + \Delta T[\rho(\mathbf{r})]
\end{aligned} \tag{1.16}$$

The sum of two correction terms  $\Delta V_{ee}[\rho(\mathbf{r})]$  and  $\Delta T[\rho(\mathbf{r})]$  is called the exchange-correlation energy functional  $E_{xc}[\rho(\mathbf{r})]$ .

$$E_{xc}[\rho(\mathbf{r})] = \Delta V_{ee}[\rho(\mathbf{r})] + \Delta T[\rho(\mathbf{r})] \tag{1.17}$$

Minimizing  $E[\rho(\mathbf{r})]$  with respect to  $\rho(\mathbf{r})$  using the Lagrange multipliers method subject to the constraint that the KS orbitals remain orthonormal, leads us to a set of equations (called KS equations),

$$\hat{h}^{KS}(\mathbf{r}_1) \psi_i^{KS}(\mathbf{r}_1) = \varepsilon_i \psi_i^{KS}(\mathbf{r}_1) \tag{1.18}$$

where

$$\hat{h}^{KS}(\mathbf{r}_1) = -\frac{1}{2} \nabla_1^2 - \sum_{A=1}^M \frac{Z_A}{r_{1A}} + \int \frac{\rho(\mathbf{r}_2)}{r_{12}} d\mathbf{r}_2 + \hat{v}_{xc}(\mathbf{r}_1) \tag{1.19}$$

and  $\hat{h}^{KS}(\mathbf{r}_1)$  is the KS one-electron operator,  $\varepsilon_i$  is the  $i^{th}$  KS orbital energy, and  $\hat{v}_{xc}(\mathbf{r})$  is the exchange-correlation potential which can be defined as the derivative of the  $E_{xc}[\rho(\mathbf{r})]$  with respect to the density  $\rho(\mathbf{r})$ .

$$\hat{v}_{xc}(\mathbf{r}) = \frac{\delta E_{xc}[\rho(\mathbf{r})]}{\delta \rho(\mathbf{r})} \tag{1.20}$$

KS equations (Equation 1.18) must be solved self-consistently like the Hartree-Fock (HF) equations. However, KS equations differ from the HF equations only by  $\hat{v}_{xc}(\mathbf{r})$ , where  $\hat{v}_{xc}(\mathbf{r})$  is computed in each self-consistent cycle [9].  $\hat{v}_{xc}(\mathbf{r})$  is an unknown term and an appropriate approximation of  $E_{xc}[\rho(\mathbf{r})]$  is needed. A number of possible approximations may be made for  $E_{xc}[\rho(\mathbf{r})]$ . The simplest known as the Local Density Approximation (LDA). In general, for a spin-unpolarized system,  $E_{xc}^{LDA}[\rho(\mathbf{r})]$  is written as

$$E_{xc}^{LDA}[\rho(\mathbf{r})] = \int \rho(\mathbf{r}) \epsilon_{xc}^{LDA}[\rho(\mathbf{r})] d\mathbf{r} \quad (1.21)$$

where  $\epsilon_{xc}^{LDA}[\rho(\mathbf{r})]$  is the exchange-correlation energy per electron of a homogeneous electron gas of density  $\rho(\mathbf{r})$ . The  $\epsilon_{xc}^{LDA}[\rho(\mathbf{r})]$  is decomposed into exchange and correlation terms linearly,

$$\epsilon_{xc}^{LDA}[\rho(\mathbf{r})] = \epsilon_x^{LDA}[\rho(\mathbf{r})] + \epsilon_c^{LDA}[\rho(\mathbf{r})] \quad (1.22)$$

The exchange terms  $\epsilon_x^{LDA}[\rho(\mathbf{r})]$  and  $E_x^{LDA}[\rho(\mathbf{r})]$  are known and given as [10, 11],

$$\epsilon_x^{LDA}[\rho(\mathbf{r})] = -\frac{3}{4} \left( \frac{3}{\pi} \right)^{1/3} \rho(\mathbf{r})^{1/3} \quad (1.23)$$

$$E_x^{LDA}[\rho(\mathbf{r})] = -\frac{3}{4} \left( \frac{3}{\pi} \right)^{1/3} \int \rho(\mathbf{r})^{4/3} d\mathbf{r} \quad (1.24)$$

However, the correlation term  $E_c^{LDA}[\rho(\mathbf{r})]$  is unknown and there are numerous different approximations. Vosko-Wilk-Nusair (VWN) [12], Perdew-Zunger (PZ81) [13], Cole-Perdew (CP) [14], and Perdew-Wang (PW92) [15] are common examples of  $E_c^{LDA}[\rho(\mathbf{r})]$  functionals.

A next level of  $E_{xc}[\rho(\mathbf{r})]$  approximations are the so-called Generalized Gradient Approximation (GGA),

$$E_{xc}^{GGA}[\rho(\mathbf{r}), \nabla\rho(\mathbf{r})] = \int \rho(\mathbf{r}) \epsilon_{xc}^{GGA}[\rho(\mathbf{r}), \nabla\rho(\mathbf{r})] d\mathbf{r} \quad (1.25)$$

The  $\epsilon_{xc}^{GGA}$  is given as a functional of both the density  $\rho(\mathbf{r})$  and its gradient  $\nabla\rho(\mathbf{r})$ . The gradient  $\nabla\rho(\mathbf{r})$  is given to account for non-homogeneity of the true electron density. LDA fails in situations where the density undergoes rapid changes such as in molecules, whereas GGA shows improvement over the LDA for calculations of molecular structures and in representing weak intermolecular bonds. Becke-88 (B88) [16], Perdew-Wang-91 (PW91) [15, 17], and Perdew-Burke-Ernzerhof (PBE) [18] are common examples of GGA functionals.

The obvious logical next step in the functional improvement is to take account of the second derivative (i.e., the Laplacian) of the density ( $\nabla^2\rho$ ). These functionals are termed as meta-GGA (mGGA). The mGGA functionals includes also the kinetic-energy density  $\tau[\rho(\mathbf{r})] = \sum_i^{occupied} \frac{1}{2} |\nabla\psi_i^{KM}(\mathbf{r})|^2$  [19].

$$E_{xc}^{mGGA}[\rho(\mathbf{r}), \nabla\rho(\mathbf{r}), \nabla^2\rho(\mathbf{r}), \tau] = \int \rho(\mathbf{r}) \epsilon_{xc}^{mGGA}[\rho(\mathbf{r}), \nabla\rho(\mathbf{r}), \nabla^2\rho(\mathbf{r}), \tau] d\mathbf{r} \quad (1.26)$$

A number of mGGA functionals for exchange, correlation, or both, have been developed such as B95 [20], B98 [21], Perdew-Kurth-Zupan-Blaha (PKZB) [22] and Lee-Yang-Parr (LYP) [23].

In another approach in 1993 Becke[24] introduced a successful hybrid functional. This functional is constructed as a linear combination of the HF exchange ( $E_x^{HF}$ , Equation 1.27)

and a number of  $E_{xc}[\rho(\mathbf{r})]$  functionals.

$$E_x^{HF} = -\frac{1}{2} \sum_{a>b} \int \int \psi_a^*(\mathbf{r}_1) \psi_b^*(\mathbf{r}_2) \frac{1}{r_{12}} \psi_b(\mathbf{r}_1) \psi_a(\mathbf{r}_2) d\mathbf{r}_1 d\mathbf{r}_2 \quad (1.27)$$

For example, the popular B3LYP (Becke, three-parameter, Lee-Yang-Parr) functional is,

$$\begin{aligned} E_{xc}^{B3LYP} = & E_x^{LDA} + a_0(E_x^{HF} - E_x^{LDA}) + a_x(E_x^{B88} - E_x^{LDA}) \\ & + E_c^{VWN} + a_c(E_c^{LYP} - E_c^{VWN}) \end{aligned} \quad (1.28)$$

where,  $a_0 = 0.20$ ,  $a_x = 0.72$ , and  $a_c = 0.81$ .  $E_x^{B88}$  is the Becke 88 GGA exchange functional [16],  $E_c^{LYP}$  is the Lee, Yang and Parr GGA correlation functional [23], and  $E_c^{VWN}$  is the Vosko, Wilk, and Nusair LDA correlation functional [12].

### 1.3 Energy Decomposition Analysis

Energy decomposition analysis (EDA) can be a valuable tool for interpreting the results of quantum chemical calculations [25]. A significant number of developments have been made in the field of EDA [26–32]. One of the earliest energy decomposition analysis schemes developed is the Kitaura-Morokuma (KM) scheme [33]. This scheme is limited to the restricted Hartree-Fock (RHF) level of theory. The KM scheme allows a partitioning of the HF energy as follows,

$$\Delta E = \Delta E_{ele} + \Delta E_{exrep} + \Delta E_{pol} + \Delta E_{chtr} + \Delta E_{mix} \quad (1.29)$$

where  $\Delta E_{ele}$  is the classical electrostatic interaction,  $\Delta E_{exrep}$  is the exchange-repulsion,  $\Delta E_{pol}$  is the polarization interaction,  $\Delta E_{chtr}$  is the charge transfer energy, and  $\Delta E_{mix}$  describes contributions to the interaction energy that are not capable of being assigned to a particular component [34]. Another scheme is the absolutely localized molecular orbital energy decomposition analysis (ALMO-EDA), in which

$$\Delta E = \Delta E_{frz} + \Delta E_{pol} + \Delta E_{chtr} \quad (1.30)$$

where  $\Delta E_{frz}$  is the frozen density component that describes the exchange and electrostatic interaction of the frozen charge densities [26, 34].

## 1.4 Atoms in Molecules

The idea of an atom or a functional group in a molecule is principal to the science of chemistry, which existed before our understanding of quantum mechanics, atomic structure, and the nature of chemical bonding. As a result understanding properties of atoms and functional groups as molecules undergo transformations and combinations has been of interest to chemists from long time.

A functional group in different molecular environments usually gives similar properties. Therefore computing molecular properties for molecules can be accomplished by assuming the transferability of the atoms in molecules (AIM) atomic properties. Thus the additivity of AIM atomic properties gives us a better understanding of molecular properties.

A lot of scientific efforts have been devoted to AIM schemes over the past decades. The Hilbert-space analysis and the analysis in the three dimensional (3D) physical space are the

two most conceptually different approaches for defining an atom within a molecule. The Hilbert-space and 3D analyses differ conceptually, and usually give significantly different results.

### 1.4.1 Hilbert-Space

In the Hilbert-space analysis each atom within a molecule is characterized with the nucleus and basis functions assigned to it [30]. The Mulliken [35] and Löwdin [36] population analyses are known examples within the Hilbert-space analysis

$$q_A^{Mulliken} = Z_A - \sum_{\mu \in A} (\mathbf{PS})_{\mu\mu} \quad (1.31)$$

$$q_A^{Löwdin} = Z_A - \sum_{\mu \in A} \left( \mathbf{S}^{1/2} \mathbf{PS}^{1/2} \right)_{\mu\mu} \quad (1.32)$$

where  $q_A$  is the net charge associated with an atomic nucleus  $A$ ,  $Z_A$  is the nuclear charge of atom  $A$ , the index of summation indicates that the sum is over the basis functions  $\mu$  centered on  $A$ ,  $\mathbf{S}$  is the overlap matrix, and  $\mathbf{P}$  is the density matrix. The results of Hilbert-space analyses greatly depend on the basis set applied [30]. This partitioning cannot be extended to the complete basis set limit because the complete basis cannot be divided into atomic subsets. Thus, the net charge from this technique is not unique [37].

### 1.4.2 Three-Dimensional (3D) Physical Space

In the 3D case, one decomposes the physical space into atomic domains with sharp (non-overlapping) or fuzzy boundaries (overlapping). In contrast to Hilbert-space analysis the



results of the 3D analysis usually have moderate basis set dependence and converge well with the increasing basis sets size, but depend greatly on the definition of the 3D domains assigned to the different atoms [30].

### 1.4.2.1 Properties of AIM with 3D Space

In general, we can associate the atomic electron density  $\rho_A(\mathbf{r})$  for every atom  $A$  in the molecule by defining a weight function  $W_A(\mathbf{r})$  such that

$$\rho_A(\mathbf{r}) = W_A(\mathbf{r})\rho(\mathbf{r}) \quad (1.33)$$

where  $\rho(\mathbf{r})$  is the molecular electron density at the position  $\mathbf{r}$ .  $\rho(\mathbf{r})$  can also be represented as a summation of all AIM densities in the molecule.

$$\rho(\mathbf{r}) = \sum_A \rho_A(\mathbf{r}) \quad (1.34)$$

The weight function  $W_A(\mathbf{r})$  should obey the condition

$$\sum_A W_A(\mathbf{r}) = 1 \quad (1.35)$$

Partitioning the molecular electron density into its atomic contributions is extensively used in the literature for different applications such as electron population, distribution analysis, and molecular energy components [38]. The identification methods of AIM within 3D analysis are categorized as disjunct and fuzzy methods.

### 1.4.2.2 Disjunct Method

In this case the partitioning weight of specific atom A is one, and only one, within its atomic basin  $\Omega_A$  and zero otherwise. This situation is described by Equation 1.36 [39]

$$W_A(\mathbf{r}) = \begin{cases} 1, & \text{if } \mathbf{r} \in \Omega_A \\ 0, & \text{otherwise} \end{cases} \quad (1.36)$$

This is characteristic for the quantum theory of atoms in molecules (QTAIM), which was developed by Richard Bader [40–42]. QTAIM recovers the electron densities of atoms by partitioning the molecular space via determination of zero flux surfaces where the gradient of electron density  $\nabla\rho(\mathbf{r})$  is zero for all points on the atomic surface, Equation 1.37.

$$\nabla\rho(\mathbf{r}) \cdot \mathbf{n}(\mathbf{r}) = 0 \quad \text{for all } \mathbf{r} \text{ on the surface} \quad (1.37)$$

where  $\nabla\rho(\mathbf{r})$  is the gradient vector of electron density and  $\mathbf{n}(\mathbf{r})$  is the unit vector normal to the atomic surface.

The disadvantages of the QTAIM method are that the electronic charge at any point is attributed to only one of the atoms in the molecule. Thus it cannot reflect directly the fact that there is an accumulation of electron density in bonding regions between atoms. Also QTAIM may lead to so called non-nuclear attractors (NNA), i.e., basins that have no nuclei associated to them. In addition, the QTAIM method requires significant computer time to investigate the topology of the electron density [38].

### 1.4.2.3 Fuzzy Method

In fuzzy atomic domains, the regions assigned to individual atoms have no sharp boundaries and show a continuous transition from one to another. Fuzzy atoms were first used by Hirshfeld [43] for calculating effective atomic charges in molecules.

#### Hirshfeld weight

The Hirshfeld weight function  $W_A(\mathbf{r})$  is computed from a so-called promolecular density  $\rho_{mol}^0(\mathbf{r})$ ,

$$W_A(\mathbf{r}) = \frac{\rho_A^0(\mathbf{r})}{\rho_{mol}^0(\mathbf{r})} = \frac{\rho_A^0(\mathbf{r})}{\sum_A \rho_A^0(\mathbf{r})} \quad (1.38)$$

where  $\rho_A^0(\mathbf{r})$  is the electron density of the isolated atom  $A$ .  $\rho_{mol}^0(\mathbf{r})$  is computed by summing overall atomic densities ( $\rho_A^0(\mathbf{r})$ ) at the same point in space. The atomic electron densities for the real molecule are then obtained via

$$\rho_A(\mathbf{r}) = \frac{\rho_A^0(\mathbf{r})}{\rho_{mol}^0(\mathbf{r})} \rho(\mathbf{r}) \quad (1.39)$$

where  $\rho(\mathbf{r})$  is the real molecular density.

The Hirshfeld weight has many shortcomings such as atomic charges tend to be virtually zero, depend on the choice of the promolecular density, are only available for neutral molecules, and many others [44]. Many extended Hirshfeld population analysis schemes were proposed to overcome original Hirshfeld drawbacks [45–48].

## Becke weight

Becke, in his seminal paper [49], decomposed the molecular integral into atomic components by assigning a weight function  $W_A(\mathbf{r})$  for each atom in the molecule. The Becke weight was originally invented for numerical integration of three-dimensional functions of molecular atomic properties. For any point  $\mathbf{r}_i$  in molecular space, the confocal elliptical coordinate  $\mu_{iAB}$  of nuclei  $A$  and  $B$  can be calculated as

$$\mu_{iAB} = \frac{r_{iA} - r_{iB}}{R_{AB}} \quad (1.40)$$

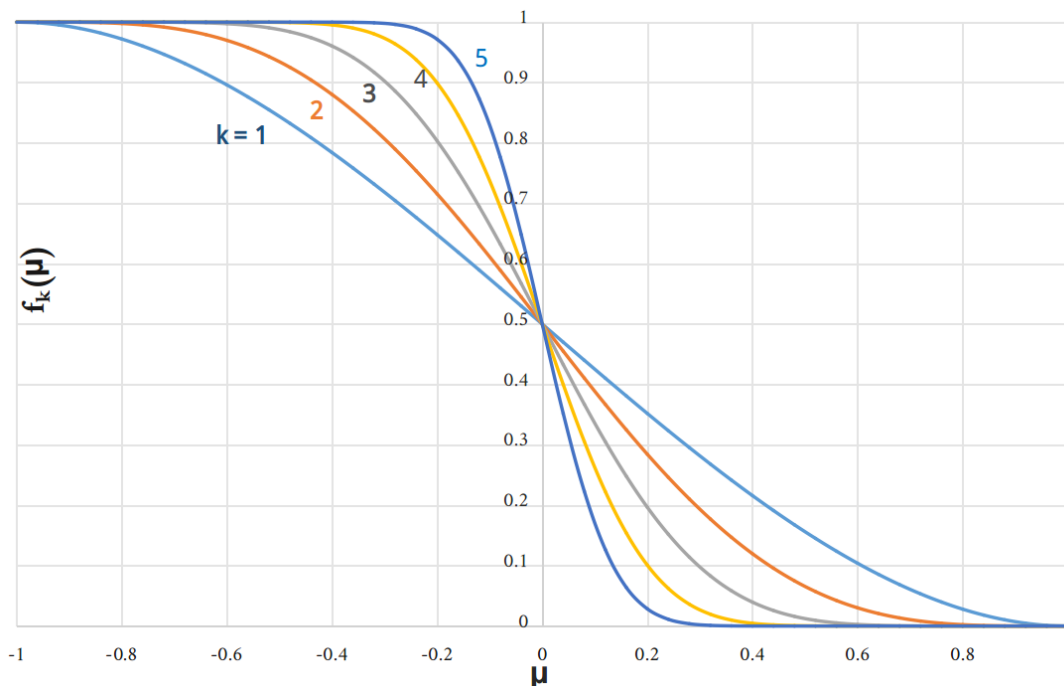
where  $r_{iA}$ ,  $r_{iB}$  denote the distances from  $\mathbf{r}_i$  to nuclei  $A$  and  $B$  respectively.  $R_{AB}$  is the internuclear distance between the atoms  $A$  and  $B$ . Values of  $\mu_{iAB}$  are always between -1 and 1. For example, if the  $\mathbf{r}_i$  point lies on atom  $A$  then  $\mu_{iAB} = -1$  and if the point lies on atom  $B$  then  $\mu_{iAB} = 1$ . Becke introduced the following continuous cubic polynomial function that varies smoothly between  $-1$  and  $+1$ ,

$$h(\mu_{iAB}) = \frac{3}{2}\mu_{iAB} - \frac{1}{2}\mu_{iAB}^3 \quad (1.41)$$

where  $h(1) = 1$  and  $h(-1) = -1$ . This function is not “step function-like”, therefore Becke iterated as

$$\begin{aligned} f_1(\mu_{iAB}) &= h(\mu_{iAB}) & k &= 1 \\ f_2(\mu_{iAB}) &= h[h(\mu_{iAB})] & k &= 2 \\ f_3(\mu_{iAB}) &= h\{h[h(\mu_{iAB})]\} & k &= 3 \\ &\vdots \end{aligned} \quad (1.42)$$

to obtain successively sharper functions. A sequence of cutoff profiles is illustrated in Figure 1.1 for  $k$  values from 1 to 5. Sharper functions are generated with increasing the iteration  $k$  value. Becke found on the basis of his experience that  $k = 3$  gives the best numerical results.



**Figure 1.1:** Cutoff profiles  $f_k(\mu_{iAB})$  of Equation 1.42 for  $k = 1$  to 5 [49].

Because the desired weight has to be in the range  $[0, 1]$ , Becke proposed  $s(\mu_{iAB})$  function

$$s(\mu_{iAB}) = \frac{1}{2} [1 - f_3(\mu_{iAB})] \quad (1.43)$$

One problem with the scheme given so far is that the space is divided equally between any two atoms. A proposed atomic size adjustment is introduced for heteronuclear molecules

by transforming  $\mu_{iAB}$  into  $v_{iAB}$  as follows,

$$v_{iAB} = \mu_{iAB} + a_{AB}(1 - \mu_{iAB}^2) \quad (1.44a)$$

$$a_{AB} = \frac{u_{AB}}{u_{AB}^2 - 1} \quad (1.44b)$$

$$u_{AB} = \frac{\chi - 1}{\chi + 1} \quad (1.44c)$$

$$\chi_{AB} = \frac{R_A}{R_B} \quad (1.44d)$$

where  $R_A$  and  $R_B$  are the Bragg-Slater radii for atoms A and B, respectively. Becke defined the Voronoi polyhedron on nucleus A by the product of all pairwise ownership contributions  $s(v_{iAB})$ .

$$P_A(\mathbf{r}) = \prod_{B=1, B \neq A}^N s(v_{iAB}) \quad (1.45)$$

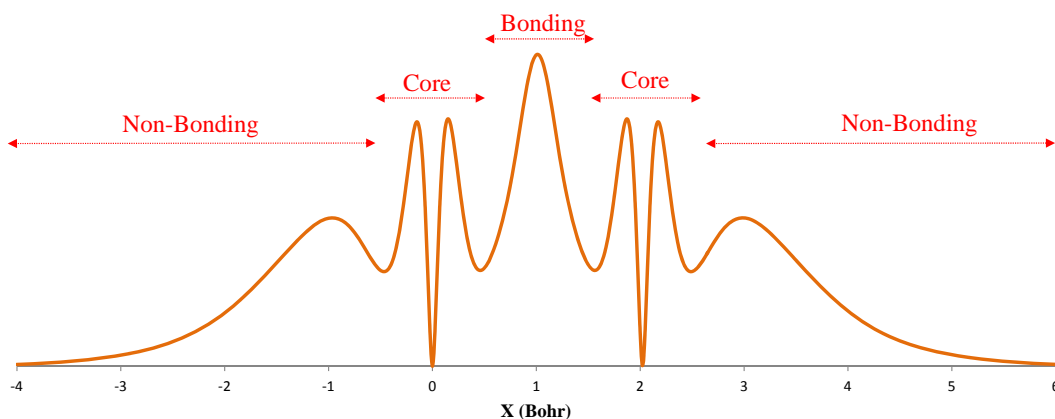
$P_A(\mathbf{r})$  is called the cell function of atom A that has a value of one at nucleus A and vanishes at all other nuclei B. The normalized Becke weight function  $W_A(\mathbf{r})$  for an atom A at a particular point  $\mathbf{r}$  can now be defined as follows.

$$W_A(\mathbf{r}) = \frac{P_A(\mathbf{r})}{\sum_n P_n(\mathbf{r})} \quad (1.46)$$

The Becke weight suffers some drawbacks such as: (i) its dependence on fixed Bragg–Slater radii makes it less useful in dealing with AIM that have different charged states, (ii) if there is a large difference between the Bragg–Slater radii of the atoms, for example, in Equation 1.44d, if the value of  $\chi_{AB}$  for two atoms A and B is larger than 2.4 or smaller than 0.41, then the ratio will be capped to these extremes [50].

## Atoms in Molecules from Radial Density

Warburton et al. [50, 51] indicated that the radial density can be used as another method for partitioning the molecule space into atomic contributions. The results show that molecules can be partitioned into three main regions: non-bonding, core, and bonding regions. Figure 1.2 shows the radial electron density of  $N_2$ , where  $N_2$  is partitioned into these regions using Becke weight. The core region of each nucleus is easily identified in this cross section as the symmetrical, volcano-shaped cones that surround the nuclei. The core is separated from the distorted valence by a minimum in radial density.



**Figure 1.2:** Radial electron density of  $N_2$ .

A partitioning weight can be used to calculate the atomic electron density associated with each AIM as follows,

$$\rho_A(\mathbf{r}_i) = W_A(\mathbf{r}_i)\rho(\mathbf{r}_i) \quad (1.47)$$

and the AIM radial density is expressed as

$$\rho_A^{rad}(\mathbf{r}_i) = \mathbf{r}_{iA}^2 \rho_A(\mathbf{r}_i) \quad (1.48)$$

The total radial density for the molecule is the sum of each of these individual radial contributions.

$$\begin{aligned}
 \rho_{rad}(\mathbf{r}_i) &= \sum_{A=1}^N W_A(\mathbf{r}_i) \mathbf{r}_{iA}^2 \rho(\mathbf{r}_i) \\
 &= \sum_{A=1}^N \mathbf{r}_{iA}^2 \rho_A(\mathbf{r}_i) \\
 &= \sum_{A=1}^N \rho_A^{rad}(\mathbf{r}_i)
 \end{aligned} \tag{1.49}$$

## 1.5 Numerical Integration

The evaluation of the 3D integral (Equation 1.50) is important in many fields. For example, some of the applications of numerical integration in physical chemistry are to calculate number of electrons ( $N_e = \int \rho(\mathbf{r}) d\mathbf{r}$ ), dipole moment ( $\mu(x) = \int x \cdot \rho(x) dx$ ), Coulomb potential ( $V_{ee} = \int \int \frac{\rho(\mathbf{r}_1)\rho(\mathbf{r}_2)}{|\mathbf{r}_{12}|} d\mathbf{r}_1 d\mathbf{r}_2$ ), potential energy ( $V_{ne} = -\sum_A Z_A \int \frac{\rho(\mathbf{r})}{|\mathbf{r}-\mathbf{R}_A|} d\mathbf{r}$ ) and many other atomic and molecular properties.

$$I = \int f(\mathbf{r}) d\mathbf{r} \tag{1.50}$$

The integration of some functions may be difficult or impossible to compute analytically, but can be computed numerically. For example, in DFT calculations, the exchange-correlation energies of many-electron approximations (Equation 1.51) cannot be evaluated analytically.

$$I = \int f(\rho, \nabla \rho, \dots) d\mathbf{r} \tag{1.51}$$



In the next sections, some of the most common numerical integration methods, Newton-Cotes formulas and Gaussian quadrature, will be illustrated.

### 1.5.1 Newton-Cotes Formulas

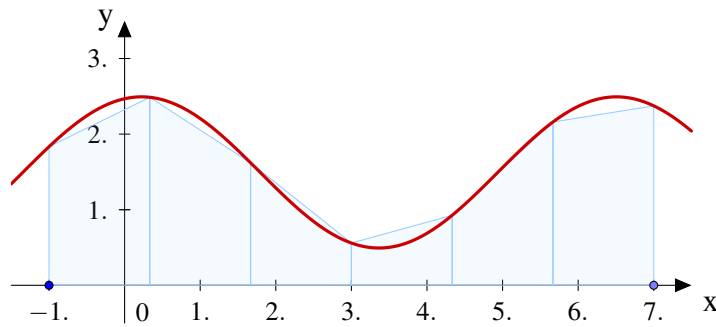
Newton-Cotes rules are used to evaluate the integral,  $\int_a^b f(x)dx$  at equally spaced points. Table 1.1 shows some common Newton-Cotes formulas (trapezoidal rule, Simpson's rule, and Euler-Maclaurin). Figure 1.3 shows an example for the trapezoidal rule. In this figure, the blue area represents the approximate area of function  $f(x) = \cos(x - 0.2) + 1.5$  in range  $[-1, 7]$  using trapezoidal rule.

**Table 1.1:** Some common Newton-Cotes formulas within range  $[a, b]$  and step size  $h = \frac{b-a}{n}$  [52, 53].

| Name                         | Formula  |
|------------------------------|--|
| Trapezoidal rule             | $h \sum_{k=1}^n \frac{f(x_{k+1}) + f(x_k)}{2}$   |
| Simpson's rule <sup>1</sup>  | $\frac{h}{3} \left[ f(x_0) + 2 \sum_{k=1}^{\frac{n}{2}-1} f(x_{2k}) + 4 \sum_{k=1}^{\frac{n}{2}} f(x_{2k-1}) + f(x_n) \right]$                     |
| Euler-Maclaurin <sup>2</sup> | $h \sum_{k=1}^{n-1} f(a + kh) - \frac{h}{2} (f(b) + f(a)) - \sum_{k=1}^m \frac{h^{2k}}{(2k)!} B_{2k} \left( f^{(2k-1)}(b) - f^{(2k-1)}(a) \right)$ |

<sup>1</sup> n is an even integer number.

<sup>2</sup>  $B_{2k}$  is a Bernoulli number, the first few even numbers are  $B_0 = 1, B_2 = \frac{1}{6}, B_4 = \frac{-1}{30}$ .



**Figure 1.3:** An example for trapezoidal rule,  $f(x) = \cos(x - 0.2) + 1.5$  where  $n = 6$ .

## 1.5.2 Gaussian Quadrature

Geometrically, quadrature means finding a square equal in area to the area of interest. It refers to any numerical approximation of the integral,  $\int_a^b f(x)dx$ , with the fewest function evaluations on finite sets of points (Equation 1.52).

$$\int_a^b f(x)dx = \sum_{i=1}^n \frac{w_i}{w(x_i)} f(x_i) = \sum_{i=1}^n \omega(x_i) f(x_i) \quad (1.52)$$

where  $x_i$  are the base points and  $w_i$  are called quadrature points, abscissas or weight factors. Practically the weights  $w_i$  and the  $x_i$  points have to be stored as fixed data in the program. Orthogonal polynomials are required to find the quadrature points where Equation 1.53 should be obeyed for a sequence of orthogonal polynomials over the range  $[a, b]$ , [54]

$$\int_a^b w(x) p_i(x) p_j(x) dx = \delta_{ij} \int_a^b w(x) p_i(x)^2 dx \quad (1.53)$$

where  $w(x)$  is the weight function, and  $p_n(x)$  is the following polynomial function of degree  $n$ ,

$$p_n(x) = a_n x^n + a_{n-1} x^{n-1} + \cdots + a_1 x + a_0 \quad (1.54)$$

The roots of  $p_n(x)$  must be real numbers and fall within the range  $[a, b]$ . Examples of orthogonal polynomials are shown in Table 1.2. For example, Laguerre functions,  $L_n^\alpha(x)$ , are orthogonal over the weight function  $x^\alpha e^{-x}$  within  $[0, \infty)$ .

If  $n \geq i \geq 1$  and  $j = 0$  in Equation 1.53, the equation becomes,

$$\int_a^b w(x) p_i(x) p_0(x) dx = 0 \quad \text{where } n \geq i \geq 1 \quad (1.55)$$

**Table 1.2:** Some of the common classical orthogonal polynomials [52, 55–57].

| Name          | $p_n(x)$                 | $w(x)$                     | $x_i$                                     | $w_i$   | Range $[a, b]$      |
|---------------|--------------------------|----------------------------|---|---|---------------------|
| Legendre      | $P_n(x)$                 | 1                          | $i^{th}$ zero of $P_n(x)$                 | $\frac{2}{(1-x_i^2)} [P_n'(x_i)]^2$                   | $[-1, 1]$           |
| Chebyshev 1st | $T_n(x)$                 | $1/\sqrt{1-x^2}$           | $\cos[\frac{(2i-1)\pi}{2n}]$              | $\pi/n$   | $(-1, 1)$           |
| Chebyshev 2nd | $U_n(x)$                 | $\sqrt{1-x^2}$             | $\cos[\frac{i\pi}{(n+1)}]$                | $\frac{\pi}{n+1} \sin^2[\frac{i\pi}{(n+1)}]$          | $[-1, 1]$           |
| Laguerre      | $L_n^\alpha(x)$          | $x^\alpha e^{-x}$          | $i^{th}$ zero of $L_n^\alpha(x)$          | $\frac{x_i}{(n+1)^2 [L_{n+1}'(x_i)]^2}$               | $[0, \infty)$       |
| Hermite       | $H_n(x)$                 | $e^{-x^2}$                 | $i^{th}$ zero of $H_n(x)$                 | $\frac{2^{n-1} n! \sqrt{\pi}}{n^2 [H_{n-1}'(x_i)]^2}$ | $(-\infty, \infty)$ |
| Jacobi        | $J_n^{\alpha, \beta}(x)$ | $(1-x)^\alpha (1+x)^\beta$ | $i^{th}$ zero of $J_n^{\alpha, \beta}(x)$ |   | $(-1, 1)$           |

since  $p_0(x)$  is constant, Equation 1.55 is written as

$$\int_a^b w(x) p_i(x) p_0(x) dx = p_0(x) \int_a^b w(x) p_i(x) dx = \int_a^b w(x) p_i(x) dx = 0 \quad \text{where } n \geq i \geq 1 \quad (1.56)$$

If  $x_1, x_2, \dots, x_n$  are denoting the roots of the polynomial,  $p_n(x)$ , one can obtain the following sum for any sets of  $x_j$

$$\sum_{j=1}^n w_j p_n(x_j) = 0 \quad (1.57)$$

It can be shown that there is a unique solution of  $w_j$  for all polynomials  $p_i(x)$  where  $n \geq i \geq 1$

$$\sum_{j=1}^n w_j p_i(x_j) = 0 \quad \text{where } n \geq i \geq 1 \quad (1.58)$$

This can be done by solving a set of polynomial  $p_i(x)$  equations simultaneously,

$$\begin{pmatrix} p_0(x_1) & \cdots & p_0(x_n) \\ \vdots & \ddots & \vdots \\ p_{n-1}(x_1) & \cdots & p_{n-1}(x_n) \end{pmatrix} \begin{pmatrix} w_1 \\ \vdots \\ w_n \end{pmatrix} = \begin{pmatrix} \int_a^b w(x)p_0(x)dx \\ \vdots \\ 0 \end{pmatrix} \quad (1.59)$$

Using Equations 1.56 and 1.58, the result is

$$\int_a^b w(x)p_i(x)dx = \sum_{j=1}^n w_j p_i(x_j) = 0 \quad \text{where } n \geq i \geq 1 \quad (1.60)$$

In order to get a non-trivial solution for Equation 1.59, Equation 1.61 should not equal zero,

$$\int_a^b w(x)p_0(x)dx = \sum_{j=1}^n w_j p_0(x_j) \neq 0 \quad (1.61)$$

Writing the function  $f(x)$  as a linear combination of  $w(x)p_i(x)$  leads to,

$$f(x) \approx \sum_{k=1}^n c_k w(x)p_k(x) \quad (1.62)$$

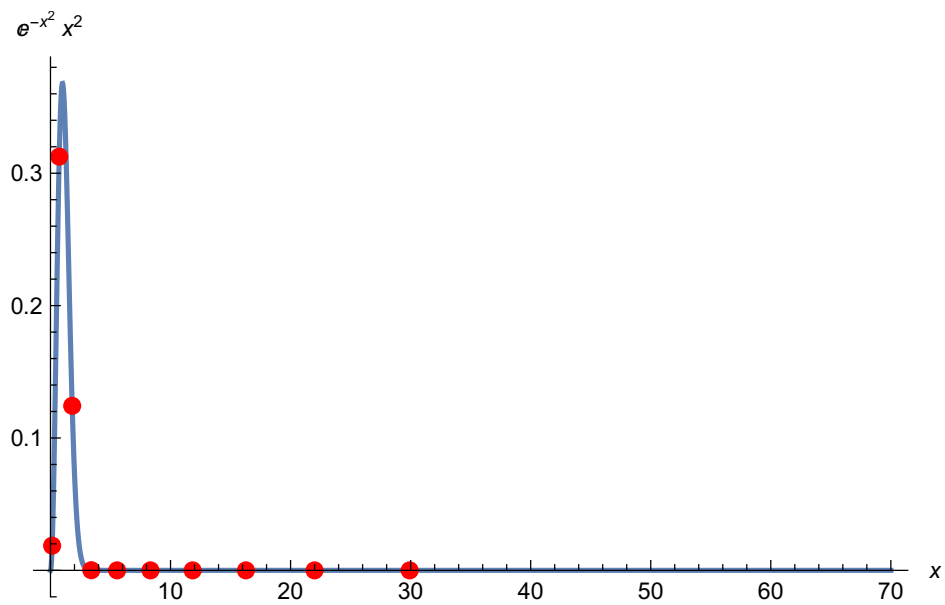
The integral of  $f(x)$  (Equation 1.52) can be derived as follows:

$$\begin{aligned}
\int_a^b f(x)dx &= \int_a^b \sum_{k=1}^b c_k w(x) p_k(x) dx && \text{using Equation 1.62} \\
&= \sum_{k=1}^b c_k \left( \int_a^b w(x) p_k(x) dx \right) \\
&= \sum_{k=1}^b c_k \left( \sum_{j=1}^n w_j p_k(x_j) \right) && \text{using Equation 1.60} \\
&= \sum_{j=1}^n w_j \sum_{k=1}^b c_k p_k(x_j) \\
&= \sum_{j=1}^n \frac{w_j}{w(x_j)} \sum_{k=1}^b c_k w(x_j) p_k(x_j) \\
&= \sum_{j=1}^n \frac{w_j}{w(x_j)} f(x_j) && \text{using Equation 1.62} \tag{1.63}
\end{aligned}$$

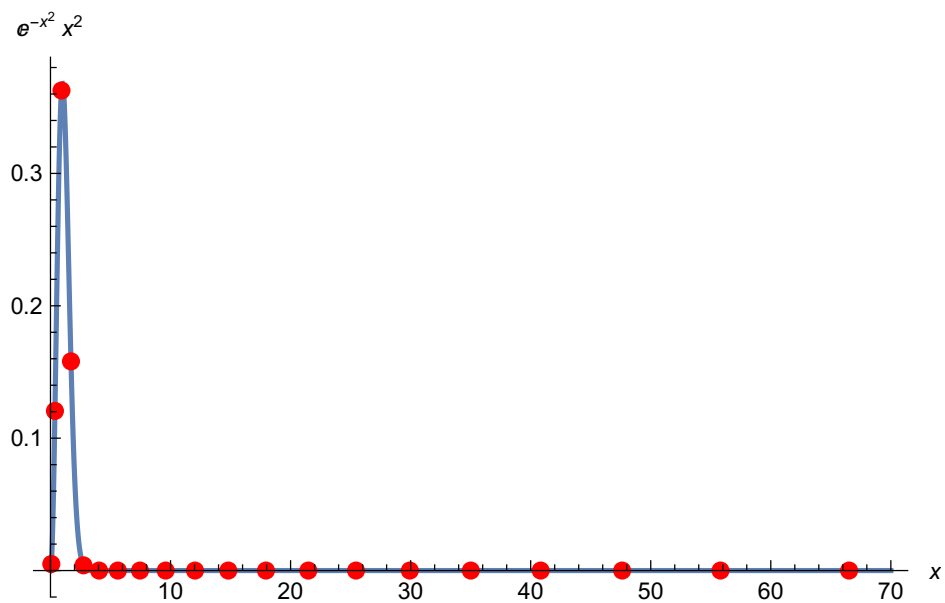
For example, the numerical integral of  $x^2 e^{-x^2}$  using the Laguerre method (Table 1.2) where  $\alpha = 0$ ,  $n = 10$  and  $n = 20$  points are displayed in Figures 1.4a and 1.4b, respectively. The integral is negligibly small beyond  $x = 4$  while the most of quadrature roots  $w_i$  are beyond  $x = 4$ . The use of 20 points instead of 10 results in increasing the number of quadrature roots out of the region of interest (Figure 1.4b).

To overcome this problem, these roots should be scaled. This can be done using the  $u$ -substitution integration technique

$$\int_a^b f(x)dx = \int_{u(a)}^{u(b)} f(g(u))g'(u)du \quad \text{where } x = g(u) \tag{1.64}$$



(a) using 10 points



(b) using 20 points

**Figure 1.4:** Gauss-Laguerre nodes for  $x^2 e^{-x^2}$  function.

A new function,  $u = \frac{x}{\lambda}$ , is proposed where  $\lambda$  is a scale factor. The integration,  $\int_a^b f(x)dx$ , is achieved by rewriting the integral as  $\int_{u(a)}^{u(b)} \lambda f(u\lambda) du$ . Therefore Equation 1.52 becomes,

$$\int_a^b f(x)dx = \lambda \int_{\frac{a}{\lambda}}^{\frac{b}{\lambda}} f(\lambda u) du = \lambda \sum_{i=1}^n \frac{w_i}{w(u_i)} f(\lambda u_i) \quad (1.65)$$

As a result the distribution of the points in  $\mathbf{r}$  space can be controlled by scaling the factor  $\lambda$  (Figures 1.5a and 1.5b).

## 1.6 Numerical Integration in Quantum Chemistry

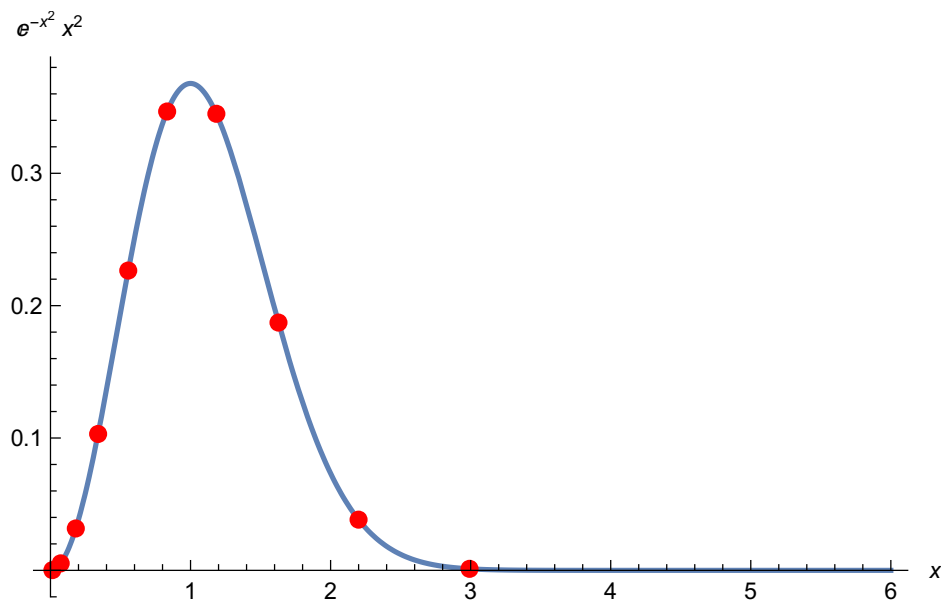
### 1.6.1 Atomic Numerical Integration

We are living in 3D world, thus the solution of triple integrals using spherical coordinates is considered to be important.

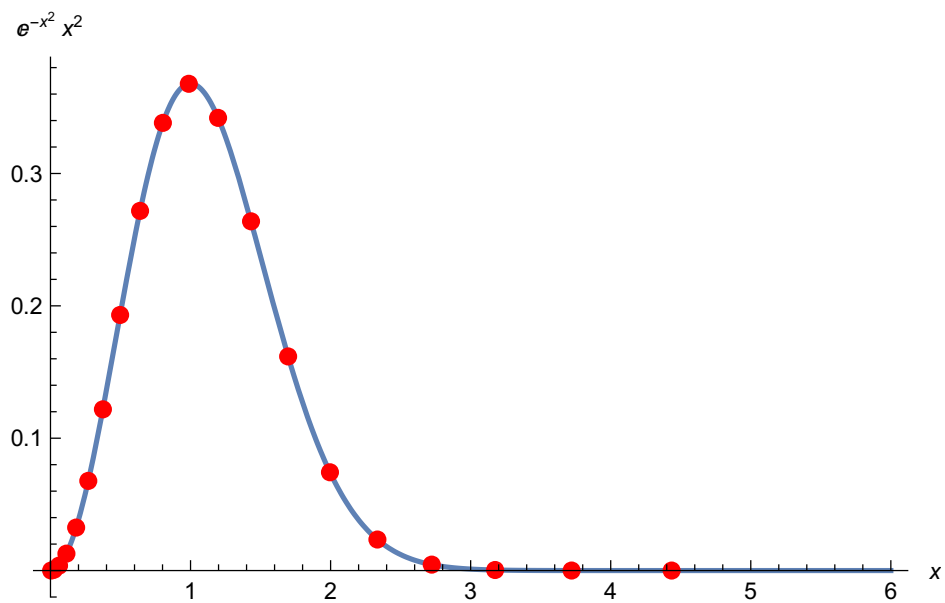
$$\int f(\mathbf{r}) d\mathbf{r} = \int_0^{2\pi} \int_0^{\pi} \int_0^{\infty} r^2 \sin(\theta) f(r, \theta, \phi) d\phi d\theta dr \quad (1.66)$$

For simplicity, the function  $f(\mathbf{r})$  is separated into radial and angular parts (e.g.  $f(\mathbf{r})=f(r)f(\theta, \phi)$ ),

$$\int f(\mathbf{r}) d\mathbf{r} = \int_0^{\infty} r^2 f(r) dr \int_0^{2\pi} \int_0^{\pi} \sin(\theta) f(\theta, \phi) d\phi d\theta \quad (1.67)$$



(a) using 10 points and rescaled using  $\lambda = 0.10$ .



(b) using 20 points and rescaled using  $\lambda = 0.067$ .

**Figure 1.5:** Rescaled Gauss-Laguerre nodes for  $x^2 e^{-x^2}$  function.



### 1.6.1.1 Integration of the Radial Part

Different algorithms have been used to evaluate the radial part,  $I(r) = \int_0^\infty r^2 f(r) dr$ , where the intervals,  $[a, b]$ , (Tables 1.1 and 1.2) were mapped to  $[0, \infty)$ .

Gauss-Laguerre and Gauss-Hermite quadratures (Table 1.2) are the most traditional choices because their radial integration intervals are over  $[0, \infty)$  and  $(-\infty, \infty)$  respectively.

For example, using Laguerre quadrature for  $\alpha = 0$  in Equation 1.65 the integral,  $I(r)$ , is evaluated as,

$$I(r) = \int_0^\infty r^2 f(r) dr = \lambda \sum_{i=1}^n \frac{w_i}{w(u_i)} (\lambda u_i)^2 f(\lambda u_i) = \lambda^3 \sum_{i=1}^n w_i u_i^2 e^{u_i} f(\lambda u_i) \quad (1.68)$$

where values of  $u_i$  are the roots of Laguerre polynomial  $L_n^\alpha(x)$ . Values of  $u_i$  and  $w_i$  can be calculated using Equation 1.59 or obtained using the formulas in Table 1.2. The parameter  $\lambda$  is utilized to adjust the radial points to a suitable physical scale.

Becke [49] used the Gauss-Chebyshev 2nd quadrature (Table 1.2) to find the radial integral  $I(r)$  by mapping the interval of  $r$  from  $[-1, 1]$  to  $[0, \infty)$ . Becke used the transform coordinate formula  $r = \alpha \frac{(1+x)}{(1-x)}$  with  $u$ -substitution integration technique (Equation 1.64) (note:  $dr = \frac{2\alpha}{(1-x)^2} dx$ ) to get:

$$\begin{aligned} \int_0^\infty r^2 f(r) dr &= \int_{-1}^1 \left( \alpha \frac{1+x}{1-x} \right)^2 \frac{2\alpha}{(1-x)^2} f\left( \alpha \frac{1+x}{1-x} \right) dx \\ &= 2\alpha^3 \int_{-1}^1 \frac{(1+x)^2}{(1-x)^4} f\left( \alpha \frac{1+x}{1-x} \right) dx \\ &= 2\alpha^3 \sum_{i=1}^n \frac{w_i}{w(x_i)} \frac{(1+x_i)^2}{(1-x_i)^4} f\left( \alpha \frac{1+x_i}{1-x_i} \right) \end{aligned} \quad (1.69)$$

Equation 1.69 can be simplified by using the information in Table 1.2, where  $w_i =$

$$\frac{\pi}{n+1} \sin^2 \left[ \frac{i\pi}{(n+1)} \right] = \frac{\pi}{n+1} (1 - x_i^2) \text{ and } w(x_i) = (1 - x_i^2)^{1/2},$$

$$\int_0^\infty r^2 f(r) dr = 2\pi\alpha^3 \sum_{i=1}^n \frac{1}{(n+1)} \frac{(1+x_i)^{\frac{5}{2}}}{(1-x_i)^{\frac{7}{2}}} f(r_i) \quad (1.70)$$

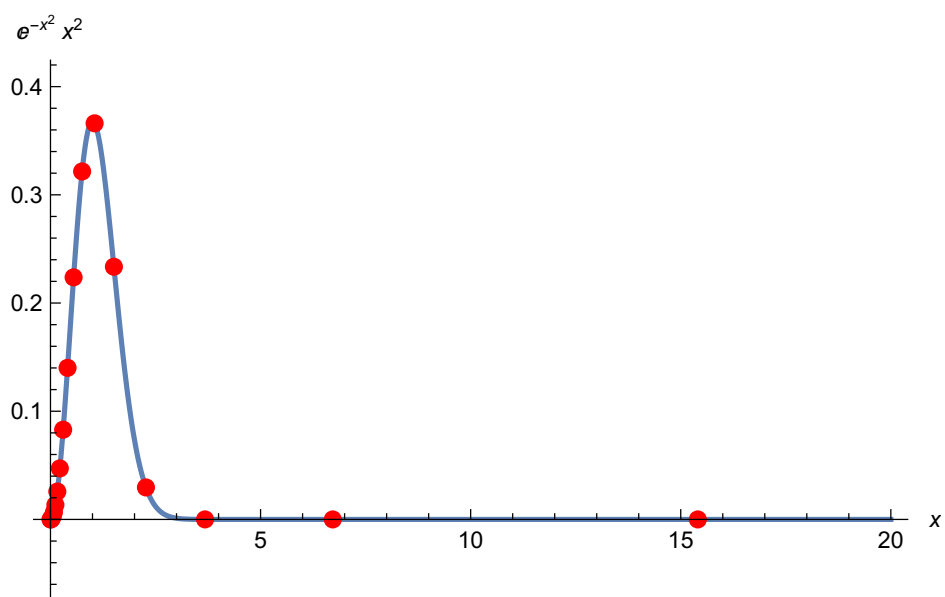
Becke assumed that parameter  $\alpha$  is equal to half of the Bragg-Slater radius of the atom, except for hydrogen. He obtained accuracy in the order of  $10^{-5}$  by using 110 angular grid points (see section 1.6.1.2 for more detail about the angular integration) and 20 radial grid points for calculating number of electrons in hydrogen. An extra 5 grid points were added for each additional atomic shell (i.e., 25 radial grid points for first-row atoms Li to Ne and 30 points for the second row atoms Na to Ar). Figure 1.6 shows that Becke grids have a large concentration near  $r = 0$ , which implies a significant waste of grid points [58].

Handy and Boys [59] approximated the Euler-Maclaurin formula within range  $[0, 1]$  (i.e.,  $a = 0$  and  $b = 1$  in Table 1.1)

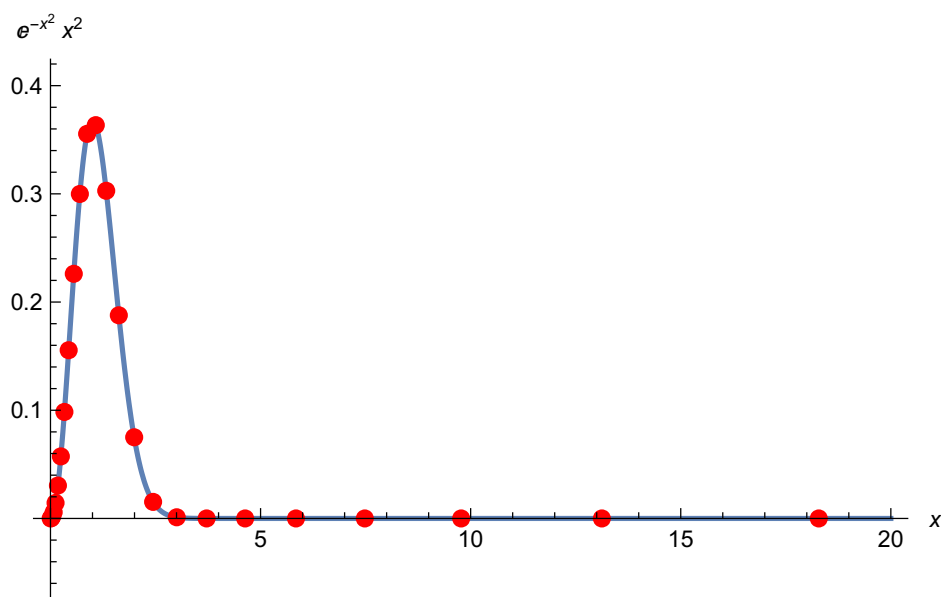
$$\int_0^1 f(q) dq = \frac{1}{n} \sum_{k=1}^{n-1} f\left(\frac{k}{n}\right) - \frac{1}{2n} (f(1) + f(0)) - \sum_{k=1}^m \frac{B_{2k}}{n^{2k}(2k)!} \left( f^{(2k-1)}(1) - f^{(2k-1)}(0) \right) \quad (1.71)$$

They showed that the integral  $\int_0^1 f(q) dq$  can be approximated by using the transform coordinate formula  $q = \alpha \left( \frac{x}{1-x} \right)^m$  to  $\frac{1}{n} \sum_{k=1}^{n-1} f\left(\frac{k}{n}\right)$ . If the summation term is to  $n$  points then the formula becomes:

$$\int_0^1 f(q) dq = \frac{1}{n+1} \sum_{k=1}^n f\left(\frac{k}{n+1}\right) \quad (1.72)$$



(a) using 20 points and 0.35 scale



(b) using 30 points and 1.8 scale

**Figure 1.6:** Some examples for Becke scheme nodes of  $x^2 e^{-x^2}$  function.

They also showed that there is no significant improvement in the results when  $m > 3$ .

Murray et al. [60] used the Handy approximation for the Euler-Maclaurin formula (Equation 1.72) and mapped it to  $[0, \infty)$  by using transform coordinate formula  $r = \alpha \left( \frac{x}{1-x} \right)^2$  ( $m = 2$ ), (note:  $dr = 2\alpha \frac{x}{(1-x)^3} dx$ ),

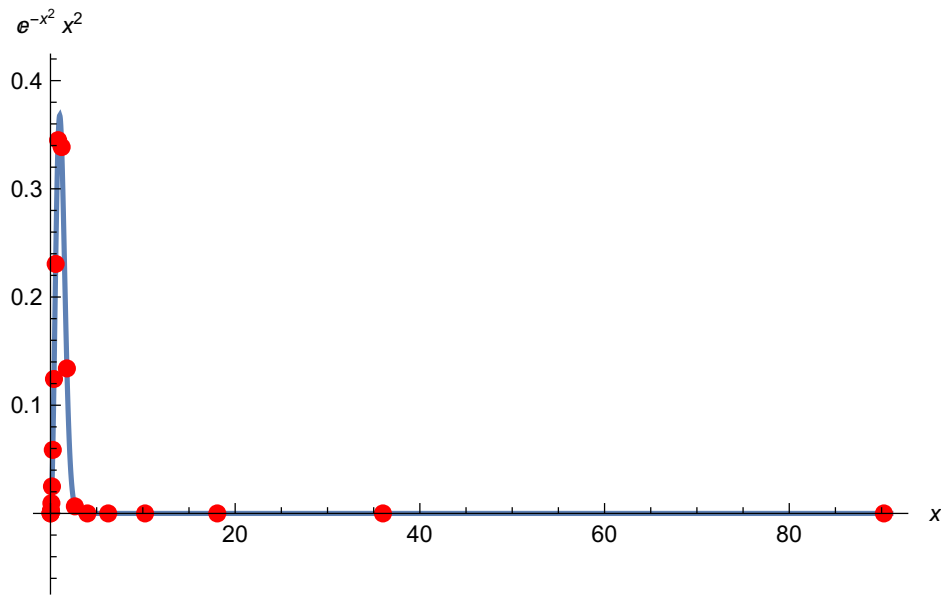
$$\begin{aligned} \int_0^\infty r^2 f(r) dr &= \int_0^1 \left( \alpha \left( \frac{x}{1-x} \right)^2 \right)^2 (2\alpha) \frac{x}{(1-x)^3} f(r) dx \\ &= 2\alpha^3 \int_0^1 \frac{x^5}{(1-x)^7} f(r) dx \\ &= 2\alpha^3 \sum_{i=1}^n \frac{1}{(n+1)} \frac{x_i^5}{(1-x_i)^7} f(r_i) \quad \text{where } x_i = \frac{i}{n+1} \end{aligned} \quad (1.73)$$

Murray et al. assumed  $\alpha$  as a scaling parameter depends on the Bragg-Slater atomic radii. They compared the accuracy for their calculations with Gauss-Laguerre points. They showed that the Euler-Maclaurin scheme is more accurate than the Gauss-Laguerre scheme. The Euler-Maclaurin scheme generates a number of points which are very far from the nucleus (Figure 1.7). These wasted points can be eliminated by dynamic pruning of the grid points [61].

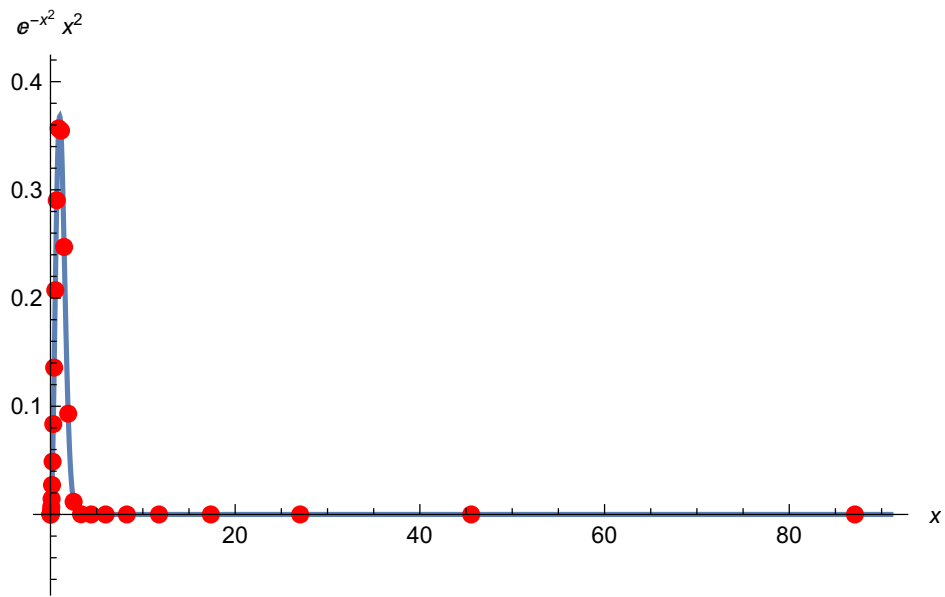
Treutler and Ahlrichs [58] used the Gauss-Chebyshev 2nd quadrature (Table 1.2) with transform coordinate formula (Equation 1.74) to find the integral  $I(r)$  by mapping the interval of  $r$  from  $[-1, 1]$  to  $[0, \infty)$ .

$$r = \alpha \frac{(1+x)^\beta}{\ln 2} \ln \left( \frac{2}{1-x} \right) \quad (1.74)$$

The best performance was obtained at  $\beta = 0.6$ . The values of  $\alpha$  for H to Kr atoms are given in reference [58]. By using the previous  $u$ -substitution integration technique (Equa-



(a) using 20 points



(b) using 30 points

**Figure 1.7:** Some examples for Euler-Maclaurin scheme nodes of  $x^2 e^{-x^2}$  function.

tion 1.64), they obtained

$$\int_0^\infty r^2 f(r) dr = \frac{\pi \alpha^3}{\ln^3 2} \sum_{i=1}^n \frac{(1+x_i)^{3\beta}}{n+1} \left[ \sqrt{\frac{1+x_i}{1-x_i}} \ln^2 \left( \frac{1-x_i}{2} \right) - \beta \sqrt{\frac{1-x_i}{1+x_i}} \ln^3 \left( \frac{1-x_i}{2} \right) \right] \quad (1.75)$$

By comparing the Treutler and Ahlrichs with Becke grids, they proved that the new scheme is superior to the Becke scheme by more than one magnitude in accuracy [58].

Mura and Knowles [61] proposed another transform coordinate formula from  $[0, 1]$  to  $[0, \infty)$  using the Euler-Maclaurin formula approximation (Equation 1.72). The new transform formula is

$$r = -\alpha \ln(1 - x^m) \quad \text{where} \quad dr = \frac{m\alpha x^{m-1}}{1 - x^m} dx \quad (1.76)$$

By combining Equation 1.72 and their transform coordinate formula (Equation 1.76) they got,

$$\begin{aligned} \int_0^\infty r^2 f(r) dr &= \int_0^1 (-\alpha \ln(1 - x^m))^2 \frac{m\alpha x^{m-1}}{1 - x^m} f(r) dx \\ &= m\alpha^3 \int_0^1 \frac{x^{m-1} \ln^2(1 - x^m)}{1 - x^m} f(r) dx \\ &= m\alpha^3 \sum_{i=1}^n \frac{x_i^{m-1} \ln^2(1 - x_i^m)}{(n+1)(1 - x_i^m)} f(r_i) \quad \text{where} \quad x_i = \frac{i}{n+1} \end{aligned} \quad (1.77)$$

where the values of  $\alpha$  are given in Mura and Knowles paper [61]. The best performance was obtained when  $m = 3$ . The results are slightly better than the Treutler and Ahlrichs grids (Equation 1.75).

Gill and Chien [62] constructed their orthogonal polynomial for weight function  $w(x) = \ln^2(x)$  by using Gram-Schmidt process (Equation 1.78) on the range of  $[0, 1]$  starting from

sequence of  $\{1, x, x^2, \dots\}$

$$p_n = x^n - \sum_{j=0}^{n-1} \frac{\langle x^n, p_j(x) \rangle}{\langle p_j(x), p_j(x) \rangle} p_j(x) \quad \text{where} \quad \langle f(x), g(x) \rangle = \int_a^b w(x) f(x) g(x) dx \quad (1.78)$$

where the nodes  $x_i$  and the weights  $w_i$  for  $n = 1 - 6, 8, 10, 15, 20$  are given in their paper [62]. They mapped the interval of  $r$  from  $[0, 1]$  to  $[0, \infty)$  by using the transform coordinate formula  $r = -\alpha \ln x$  (note  $dr = -\alpha x^{-1} dx$ ) as follows,

$$\begin{aligned} \int_0^\infty r^2 f(r) dr &= \int_1^0 -\alpha x^{-1} (-\alpha \ln(x))^2 f(r) dx \\ &= \alpha^3 \int_0^1 \ln^2(x) \frac{f(r)}{x} dx = \alpha^3 \sum_{i=1}^n \frac{w_i}{x_i} f(r_i) \end{aligned} \quad (1.79)$$

The new Gill and Chien scheme is called the "MultiExp quadrature".

### 1.6.1.2 Integration of the Angular Part

One of the most common angular integration quadrature (Equation 1.80) is the Lebedev quadrature [63–66].

$$I(\theta, \phi) = \int_0^{2\pi} \int_0^\pi \sin(\theta) f(\theta, \phi) d\theta d\phi \quad (1.80)$$

Lebedev developed a high performance angular quadrature. The grid points are constructed to lie on the surface of the unit sphere  $S: x^2 + y^2 + z^2 = 1$ . The integral can be written as

$$I(\theta, \phi) = 4\pi I(f) = \int_S f(s) ds \quad (1.81)$$

where  $s$  represents the Cartesian coordinates of vectors pointing to the surface of a unit sphere,  $ds = \sin(\theta)d\theta d\phi$  and  $I(1) = 1$ . Lebedev used the quadrature scheme below to evaluate the integral  $I(f)$ .

$$I(f) = A_1 \sum_{i=1}^6 f(a_i^1) + A_2 \sum_{i=1}^{12} f(a_i^2) + A_3 \sum_{i=1}^8 f(a_i^3) + \sum_{k=1}^{N_1} B_k \sum_{i=1}^{24} f(b_i^k) + \sum_{k=1}^{N_2} C_k \sum_{i=1}^{24} f(c_i^k) + \sum_{k=1}^{N_3} D_k \sum_{i=1}^{48} f(d_i^k). \quad (1.82)$$

The weights  $(A_1, A_2, A_3, B_k, C_k, D_k)$  and the base points  $(a_i^1, a_i^2, a_i^3, b_i^k, c_i^k, d_i^k)$  are invariant with respect to the inversion of the octahedral group. Table 1.3 shows how the base points are represented geometrically on the surface of the unit sphere.

**Table 1.3:** The Lebedev grid points distribution on octahedral point group

|         | location on octahedron | coordinates (permutations)              | constraint  | number of points |
|---------|------------------------|---|---|------------------|
| $a_i^1$ | vertices               | $(\pm 1, 0, 0)$                         |   | 6                |
| $a_i^2$ | mid-point of edges     | $2^{-\frac{1}{2}}(\pm 1, \pm 1, 0)$     |   | 12               |
| $a_i^3$ | centres of faces       | $3^{-\frac{1}{2}}(\pm 1, \pm 1, \pm 1)$ |   | 8                |
| $b_i^k$ | bisectors of faces     | $(\pm l_k, \pm l_k, \pm m_k)$           | $l_k = 2^{-\frac{1}{2}}(1 - m_k^2)^{\frac{1}{2}}$ | 24               |
| $c_i^k$ | edges of faces         | $(\pm p_k, \pm q_k, 0)$                 | $q_k = (1 - p_k^2)^{\frac{1}{2}}$                 | 24               |
| $d_i^k$ | general position       | $(\pm r_k, \pm s_k, \pm t_k)$           | $r_k^2 + s_k^2 + t_k^2 = 1$                       | 48               |

## 1.6.2 Performance and Accuracy of Numerical Integrations

The performance and the accuracy of the integration quadrature scheme depends on the molecule of interest; the type of atoms, the type of bonds, the structure of the molecule or the desired property.



Gill et al. [67] suggested that the acceptance error in molecular energy should be  $\approx 300 \mu\text{hartree}$  for a medium-sized molecule. They modified the standard grid (SG1) where different angular points were used on different radial points (i.e., partitioning the atom into regions from innermost to outermost spheres with 6, 38, 86, 194 and 86 angular points). This strategy is called "grid pruning". They showed that SG1 (which is about 3750 grid points per atom) yields acceptable grid errors compared with a benchmark grid that consists of 50 Euler-Maclaurin radial grids (Equation 1.73) and combined with 194 Lebedev angular grids (i.e., the total is 9700 grid points per atom).

Later, Chien and Gill [68] developed another standard grid (SG0) which is half of SG1 (i.e., SG0 is about 1500 grid points per atom) but somewhat less accurate than SG1. They chose the radial scheme (Equation 1.79) as a base for SG0 and they decided to select the angular grid independently for each radial point.

El-Sherbiny and Poirier [69] tested the MultiExp grids (Equation 1.79) in terms of accuracy and efficiency. They found that dividing the atomic space into three regions (core, middle, and outer) gave errors that are well within the acceptable error. They showed even for a small grid (6, 6, 8) with 6-86-194 angular grid that it performs well compared with the much larger SG1 grid, and is better than the Becke grid. Also, they found that dividing the atomic space of the Treutler scheme (Equation 1.75) into three regions improved the accuracy more than one order of magnitude even by using fewer grids for both the radial and the angular parts.

El-Sherbiny and Poirier [70] tested different numerical methods by computing different molecular properties. They found that SG1 and their new Treutler schemes gave the most accurate results. All tested grids were inaccurate in calculating the potential energy and there was inconsistent behaviour in Coulomb energy calculations.

### 1.6.3 Molecular Numerical Integration (Multi-Center Integration)

In molecules, the numerical integration of  $f(\mathbf{r})$  is not straightforward. The popular solution for this involves partitioning of molecular space into cells or discrete regions, where the straightforward numerical integration can be carried out within each region. Bader et al. [40] have defined the atomic regions within the molecules on the basis of the topology of the electron density.

The molecular function,  $f(\mathbf{r})$ , can be decomposed into a sum of atomic functions,  $f_a(\mathbf{r})$ ,

$$f(\mathbf{r}) = \sum_a f_a(\mathbf{r}), \quad f_a(\mathbf{r}) = W_a(\mathbf{r})f(\mathbf{r}) \quad (1.83)$$

Here  $W_a$  is the partition function that satisfies the conditions;  $\sum_a W_a(\mathbf{r}) = 1$  and  $W_a(\mathbf{r}) \geq 0$ .

The sum is over all atoms in the molecule.  $W_a(\mathbf{r})$  has a value of unity near the nucleus  $a$ , but vanishes at other nuclei. The integral  $I$  in Equation 1.50 can be calculated as a sum of atomic integral contributions. Thus the problem is reduced from multicentre integration to the sum of single centre integrals (i.e.,  $I = \sum_{i=1}^N I_i$ , where  $N$  is the number of atoms within the molecule).

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## Chapter 2

# Proposition of New-Partitioning Weights

*“If you think you understand quantum mechanics, you don’t understand quantum mechanics.”*

— Richard Feynman

## 2.1 New-Partitioning Weights

In this chapter the development of our three new partitioning models will be explained. These models are Fermi-Dirac, triangle, and Awad. The goal is to develop new weights in which the core electrons close to a nucleus of an atom should be assigned to that atom and not to the core or the bond of other atoms.

### 2.1.1 Model One: Fermi-Dirac Weight

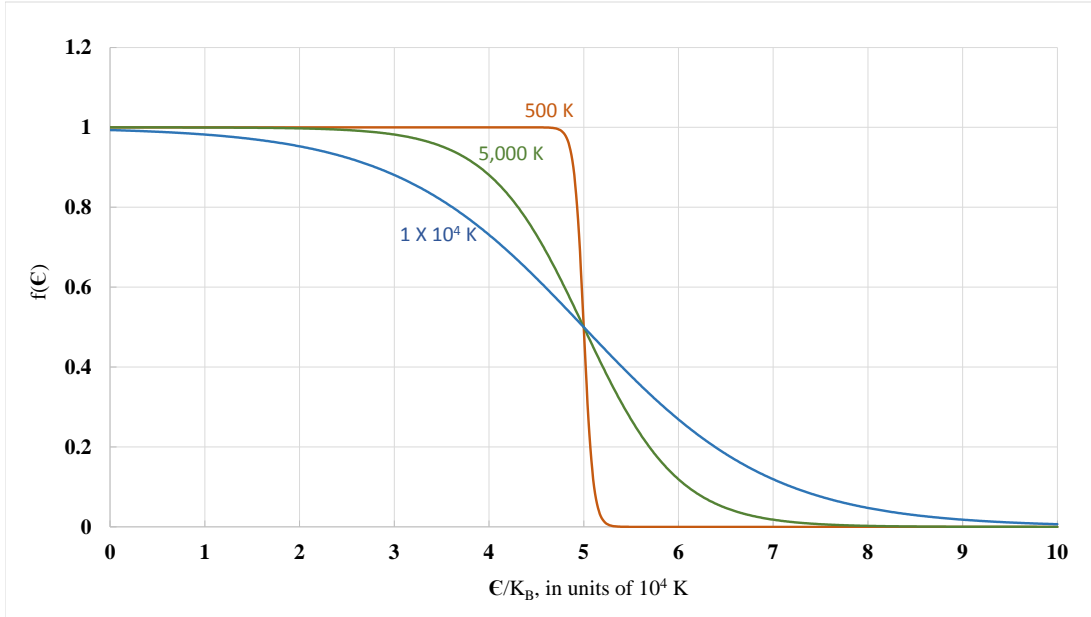
The Fermi-Dirac distribution function  $f(\epsilon)$  gives the probability of occupancy of energy levels by fermions, i.e., particles that have half-integer spin and behave in accordance to the Pauli exclusion principle,

$$f(\epsilon) = \frac{1}{e^{(\epsilon-\mu)/k_B T} + 1} \quad (2.1)$$

where  $\mu$  is the chemical potential,  $\epsilon$  is the Fermi energy that is the energy of the top most filled level at 0 K,  $k_B$  is the Boltzmann constant which equals  $1.38 \times 10^{-23} \text{ J}\cdot\text{K}^{-1}$ , and  $T$  is the temperature in Kelvin. At absolute zero, the probability is equal to one for energies less than or equal to the Fermi energy  $\epsilon$  and zero for energies greater than the Fermi energy.

The chemical potential  $\mu$  equals the Fermi energy at  $T = 0 \text{ K}$ .

Figure 2.2 is a schematic representation of two atoms A and B in a molecule. In this figure, A denotes the atom of interest and B denotes the other atoms in the molecule. Green and orange circles are the cores of atoms A and B, respectively.  $r_0^A$  and  $r_0^B$  are the radii of the cores of atoms A and B, respectively.  $r_i^A$  and  $r_i^B$  are the distances between the grid point ( $i$ ) and nuclei of atoms A and B, respectively.  $R_{AB}$  is the distance between A and B nuclei.



**Figure 2.1:** Fermi-Dirac distribution function (Equation 2.1) at the various labeled temperatures. The total number of particles is a constant and independent of temperature [1].

For simplicity, atom  $B$  is shifted to a new position in which coordinates of the grid point ( $i$ ) and nuclei  $A$  and  $B$  lay on a straight line. The distance  $r_i^B$  must remain the same before and after the shifting.  $\beta^{AB}$  is the distance between the outer surfaces for cores of atoms  $A$  and  $B$  after shifting. It is clear from Figure 2.2 that  $\beta^{AB}$  can be written as,

$$\beta^{AB} = r_i^A + r_i^B - r_0^A - r_0^B \quad (2.2)$$

The derivation of our new weight (model one) starts with an expression similar to the Fermi-Dirac distribution function (Equation 2.1), i.e.,

$$w^{AB}(r_i^A) = \frac{1}{e^{\alpha(r_i^A - \beta^{AB})^n} + 1} \quad (2.3)$$

Here  $w^{AB}(r_i^A)$  is the our partitioning weight for the atom A at specific radial grid point ( $i$ ),  $r_i^A$  is the distance between the nucleus of atom A and the grid point  $i$ ,  $\alpha$  is a real constant,  $n$  is an integer number, and  $v^{AB}$  is the distance between the nucleus of atom A and the half distance between the core of atom A and the core of other atoms B after shifting

$$v^{AB} = r_0^A + \frac{\beta^{AB}}{2} \quad (2.4)$$

By substituting  $v^{AB}$  into the Equation 2.3, we get

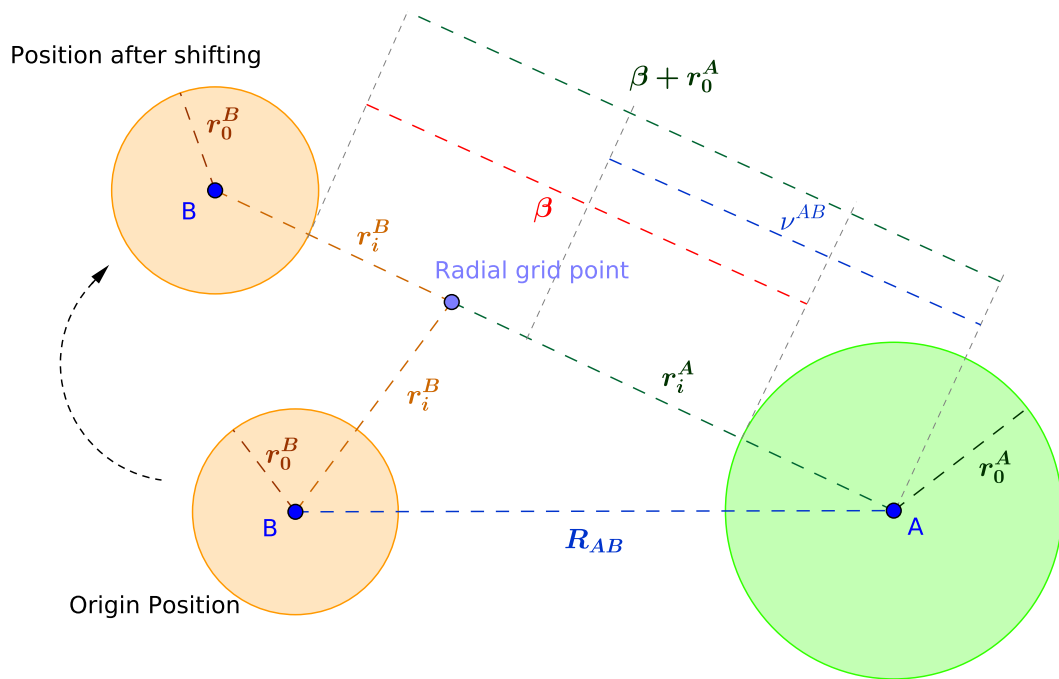
$$w^{AB}(r_i^A) = \frac{1}{e^{\alpha \left( r_i^A - \left( r_0^A + \frac{\beta^{AB}}{2} \right) \right)^n} + 1} \quad (2.5)$$

Figure 2.3 shows the partitioning weight of atom A as a function of the distance from the nucleus A along the AB axis. In this figure,  $w^{AB}(r_0^A)$  and  $w^{AB}(r_0^A + \beta^{AB})$  are the weights of atom A on the surfaces of the cores for atoms A and B, respectively.

The different between  $w^{AB}(r_0^A)$  and  $w^{AB}(r_0^A + \beta^{AB})$  is defined as follows,

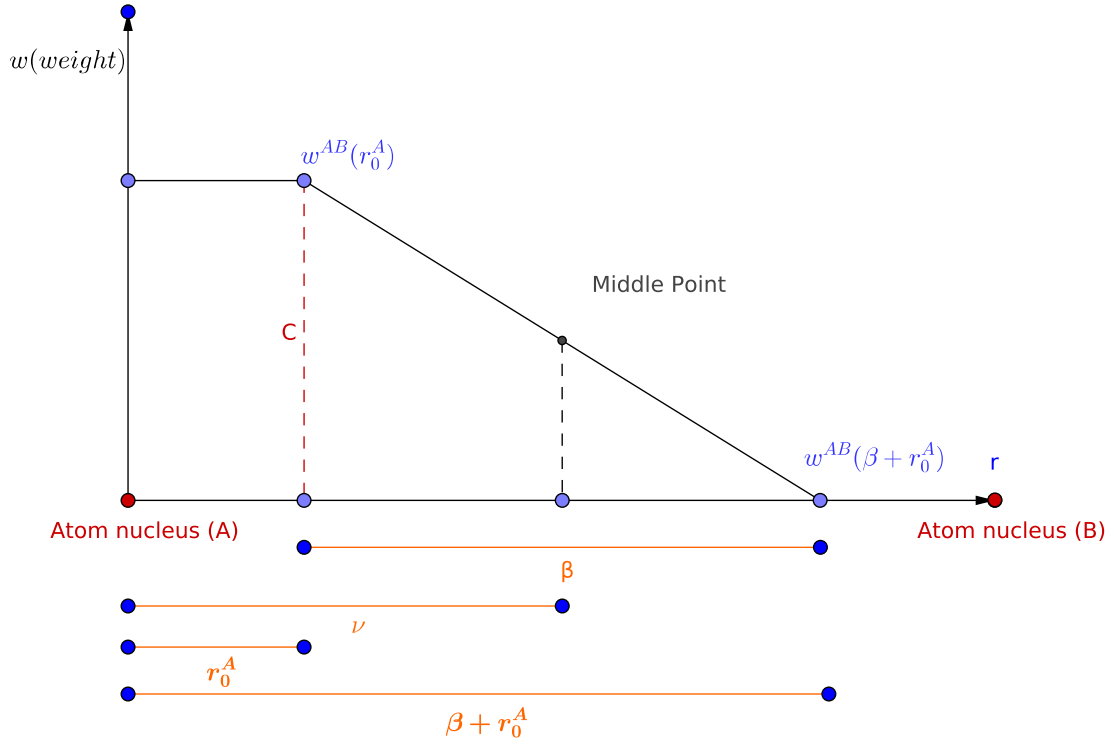
$$C = w^{AB}(r_0^A) - w^{AB}(r_0^A + \beta^{AB}) \quad \text{should be equal to one} \quad (2.6)$$

In order to define a correct partitioning weight function, the following conditions must be met  $w^{AB}(r_0^A) = 1$ ,  $w^{AB}(r_0^A + \beta^{AB}) = 0$  and  $C = 1$ . Using Equation 2.5, the weight of atom



**Figure 2.2:** Fermi-Dirac model (model one), atom A represents the atom of interest and atom B represents the other atoms. The green and the orange circles are the cores of atoms A and B respectively.





**Figure 2.3:** Fermi-Dirac model (model one).

A on its core's surface (i.e., at position  $r_0^A$ ) is,

$$\begin{aligned}
 w^{AB}(r_0^A) &= \frac{1}{e^{\alpha \left( r_0^A - \left( r_0^A + \frac{\beta^{AB}}{2} \right) \right)^n} + 1} \\
 &= \frac{1}{e^{(-1)^n \alpha \left( \frac{\beta^{AB}}{2} \right)^n} + 1}
 \end{aligned} \tag{2.7}$$

and the weight of atom A on the core's surface of atom B (i.e. at position  $r_0^A + \beta^{AB}$ ) is,

$$w^{AB}(r_0^A + \beta^{AB}) = \frac{1}{e^{\alpha \left( \frac{\beta^{AB}}{2} \right)^n} + 1} \tag{2.8}$$

The substitution of Equations 2.7 and 2.8 into Equation 2.6 leads to,

$$C = \frac{1}{e^{(-1)^n \alpha \left(\frac{\beta^{AB}}{2}\right)^n} + 1} - \frac{1}{e^{\alpha \left(\frac{\beta^{AB}}{2}\right)^n} + 1} \quad (2.9)$$

It is clear that if  $n$  is an even number then  $C = 0$ , whereas if  $n$  is an odd value  $C$  can be computed as follows:

$$\begin{aligned} C &= \frac{1}{e^{-\alpha \left(\frac{\beta^{AB}}{2}\right)^n} + 1} - \frac{1}{e^{\alpha \left(\frac{\beta^{AB}}{2}\right)^n} + 1} \quad n \text{ is an odd number} \\ &= \frac{e^{\alpha \left(\frac{\beta^{AB}}{2}\right)^n} - 1}{e^{\alpha \left(\frac{\beta^{AB}}{2}\right)^n} + 1} \end{aligned} \quad (2.10)$$

From the above equation,  $\alpha$  is,

$$\begin{aligned} C \left( e^{\alpha \left(\frac{\beta^{AB}}{2}\right)^n} + 1 \right) &= e^{\alpha \left(\frac{\beta^{AB}}{2}\right)^n} - 1 \\ C e^{\alpha \left(\frac{\beta^{AB}}{2}\right)^n} + C &= e^{\alpha \left(\frac{\beta^{AB}}{2}\right)^n} - 1 \\ e^{\alpha \left(\frac{\beta^{AB}}{2}\right)^n} - C e^{\alpha \left(\frac{\beta^{AB}}{2}\right)^n} &= 1 + C \\ e^{\alpha \left(\frac{\beta^{AB}}{2}\right)^n} (1 - C) &= 1 + C \\ \alpha &= \left( \frac{2}{\beta^{AB}} \right)^n \ln \left( \frac{1+C}{1-C} \right) \end{aligned} \quad (2.11)$$

As mentioned before the variable  $C$  must be one. However, from Equation 2.11  $C$  can not equal one, and so we assume  $C$  is a number very close to one.

For simplicity,  $\alpha$  can be written as,

$$\alpha = k \left( \frac{2}{\beta^{AB}} \right)^n \quad \text{where} \quad k = \ln \left( \frac{1+C}{1-C} \right) \quad (2.12)$$

Substituting  $\alpha$  into Equation 2.5 gives

$$w^{AB}(r_i^A) = \frac{1}{e^{k \left( \frac{2(r_i^A - r_0^A)}{\beta^{AB}} - 1 \right)^n} + 1} \quad (2.13)$$

Because  $n$  must be an odd number and for simplicity through this thesis, we will consider  $n = 1$ . Note that all variables in Equation 2.13 are given or easily calculated. Equation 2.13 is the final form of the partitioning weight for atom  $A$ . In order to determine the amount of space owned by atom  $A$  in the entire molecule, the following pairwise ownership has been defined between atom  $A$  and all other atoms.

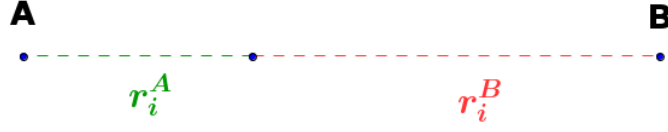
$$w^A(\mathbf{r}) = \prod_{B=1, B \neq A}^N w^{AB}(r_i^A) \quad (2.14)$$

The normalized weight function  $W^A(\mathbf{r})$  for an atom  $A$  at a particular point  $\mathbf{r}$  can now be defined as follow.

$$W^A(\mathbf{r}) = \frac{w^A(\mathbf{r})}{\sum_n w^n(\mathbf{r})} \quad (2.15)$$

### 2.1.2 Model Two: Triangle Weight

Figure 2.4 is a schematic representation of two atoms  $A$  and  $B$  in a molecule, where  $A$  denotes the atom of interest and  $B$  denotes the other atoms in the molecule. If we let a radial grid point  $i$  lie between these two atoms, then we can define the following partitioning ratio



**Figure 2.4:** Model two; first assumption.

$\mu_i^{AB}$ ,

$$\mu_i^{AB} = \frac{r_i^B - r_i^A}{r_i^B + r_i^A} \quad (2.16)$$

where  $r_i^A$  and  $r_i^B$  are the distances between the grid point  $i$  and the nuclei of  $A$  and  $B$ , respectively. Note that when  $i$  lies on the nucleus of atom  $A$  (i.e.,  $r_i^A \rightarrow 0$ ) then  $\mu_i^{AB} \rightarrow +1$  and when  $i$  lies on the nucleus of atom  $B$  (i.e.,  $r_i^B \rightarrow 0$ ) then  $\mu_i^{AB} \rightarrow -1$ . If  $i$  is not on a nucleus then  $\mu_i^{AB}$  takes values between  $-1$  and  $+1$ . The new partitioning weight  $w_i^{AB}$  can be defined using  $\mu_i^{AB}$  as follows,

$$\begin{aligned} w_i^{AB} &= \frac{1}{2} (\mu_i^{AB} + 1) \\ &= \frac{1}{2} \left( \frac{r_i^B - r_i^A}{r_i^B + r_i^A} + 1 \right) \end{aligned} \quad (2.17)$$

when  $r_i^A \rightarrow 0$  then  $w_i^A \rightarrow +1$ , and when  $r_i^B \rightarrow 0$  then  $w_i^A \rightarrow 0$ . If the radial grid point is not on a nucleus then  $w_i^{AB}$  takes values between  $0$  and  $+1$ .

$\mu_i^{AB}$  and  $w_i^{AB}$  can be manipulated by replacing the values of  $r_i^A$  and  $r_i^B$  in Equation 2.17 with  $(r_i^A - r_0^A)$  and  $(r_i^B - r_0^B)$ , respectively. Here  $r_0^A$  and  $r_0^B$  are the atomic core radii of  $A$  and  $B$ , respectively.

$$\mu_i^{AB} = \frac{(r_i^B - r_0^B) - (r_i^A - r_0^A)}{(r_i^B - r_0^B) + (r_i^A - r_0^A)} \quad (2.18)$$

$$w_i^{AB} = \frac{1}{2} \left( \frac{(r_i^B - r_0^B) - (r_i^A - r_0^A)}{(r_i^B - r_0^B) + (r_i^A - r_0^A)} + 1 \right) \quad (2.19)$$

In this case we assume that the atomic cores have spherical shapes. When  $i$  lies on the core surface of atom  $A$  (i.e.,  $r_i^A \rightarrow r_0^A$ ) then  $w_i^{AB} \rightarrow +1$ , and when  $i$  lies on the core surface of other atoms  $B$  (i.e.,  $r_i^B \rightarrow r_0^B$ ) then  $w_i^{AB} \rightarrow 0$ .

In order to make the weight  $w_i^{AB}$  of  $A$  equals to one within its core and equal to zero within the core of  $B$ , Equation 2.19 can be modified by utilizing the absolute value as follows,

$$w_i^{AB} = \frac{1}{2} \left( \frac{|r_i^B - r_0^B| - |r_i^A - r_0^A|}{(r_i^B - r_0^B) + (r_i^A - r_0^A)} + 1 \right) \quad (2.20)$$

All variables in Equation 2.20 are given or easily calculated. Once again, in order to determine the amount of space owned by particular atom,  $A$ , in the entire molecule, the following pairwise ownership has been used,

$$w^A(\mathbf{r}) = \prod_{B=1, B \neq A}^N w_i^{AB}(\mathbf{r}) \quad (2.21)$$

The normalized weight function  $W^A(\mathbf{r})$  for an atom  $A$  at a particular point  $\mathbf{r}$  can now be defined as,

$$W^A(\mathbf{r}) = \frac{w^A(\mathbf{r})}{\sum_n w^n(\mathbf{r})} \quad (2.22)$$

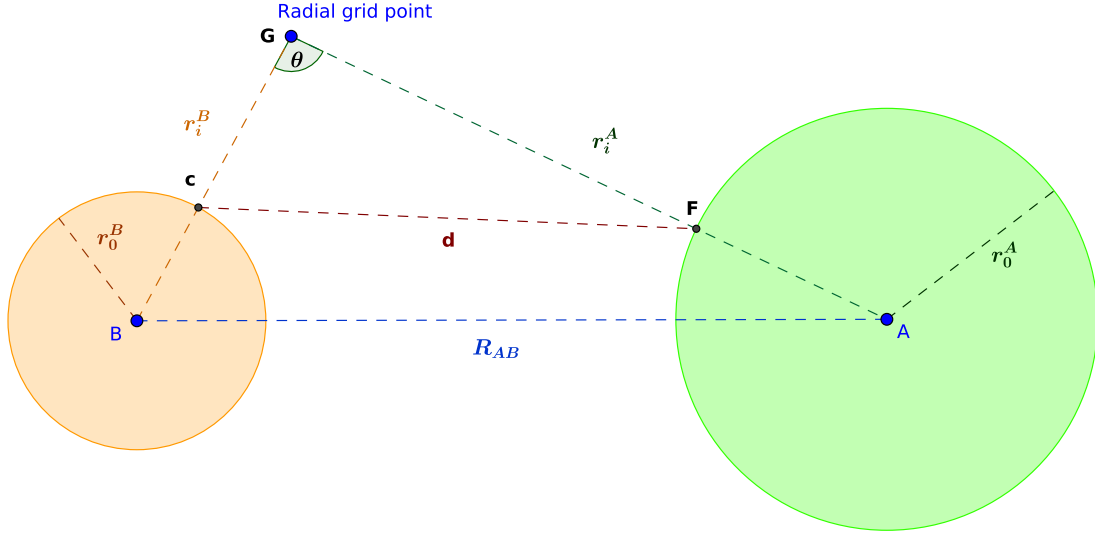
### 2.1.3 Model Three: Awad Weight

In this model, we use the same principle that is used in model two (subsection 2.1.2).

However, the denominator of Equation 2.18 is modified as follows

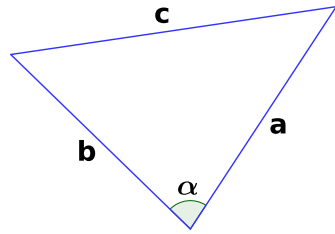
$$\mu_i^{AB} = \frac{(r_i^B - r_0^B) - (r_i^A - r_0^A)}{d} \quad (2.23)$$

where  $d$  is the distance between points  $C$  and  $F$  in Figure 2.5. The points  $C$  and  $F$  represent the intersection of lines that connect the atomic nuclei and the grid point  $i$  with the atomic core surfaces of A and B, respectively. In order to derive an expression for  $d$ , we apply the



**Figure 2.5:** Awad model (model three).

cosine rule,

$$\cos(\alpha) = \frac{a^2 + b^2 - c^2}{2ab} \quad (2.24)$$


As shown in Figure 2.5, triangles  $GAB$  and  $GFC$  share the angle  $\theta$ . By applying the cosine rule (Equation 2.24) on  $GAB$  and  $GFC$ , we get

$$\cos(\theta) = \frac{(r_i^A - r_0^A)^2 + (r_i^B - r_0^B)^2 - d^2}{2(r_i^A - r_0^A)(r_i^B - r_0^B)} = \frac{r_i^{A^2} + r_i^{B^2} - R_{AB}^2}{2r_i^A r_i^B} \quad (2.25)$$

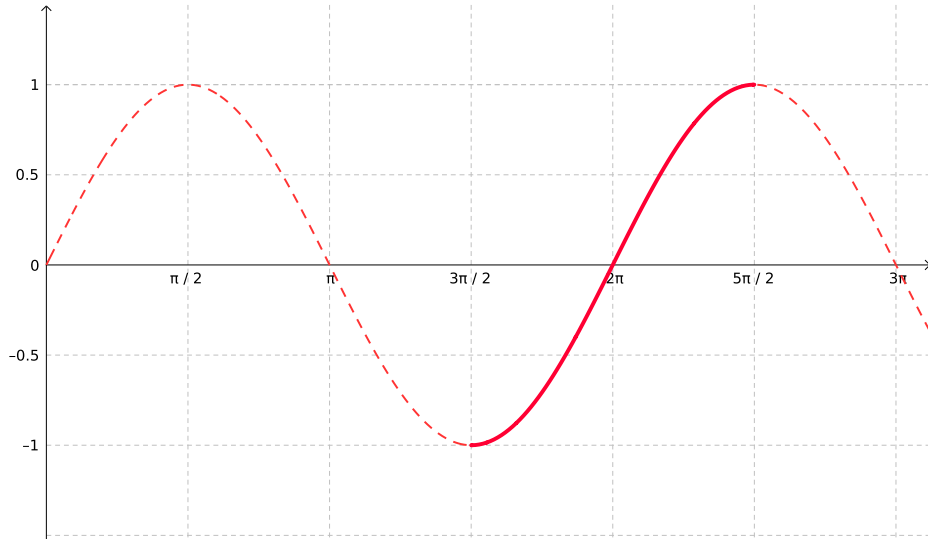
The rearrangement of Equation 2.25 gives,

$$d^2 = (r_i^A - r_0^A)^2 + (r_i^B - r_0^B)^2 - \frac{(r_i^{A^2} + r_i^{B^2} - R_{AB}^2)(r_i^A - r_0^A)(r_i^B - r_0^B)}{r_i^A r_i^B} \quad (2.26)$$

Substitution of  $d$  in Equation 2.23 gives,

$$\mu_i^{AB} = \frac{(r_i^B - r_0^B) - (r_i^A - r_0^A)}{\left[ (r_i^A - r_0^A)^2 + (r_i^B - r_0^B)^2 - (r_i^{A^2} + r_i^{B^2} - R_{AB}^2) \frac{(r_i^A - r_0^A)(r_i^B - r_0^B)}{r_i^A r_i^B} \right]^{\frac{1}{2}}} \quad (2.27)$$

Note that when  $i$  lies on the core surface of atom A (i.e.,  $r_i^A \rightarrow r_0^A$ ) then  $\mu_i^{AB} \rightarrow +1$  and when  $i$  lies on the core surface of atom B (i.e.,  $r_i^B \rightarrow r_0^B$ ) then  $\mu_i^{AB} \rightarrow -1$ . It should be noted that Equation 2.27 is invalid within the cores of atoms A and B. However,  $\mu_i^{AB}$  outside the core regions is a linear function. To replace  $\mu_i^{AB}$  by a smooth function, one can benefit from the sine function graph (shown in Figure 2.6).



**Figure 2.6:** Graph of the sine function  $\sin(x)$ .

Using Taylor expansion for the sine function of  $x$  in radians,

$$\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \dots \quad (2.28)$$

Our aim is to use the sine function shape in  $[3\pi/2, 5\pi/2]$  (solid line in Figure 2.6). This shape can be obtained from using the first two terms in the expansion,

$$h(x) = ax - bx^3 \quad (2.29)$$

where  $h(x)$  is the truncated Taylor expansion that has the desired shape with unknown coefficients  $a$  and  $b$ . To derive the desired function, the conditions:  $h(1) = 1$  and  $h(-1) = -1$ ,

$$\begin{aligned} h(\pm 1) &= a(\pm 1) - b(\pm 1)^3 = \pm 1 \\ \Rightarrow a - b &= 1 \end{aligned} \quad (2.30)$$

and the value of  $h'(\pm 1) = 0$  should be held,

$$\begin{aligned} h'(\pm 1) &= a - 3b(\pm 1)^2 = 0 \\ \Rightarrow a &= 3b \end{aligned} \quad (2.31)$$

from Equations 2.30 and 2.31,  $a = \frac{3}{2}$  and  $b = \frac{1}{2}$ , and  $h(x)$  becomes,

$$h(x) = 1.5x - 0.5x^3 \quad (2.32)$$



Equation 2.32 represents the maximum smoothing for  $h(x)$ , whereas when  $b = 0$  Equation 2.29 becomes  $h(x) = x$  and represents the minimum smoothing. Therefore, we can control the steepness of the curve by changing the value of  $b$  from 0 to  $\frac{1}{2}$ , or mathematically,

$$h(x) = (b + 1)x - bx^3 \quad \text{where} \quad 0 \leq b \leq \frac{1}{2} \quad (2.33)$$

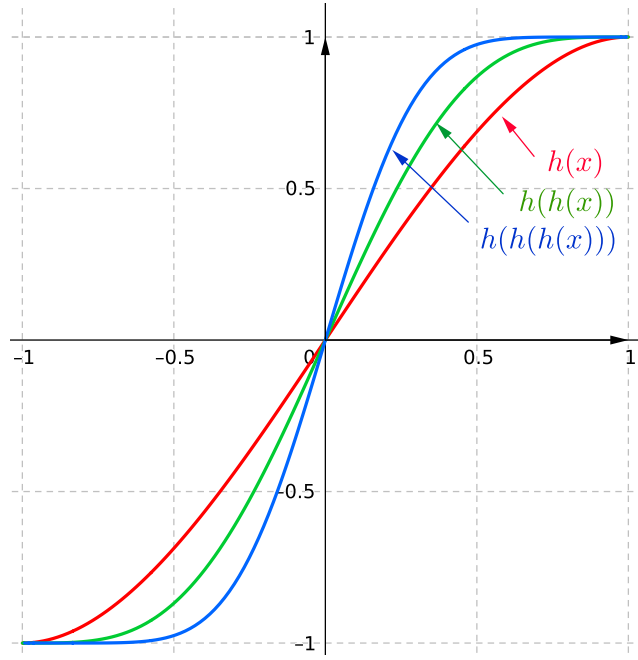
The substitution of  $\mu_i^{AB}$  in Equation 2.33 leads to,

$$h(\mu_i^{AB}) = (b + 1)\mu_i^{AB} - (b)\mu_i^{AB^3} \quad (2.34)$$

This function  $h(\mu_i^{AB})$  is still not smooth enough. Thus  $h(\mu_{Bi}^A)$  is iterated three times (i.e., same as Becke) to obtain the desired smoothness.

$$\begin{aligned} f_1(\mu_i^{AB}) &= h(\mu_i^{AB}) \\ f_2(\mu_i^{AB}) &= h[h(\mu_i^{AB})] \\ f_3(\mu_i^{AB}) &= h\{h[h(\mu_i^{AB})]\} \end{aligned} \quad (2.35)$$

The outcome of above three iterations is illustrated in Figure 2.7.



**Figure 2.7:** Profiles of functions  $f_1(\mu_i^{AB})$ ,  $f_2(\mu_i^{AB})$ , and  $f_3(\mu_i^{AB})$ .

The Awad weight  $w_i^{AB}$  can be defined using  $f_3(\mu_i^{AB})$  as follows,

$$w_i^{AB} = \frac{1}{2} \left[ f_3(\mu_i^{AB}) + 1 \right] \quad (2.36)$$

Once again, in order to determine the amount of space owned by particular atom,  $A$ , in the entire molecule, the following pairwise ownership has been used,

$$w^A(\mathbf{r}) = \prod_{B=1, B \neq A}^N w_i^{AB}(\mathbf{r}) \quad (2.37)$$

The normalized weight function  $W^A(\mathbf{r})$  for an atom  $A$  at a particular point  $\mathbf{r}$  can now be defined as,

$$W^A(\mathbf{r}) = \frac{w^A(\mathbf{r})}{\sum_n w^n(\mathbf{r})} \quad (2.38)$$

# Bibliography

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## Chapter 3

# New-Partitioning Weights Calculations and Results

*“Einstein said that if quantum mechanics were correct then the world would be crazy.  
Einstein was right - the world is crazy.”*

— Daniel M. Greenberger

### 3.1 Computational Method

All the calculations were performed using the MUNgauss package [1]. The results were calculated at the HF/6-31G(d)//HF/6-31G(d) level unless otherwise specified. The atomic radial densities for the isolated atoms from hydrogen to argon are computed at HF/6-311++G(d,p) and employ a mesh size of 0.00001 bohr. The results of numerical integration were calculated using the 6-31G(d) basis sets on HF/6-31G(d) optimized structures, and the benchmark grid is used in the calculations to decrease the error that comes from the grid. All of the visual aids, including contour, relief, and the 3D plots were created using Mathematica Version 11.2 graphing package [2].

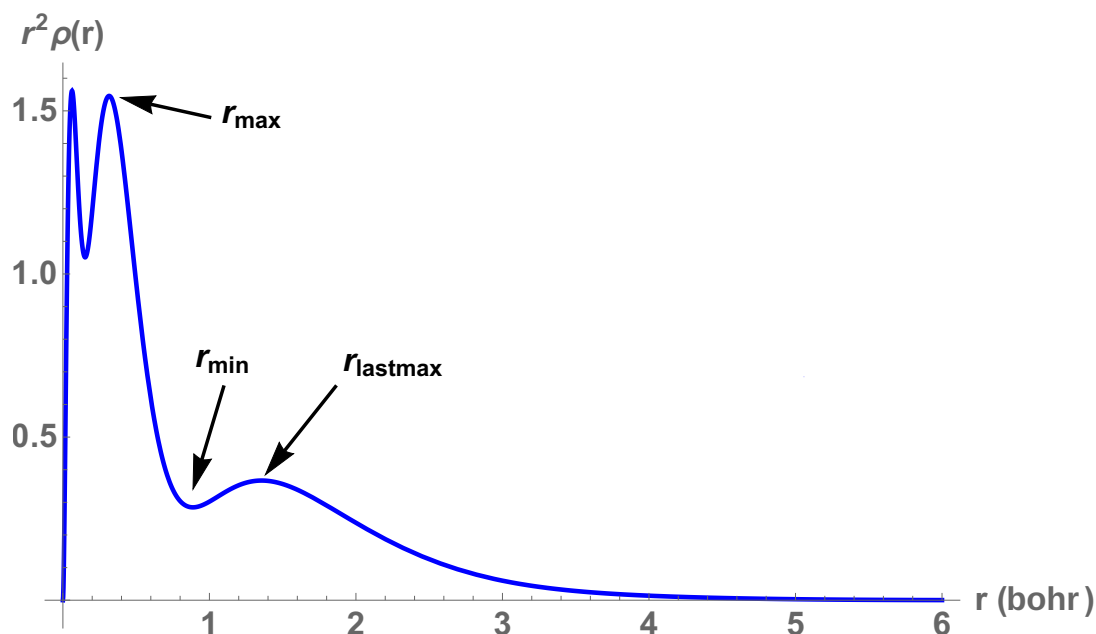
Throughout this thesis, the length is given in bohr, radial density  $\rho_{rad}$  is given in  $e/\text{bohr}$  and electron density is in  $e/\text{bohr}^3$ , unless otherwise specified.

## 3.2 Results of the New Partitioning Weight Models

The derivation of three new partitioning weight models (Fermi-Dirac, triangle, and Awad) are shown in the previous chapter. In this chapter, we will compute some of molecular properties using the new partitioning weight models. The visualization of radial and bond electron densities using these models will be discussed, then the molecular properties will be calculated numerically using the new partitioning weight models. The results will be compared with other popular weight schemes such as the Becke weight.

### 3.2.1 Suggested Cores Sizes

As mentioned in chapter 2, the Fermi-Dirac, triangle, and Awad models (for example Equation 2.27) require the atomic core sizes. The question is, what are the atomic core sizes to be used in these models. There is an enormous number of definition for atomic radii, such as a covalent, ionic, metallic, and Bragg Slater radii. Because the new weights are developed for molecular radial electron density, the atomic radial densities for the isolated atoms from hydrogen to argon are computed at HF/6-311++G(d,p). The minimum radial density, maximum radial density and the last core maximum were found using Mathematica's FindMaximum and FindMinimum functions, and have an accuracy of  $\pm 0.00001$  bohr based on using a mesh size of 0.00001 bohr between points. For example, the molecular radial electron density (RDEN) for Cl atom is shown in Figure 3.1, where  $r_{max}$  is the last core maximum,  $r_{lastmax}$  is the last maximum and  $r_{min}$  is the last minimum. In addition, the expectation value for the radii of the last core atomic shells  $r_{\langle r \rangle}$  of the isolated atoms from hydrogen to argon were calculated numerically using SG1 radial grid at HF/6-311++G(d,p). Table 3.1 shows values of  $r_{min}$ ,  $r_{max}$ ,  $r_{lastmax}$ , and  $r_{\langle r \rangle}$  for isolated atoms



**Figure 3.1:** Radial electron density for Cl atom using HF/6-311++G(d,p).  $r_{max}$  is the last core maximum,  $r_{lastmax}$  is the last maximum and  $r_{min}$  is the last minimum.

from hydrogen to argon in bohr. It is clear from Table 3.1 that  $r_{lastmax}$  cannot be used to define atomic cores as their values are so large. For example, the bond length of the LiCl molecule is about 3.849 bohr, where the  $r_{lastmax}$  of Li is 3.081 bohr and  $r_{lastmax}$  of Cl is 1.356 bohr; their sum is larger than bond length of LiCl. It should be noted that differences between cores in metal atoms (e.g., Li and Na) are larger compared to non-metal atoms (e.g., F and S). Therefore, in molecules containing metal atoms the choice of the core is of great importance as it affects significantly both RDEN and bond electron density (BDEN) of atoms in the molecule.

### 3.2.2 Visualization of the New Weights

In this section, some of the advantages and the drawbacks of Fermi-Dirac, triangle, and Awad models will be illustrated visually. The weight of the C atom within the CO molecule

**Table 3.1:** The cores values (bohr), minimum radial density ( $r_{min}$ ), maximum radial density ( $r_{max}$ ), the last core maximum ( $r_{lastmax}$ ), and the average radial value of the last core shell ( $r_{\langle r \rangle}$ ) of isolated atoms calculated using HF/6-311++G(d,p)

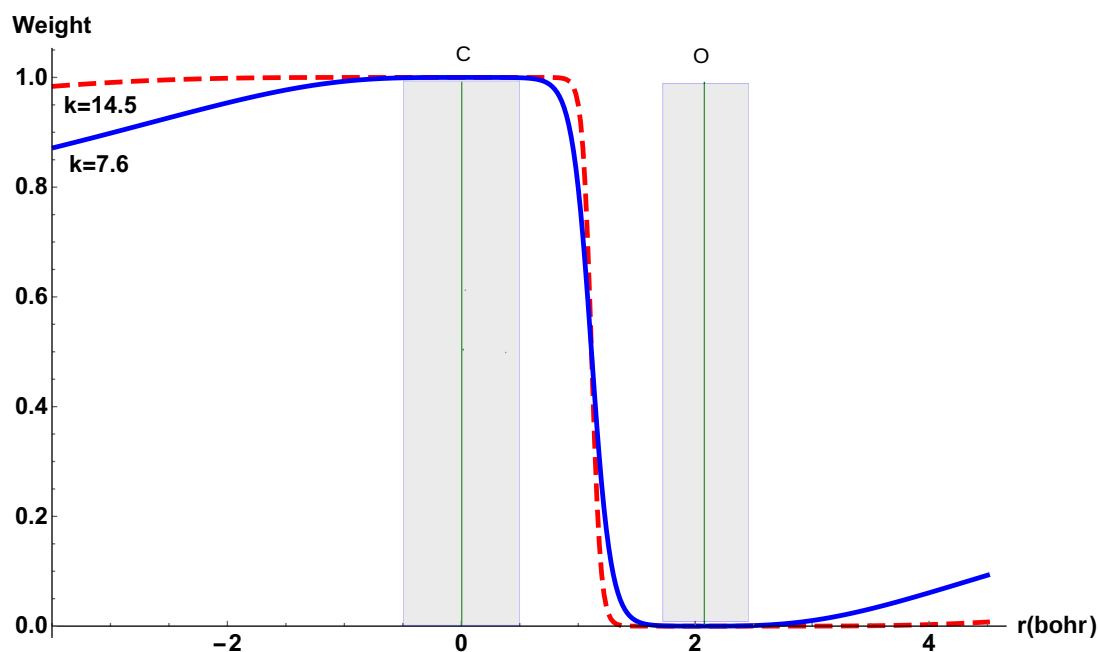
|           | $r_{min}$ | $r_{max}$ | $r_{lastmax}$ | $r_{\langle r \rangle}$ |
|-----------|-----------|-----------|---------------|-------------------------|
| <b>H</b>  | 0.00000   | 0.00000   | 1.00784       | 0.00000                 |
| <b>He</b> | 0.00000   | 0.00000   | 0.57697       | 0.00000                 |
| <b>Li</b> | 1.77050   | 0.36501   | 3.08074       | 1.14660                 |
| <b>Be</b> | 1.09369   | 0.26629   | 2.05757       | 0.83013                 |
| <b>B</b>  | 0.79765   | 0.21031   | 1.53395       | 0.65175                 |
| <b>C</b>  | 0.61976   | 0.17398   | 1.21493       | 0.53690                 |
| <b>N</b>  | 0.50088   | 0.14875   | 1.00110       | 0.45661                 |
| <b>O</b>  | 0.41737   | 0.13009   | 0.84941       | 0.39719                 |
| <b>F</b>  | 0.35450   | 0.11581   | 0.73676       | 0.35151                 |
| <b>Ne</b> | 0.30578   | 0.10451   | 0.65015       | 0.31527                 |
| <b>Na</b> | 3.40265   | 0.57464   |               | 1.59791                 |
| <b>Mg</b> | 2.10027   | 0.50602   | 2.42357       | 1.38154                 |
| <b>Al</b> | 1.63344   | 0.45278   | 2.12579       | 1.24024                 |
| <b>Si</b> | 1.34635   | 0.40929   | 1.87480       | 1.12594                 |
| <b>P</b>  | 1.14513   | 0.37240   | 1.66778       | 1.03142                 |
| <b>S</b>  | 0.99901   | 0.34217   | 1.48923       | 0.95160                 |
| <b>Cl</b> | 0.88750   | 0.31578   | 1.35608       | 0.88353                 |
| <b>Ar</b> | 0.79563   | 0.29320   | 1.23988       | 0.82465                 |



along the bond using  $r_{\langle r \rangle}$  as cores for C and O atoms will be used as an example. Figures 3.2, 3.3, and 3.4 show Fermi-Dirac, triangle, and Awad models, respectively.

### 3.2.2.1 Fermi-Dirac Weight

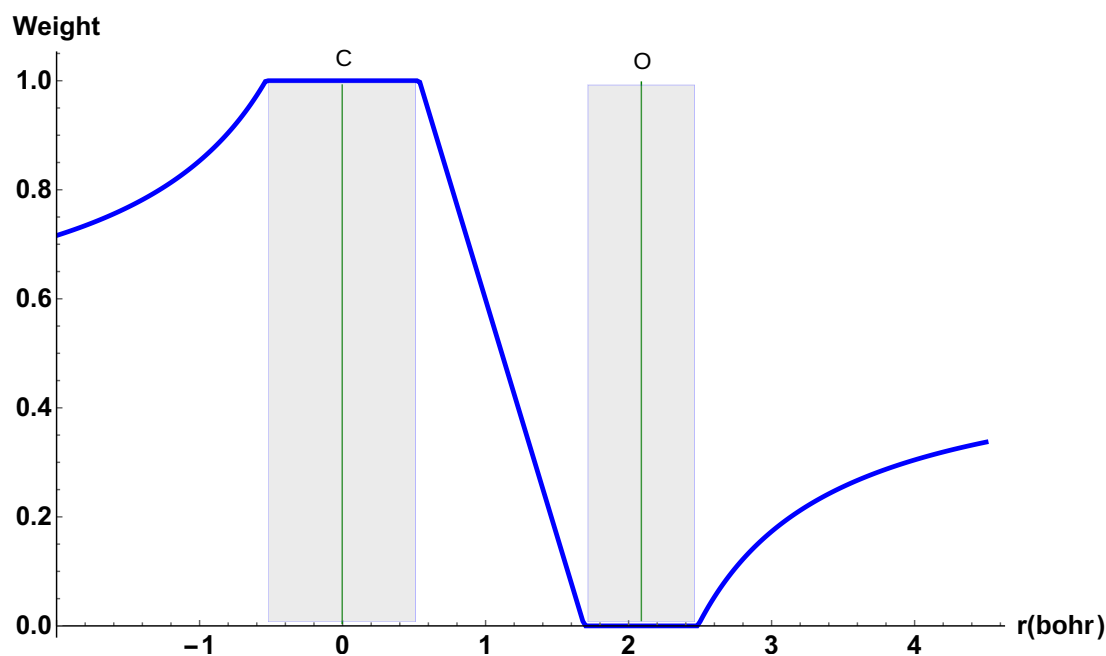
Figure 3.2 shows the Fermi-Dirac weight for C atom ( $W^C(r)$ ) within the CO molecule along the bond at  $C$  values of 0.999 and 0.999999, i.e.,  $k = 7.6$  and 14.5. As it can be seen in this figure, the weight shows some drawbacks: it goes to 0.5 when  $r$  approaches  $-\infty$  or  $\infty$  along the bond, and it is not exactly one (less than one) within the atom's core. It is clear that when changing the  $k$  value from 7.6 to 14.5 the weight function becomes similar to a step function.



**Figure 3.2:** The Fermi-Dirac weight  $W(\mathbf{r})$  with cores given by  $r_{\langle r \rangle}$  for C atom within the CO molecule along the bond. The red dashed line is obtained at  $k = 14.5$  and the blue solid line is obtained at  $k = 7.6$  (Equation 2.12). The C atom located at the origin, the C-O bond length is 2.0871 bohr,  $r_{\langle r \rangle}$  for C is 0.53690 bohr, and  $r_{\langle r \rangle}$  for O is 0.39719 bohr.

### 3.2.2.2 Triangle Weight

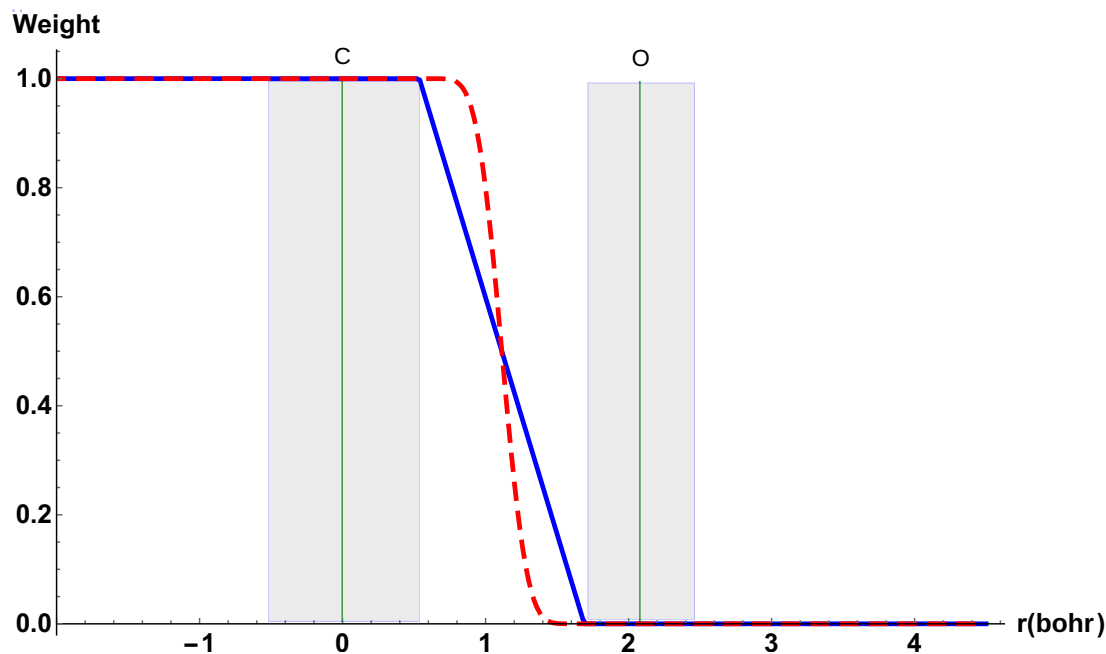
In contrast to Fermi-Dirac weight, the triangle weight of the C atom in the CO molecule shows that the weight is exactly one within the core of C atom and exactly zero within the core of the other atom (in this case O atom) (Figure 3.3). However, similar to Fermi-Dirac weight, triangle weight goes quickly to 0.5 when  $r$  approaches  $-\infty$  or  $\infty$  along the bond. Another drawback is that the weight does not give a smooth curve because it has four singular points.



**Figure 3.3:** The triangle weight  $W(\mathbf{r})$  with cores given by  $r_{\langle r \rangle}$  for C atom within CO molecule along the bond. The C atom located at the origin, the C-O bond length is 2.0871 bohr,  $r_{\langle r \rangle}$  for C is 0.53690 bohr, and  $r_{\langle r \rangle}$  for O is 0.39719 bohr.

### 3.2.2.3 Awad Weight

The Awad weight gives exactly one within the core of C atom and exactly zero within the core of other atoms (Figure 3.4). In addition, this weight goes to one and zero when  $r$  approaches  $-\infty$  or  $\infty$ , respectively. Moreover, Awad weight has the flexibility to change the steepness of the curve by controlling the value of steepness  $b$  (Equation 2.34). For example as shown in Figure 3.4,  $W^C(\mathbf{r})$  within the CO molecule is computed using two values of steepness  $b$ ; the red dashed line is obtained at  $b = 0.5$  and the blue solid line is obtained at  $b = 0$ .



**Figure 3.4:** The Awad weight for C atom within the CO molecule along the bond using a difference steepness value. The red dashed line is obtained at  $b = 0.5$  and the blue solid line is obtained at  $b = 0$  (Equation 2.34). The C atom is located at the origin, the C-O bond length is 2.0871 bohr,  $r_{\langle r \rangle}$  for C is 0.53690 bohr, and  $r_{\langle r \rangle}$  for O is 0.39719 bohr.

### 3.2.3 Effect of Choosing the Core Size

The Fermi-Dirac, triangle, and Awad weights depend on the core sizes of the isolated atoms. In subsection 3.2.1,  $r_{max}$ ,  $r_{min}$ , and  $r_{\langle r \rangle}$  have been suggested as core sizes for Fermi-Dirac, triangle, and Awad models. In this section, we study the effect of choosing the core size on Awad weight visually and numerically. The visualization study includes RDEN and BDEN, whereas the numerical study includes comparing the following molecular properties: number of electrons, potential energy, and Coulomb energy.

#### 3.2.3.1 Molecular Radial and Bond Electron Density

Besaw et al. [3] defined a bond as the region of space between a pair of atoms where the joint probability of finding electron density owned by each atom is nonzero. In the bonding region, the weighting functions of both nuclei ( $W_A$  and  $W_B$ ) must be nonzero, since both the space and electron density are shared. Using  $W_A$  and  $W_B$ , the BDEN ( $\rho_{A-B}(\mathbf{r})$ ) can be defined as,

$$\rho_{A-B}(\mathbf{r}) = \sqrt{W_A(\mathbf{r})W_B(\mathbf{r})}\rho(\mathbf{r}) \quad (3.1)$$

where  $\rho(\mathbf{r})$  is the molecular electron density at  $\mathbf{r}$  and the radial bond electron density ( $\rho_{\text{rad}A-B}(\mathbf{r})$ ) is defined as follows,

$$\rho_{\text{rad}A-B}(\mathbf{r}) = \sqrt{W_A(\mathbf{r})W_B(\mathbf{r})}\rho_{\text{rad}}(\mathbf{r}) \quad (3.2)$$

where  $\rho_{\text{rad}}(\mathbf{r})$  is the molecular radial electron density at  $\mathbf{r}$ .

We used two main ways of visualizing RDEN and BDEN in this thesis: the relief plot and the contour plot. The relief plot is a 3D graph that represents the cartesian location along

the plane of the molecule in the  $x$  and  $y$  directions, and the radial or bond electron densities in the  $z$ -direction. The contour plots are 2D figures that display curves of constant radial or bond electron densities. Since this section has many plots, some plots have been placed in Appendix A.

The studied molecules include CO, H<sub>2</sub>, N<sub>2</sub>, Cl<sub>2</sub>, FCl, LiCl, LiF, HCl, HF, LiH, and NaH. These molecules are divided into two groups, the homonuclear diatomic molecules (H<sub>2</sub>, N<sub>2</sub>, and Cl<sub>2</sub>) and the heteronuclear diatomic molecules (CO, FCl, HF, HCl, LiCl, LiF, LiH and NaH). Geometries of these molecules are optimized using HF/6-31G(d). The RDEN and BDEN of the studied molecules using the Awad weight are calculated on a mesh size of 0.02 bohr.

Besaw et al. [3] showed that RDEN is rich in topological features such as critical points, critical rings, and critical spheres. The classification (point, ring, sphere) and magnitude (maximum, minimum, or saddle) of a critical feature can be found by appraising the diagonalized Hessian of radial density at these points to yield three characteristic eigenvalues ( $\lambda_1, \lambda_2, \lambda_3$ ). Table 3.2 summarizes the critical architectures of RDEN. In this table, the rank ( $r$ ) and the signature ( $s$ ) are the number of nonzero eigenvalues and the sum of the signs of the three eigenvalues (+1 for a positive eigenvalue,  $-1$  for a negative eigenvalue, or zero for a vanishing eigenvalue) respectively. A rank of one, two, or three represents a critical sphere, ring, and point, respectively. The signature relative to the rank determines whether the topological feature is a maximum, minimum, or saddle. If  $s = +r$  a minimum results;  $s = -r$  a maximum results; if  $-r < s < r$ , a saddle results. The trivial case occurs when both  $s$  and  $r = 0$ .

Table 3.3 gives nuclear coordinates for atoms of the studied molecules and the magnitudes and positions of maxima and minima for the molecular radial density in the bonding region

**Table 3.2:** Molecular radial density: Critical architectures [3].

| Architecture | Name             | Acronym | $\lambda_1$ | $\lambda_2$ | $\lambda_3$ | r | s  |
|--------------|------------------|---------|-------------|-------------|-------------|---|----|
| Point        | Max point        | PX      | -           | -           | -           | 3 | -3 |
|              | Saddle max point | PSX     | -           | -           | +           | 3 | -1 |
|              | Saddle min point | PSM     | -           | +           | +           | 3 | +1 |
|              | Min point        | PM      | +           | +           | +           | 3 | +3 |
| Ring         | Max ring         | RX      | -           | -           | 0           | 2 | -2 |
|              | Saddle ring      | RS      | -           | 0           | +           | 2 | 0  |
|              | Min ring         | RM      | 0           | +           | +           | 2 | +2 |
| Sphere       | Max sphere       | SX      | -           | 0           | 0           | 1 | -1 |
|              | Min sphere       | SM      | 0           | 0           | +           | 1 | +1 |
| Trivial Case | Infinite point   | IP      | 0           | 0           | 0           | 0 | 0  |

along the internuclear axis obtained at HF/6-31G(d) using the Awad weight with  $r_{min}$ ,  $r_{max}$ , and  $r_{\langle r \rangle}$ . The maximum and minimum points along the bond have been determined accurately using Mathematica's FindMaximum and FindMinimum functions. The results in Table 3.3 show that positions and values of maximum peaks for RDENs in bonding regions for homonuclear molecules do not depend on the derived core. Whereas for heteronuclear molecules (one exception is HF), values of maximum peaks for RDENs are directly proportional to the size of cores, in which the  $r_{min}$  core have the largest values and  $r_{max}$  core have the smallest values.

**Homonuclear Diatomic Molecules:** Three homonuclear diatomic molecules;  $H_2$ ,  $N_2$ , and  $Cl_2$  are considered in this study. For the  $H_2$  molecule, RDEN and BDEN do not depend on the type of the core chosen since the H atom does not have a core and only has a valence shell. Therefore, RDEN and BDEN of  $H_2$  are composed of only distorted valence

**Table 3.3:** The maximum and minimum of molecular radial electron density (RDEN) along internuclear axis of the studied diatomic molecules in bonding region. These maximum and minimum are obtained using  $r_{min}$ ,  $r_{max}$ , and  $r_{r'}$ .

| Molecule<br>(Coordinates, bohr)     | $r_{min}$                 |                                     | $r_{max}$                 |                                     | $r_{r'}$                  |                                     |
|-------------------------------------|---------------------------|-------------------------------------|---------------------------|-------------------------------------|---------------------------|-------------------------------------|
|                                     | Maximum<br>(Coordinates*) | Minimum<br>(Coordinates*)           | Maximum<br>(Coordinates*) | Minimum<br>(Coordinates*)           | Maximum<br>(Coordinates*) | Minimum<br>(Coordinates*)           |
| N <sub>2</sub> (N 0.0, N 2.0229)    | 0.7403 (1.0115)           | 0.2225 (0.4675)                     | 0.7403 (1.0115)           | 0.2258 (0.4612)                     | 0.7403 (1.0115)           | 0.2225 (0.4675)                     |
| Cl <sub>2</sub> (Cl 0.0, Cl 3.7600) | 0.5530 (1.8800)           | 0.2498 (0.9122)                     | 0.5530 (1.8800)           | 0.2538 (0.9035)                     | 0.5530 (1.8800)           | 0.2498 (0.9122)                     |
| CO (C 0.0, O 2.0871)                | 0.7848 (1.1431)           | 0.1857 (0.5388),<br>0.2601 (1.6736) | 0.7408 (1.1359)           | 0.1896 (0.5300),<br>0.2621 (1.6771) | 0.7598 (1.1301)           | 0.1857 (0.5388),<br>0.2601 (1.6736) |
| FCI (F 3.0503, Cl 0.0)              | 0.6978 (1.8308)           | 0.2278 (0.9265),<br>0.2486 (2.6632) | 0.5980 (1.8206)           | 0.2349 (0.9136),<br>0.2489 (2.6637) | 0.6974, (1.8314)          | 0.2278 (0.9265),<br>0.2486 (2.6632) |
| HCl (H 2.4000, Cl 0.0)              | 0.4966 (1.4471)           | 0.2826 (0.8767)                     | 0.4140 (1.2451)           | 0.2935 (0.8643)                     | 0.4957 (1.4453)           | 0.2826 (0.8767)                     |
| HF (H 1.6960, F 0.0)                | 0.5082 (0.8404)           | 0.2760 (0.3770)                     | 0.5358 (0.7630)           | 0.2780 (0.3738)                     | 0.5082 (0.8394)           | 0.2760 (0.3770)                     |
| LiH (Li 0.0, H 3.0390)              | 0.4281 (2.2964)           | 0.0588 (1.2002)                     | 0.1117 (1.8370)           | 0.0628 (1.1835)                     | 0.2009 (2.0446)           | 0.0588 (1.2002)                     |
| Na (Na 0.0, H 3.6261)               | 3.0347 (3.4634)           | 0.1021 (1.7602)                     | 0.1155 (2.2375)           | 0.1032 (1.8243)                     | 0.2507 (2.5417)           | 0.1021 (1.7602)                     |
| H                                   |                           |                                     |                           |                                     |                           |                                     |
| LiCl (Li 0.0, Cl 3.8492)            | 0.7249 (2.3201)           | 0.0618 (1.1697),<br>0.2884 (2.9704) | 0.4820 (2.3546)           | 0.0652 (1.1462),<br>0.2917 (2.9772) | 0.4610 (2.2367)           | 0.0618 (1.1697),<br>0.2884 (2.9704) |
| LiF (Li 0.0, F 2.9777)              | 2.5613 (2.1282)           | 0.0847 (1.0296),<br>0.3195 (2.6159) | 0.6275 (2.0087)           | 0.0886 (1.0114),<br>0.3197 (2.6162) | 0.9107 (1.9374)           | 0.0847 (1.0296),<br>0.3195 (2.6158) |

\* All the coordinates (bohr) are along internuclear axis.

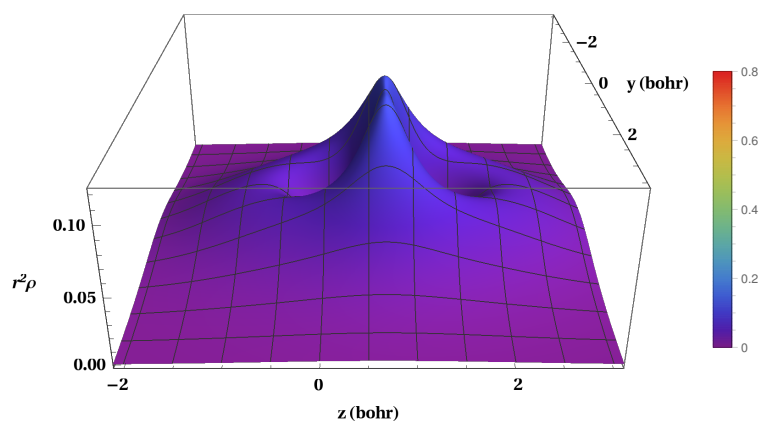
shells as shown in Figure 3.5. Figure 3.5a shows a relief map of the RDEN in the plane of  $H_2$ . Figure 3.5b is another way of displaying these results as a contour plot. Figure 3.5c shows 1D-plot of the RDEN along the internuclear axis. In these figures the RDEN of  $H_2$  molecule was obtained using the Awad weight and the atomic coordinates of the two H atoms are 0.0 and 1.3895 bohr, respectively.

The next two molecules studied are  $N_2$ , and  $Cl_2$ . Unlike the H atom, the atomic radial density of the individual N and Cl atoms possess both valence and core shells. The radial density of a single N atom has one core shell ( $n = 1$ ) and one valence shell ( $n = 2$ ), whereas the radial density of a single chlorine atom has two core shells ( $n = 1, 2$ ) and a valence shell ( $n=3$ ). Therefore, it is expected that the RDEN of  $N_2$  and  $Cl_2$  would differ from that of  $H_2$  whose atomic constituents only have a valence shell.

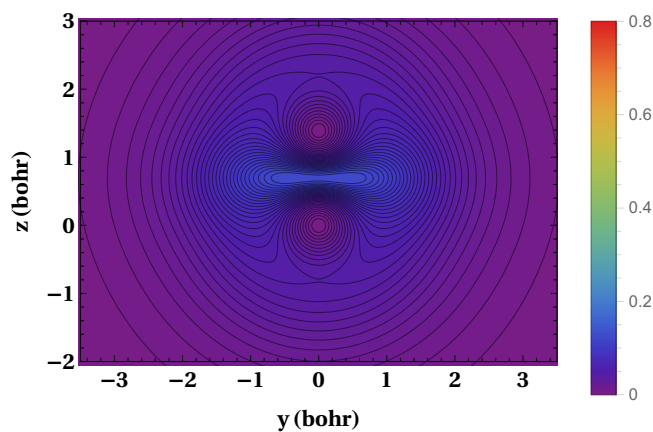
Figure 3.6a, b, and c represent contour plots and 1D-plots (along the internuclear axis) of molecular radial electron density for  $N_2$  obtained using the Awad weight at  $r_{max}$ ,  $r_{min}$ , and  $r_{\langle r \rangle}$ , respectively. For  $N_2$ , the values of  $r_{max}$ ,  $r_{min}$ , and  $r_{\langle r \rangle}$  are 0.148748, 0.500878, and 0.456610 bohr, respectively. In this figure, N atoms of the optimized  $N_2$  structure are located at 0.0 bohr and 2.0229 bohr along the  $z$  axis.

As mentioned in section 1.4.2.3, any molecule can be partitioned using RDEN to three regions: non-bonding, core, and bonding regions. As shown in Figure 3.6 the non-bonding region of  $N_2$  molecule has one maximum point on each atom and the positions and RDEN at these points remain invariant when using  $r_{min}$ ,  $r_{\langle r \rangle}$ , and  $r_{max}$  because of Awad weight is always one or zero in this region. The core region of  $N_2$  molecule contains a pair of maxima peaks adjacent to each nucleus. These peaks located at  $z = -0.1491, 0.1504, 1.8725$ , and  $2.1720$  bohr with  $0.5841, 0.5908, 0.5908$ , and  $0.5841$  e/bohr radial density, respectively by using  $r_{min}$ ,  $r_{\langle r \rangle}$ , and  $r_{max}$ . The bonding region contains one maximum located exactly

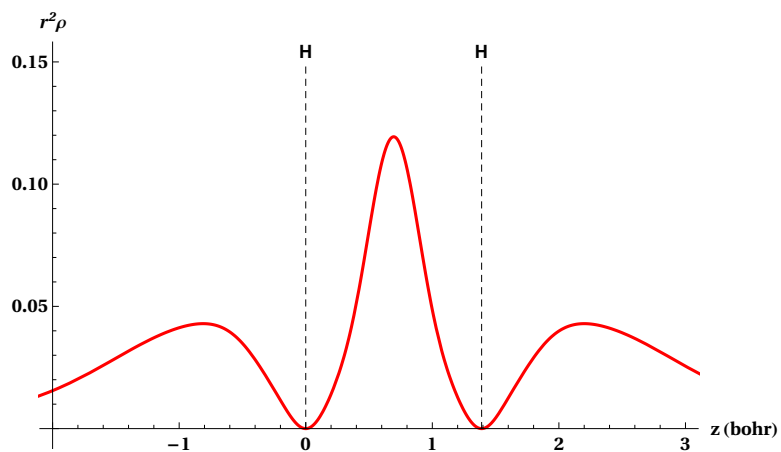




(a) 3D plot

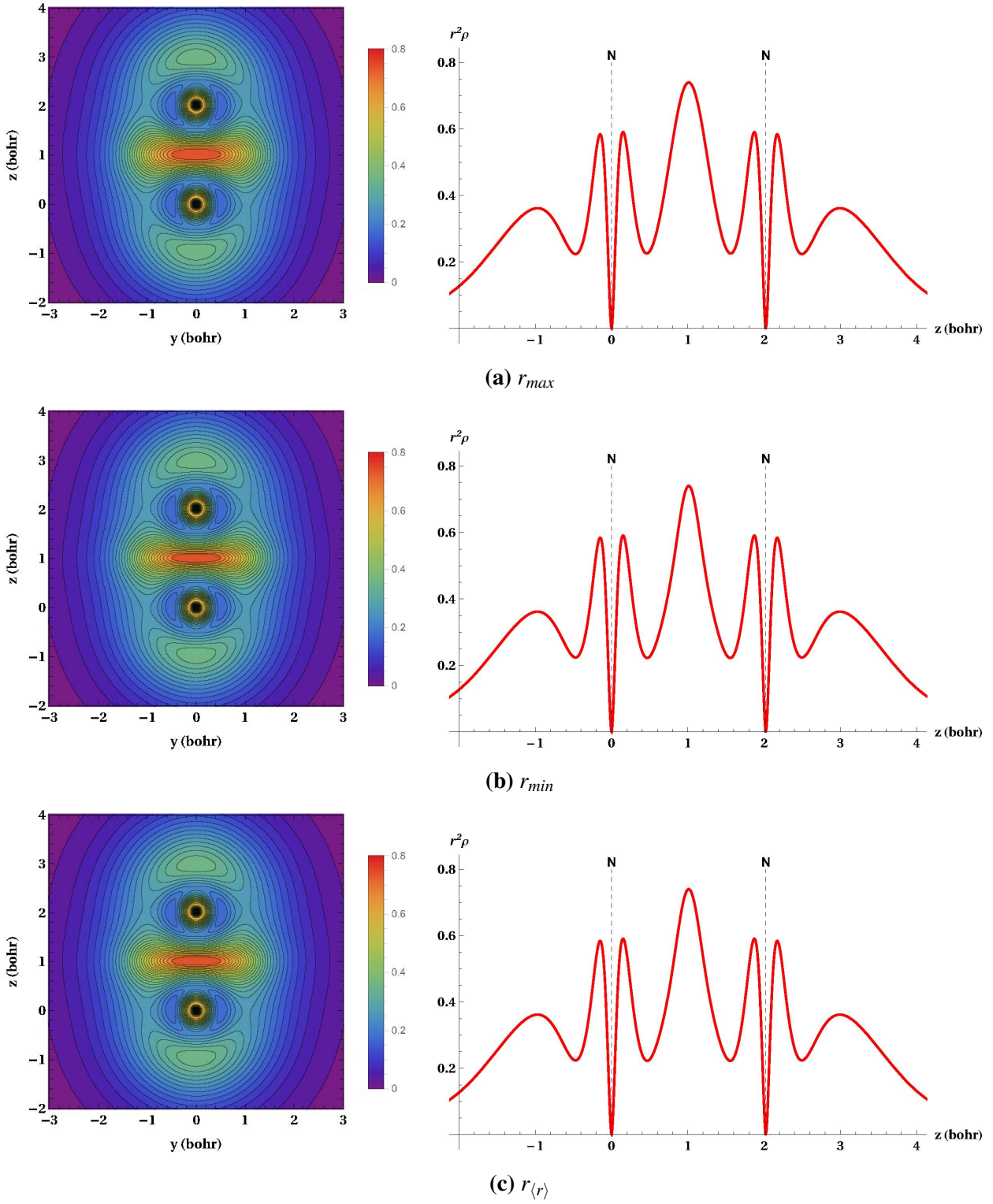


(b) Contour plot.



(c) Along the internuclear axis.

**Figure 3.5:** Molecular radial electron density (RDEN) of  $\text{H}_2$  molecule. The atomic coordinates of the two H atoms are 0.0 and 1.3895 bohr along  $z$  axis.



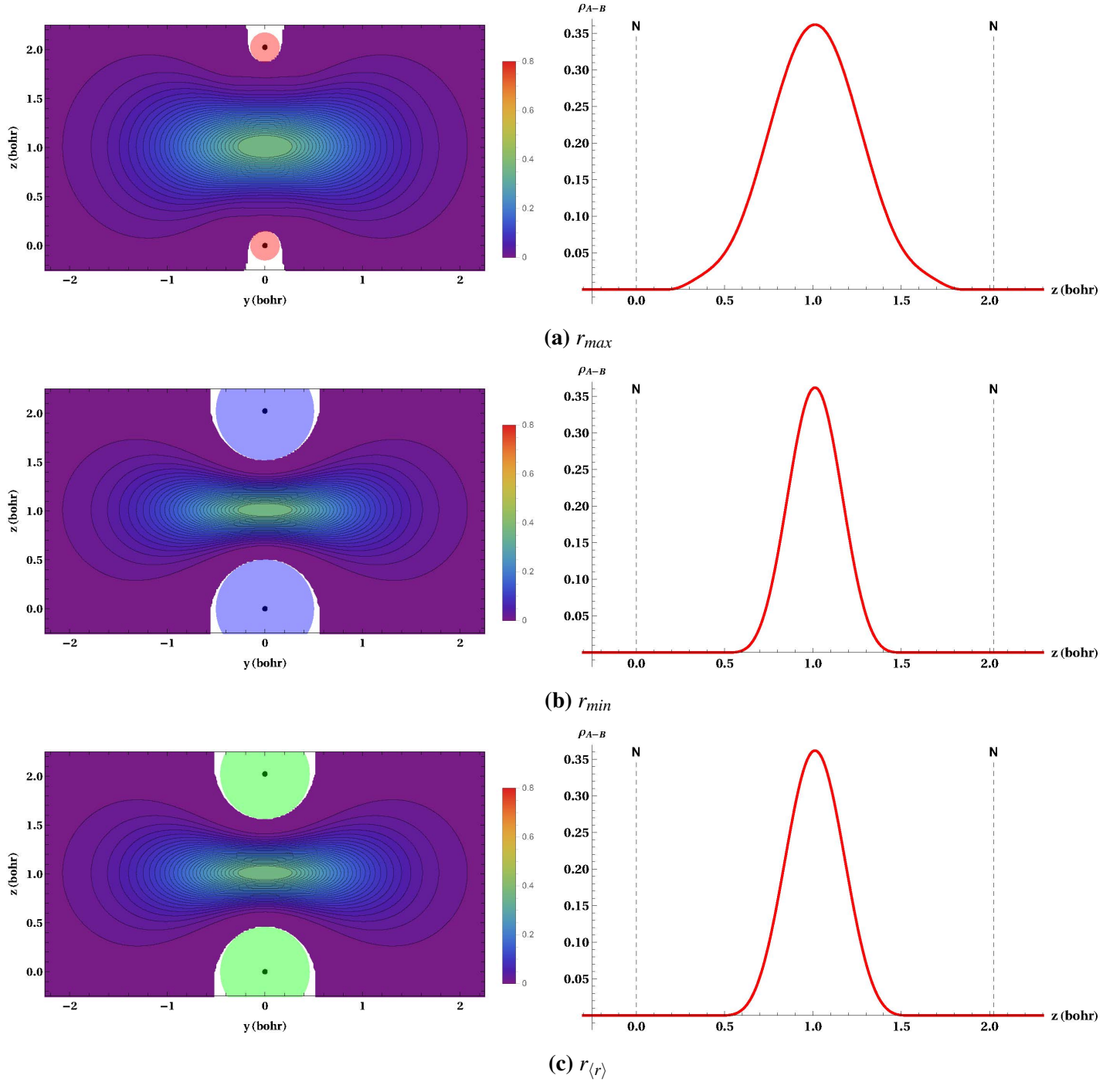
**Figure 3.6:** Molecular radial electron density (RDEN) for  $N_2$  is obtained using the Awad weight at: (a)  $r_{max}$ , (b)  $r_{min}$ , and (c)  $r_{(r)}$ . The atomic coordinates of the two N atoms are 0.0 and 2.0229 bohr along  $z$  axis.

between the two nitrogen nuclei at  $z = 1.0114$  bohr with radial density of 0.7492 e/bohr using all  $r_{min}$ ,  $r_{\langle r \rangle}$ , and  $r_{max}$ . The values and positions of these maxima are expected to be the same for  $r_{min}$ ,  $r_{\langle r \rangle}$ , and  $r_{max}$ , due to the symmetry of  $N_2$  and the Awad weight is exactly 0.5 at the middle distance of the  $N_2$  bond. Unlike positions of minima located at the non-bonding region, the positions of the two minima located between the nuclei and hence their respective RDENs vary by changing the suggested cores. When  $r_{max}$  is used as a core, the points appear at  $z = 0.4612$  and  $1.5617$  bohr with the same radial density of 0.2258 e/bohr, whereas their positions become at  $z = 0.4675$  and  $1.5554$  bohr with 0.2225 e/bohr radial density when using  $r_{\langle r \rangle}$  and  $r_{min}$ .

Figure 3.7a, b, and c show contour plots and 1D-plots (along the internuclear axis) of the BDEN for  $N_2$  molecule obtained using the Awad weight with cores defined at  $r_{max}$  (red disk),  $r_{min}$  (blue disk), and  $r_{\langle r \rangle}$  (green disk), respectively. It should be noted that  $r_{min}$ ,  $r_{\langle r \rangle}$ , and  $r_{max}$  of the  $N_2$  radial densities have smooth curves with only one maximum for each curve located exactly at the middle of the internuclear axis with 0.3662 e/bohr radial density. It is obvious from Figure 3.7 that using the smallest suggested core size ( $r_{max}$ ) gives the widest BDEN's peak relative to those of the other two cores.

Figure 3.8a, b, and c represent contour plots and 1D-plots (a long the internuclear axis) of RDEN for  $Cl_2$  obtained using Awad weight with cores defined at  $r_{max}$ ,  $r_{min}$ , and  $r_{\langle r \rangle}$ , respectively. Since  $Cl_2$  and  $N_2$  are both homonuclear diatomic molecules, it is expected that they will exhibit similar molecular radial density distributions. One of the expected differences is that Cl atom has two core shells, whereas N atom has only a single core shell. As shown in Figure 3.8, the chlorine nuclei are located at  $z = 0.0$  and  $z = 3.7600$  bohr.

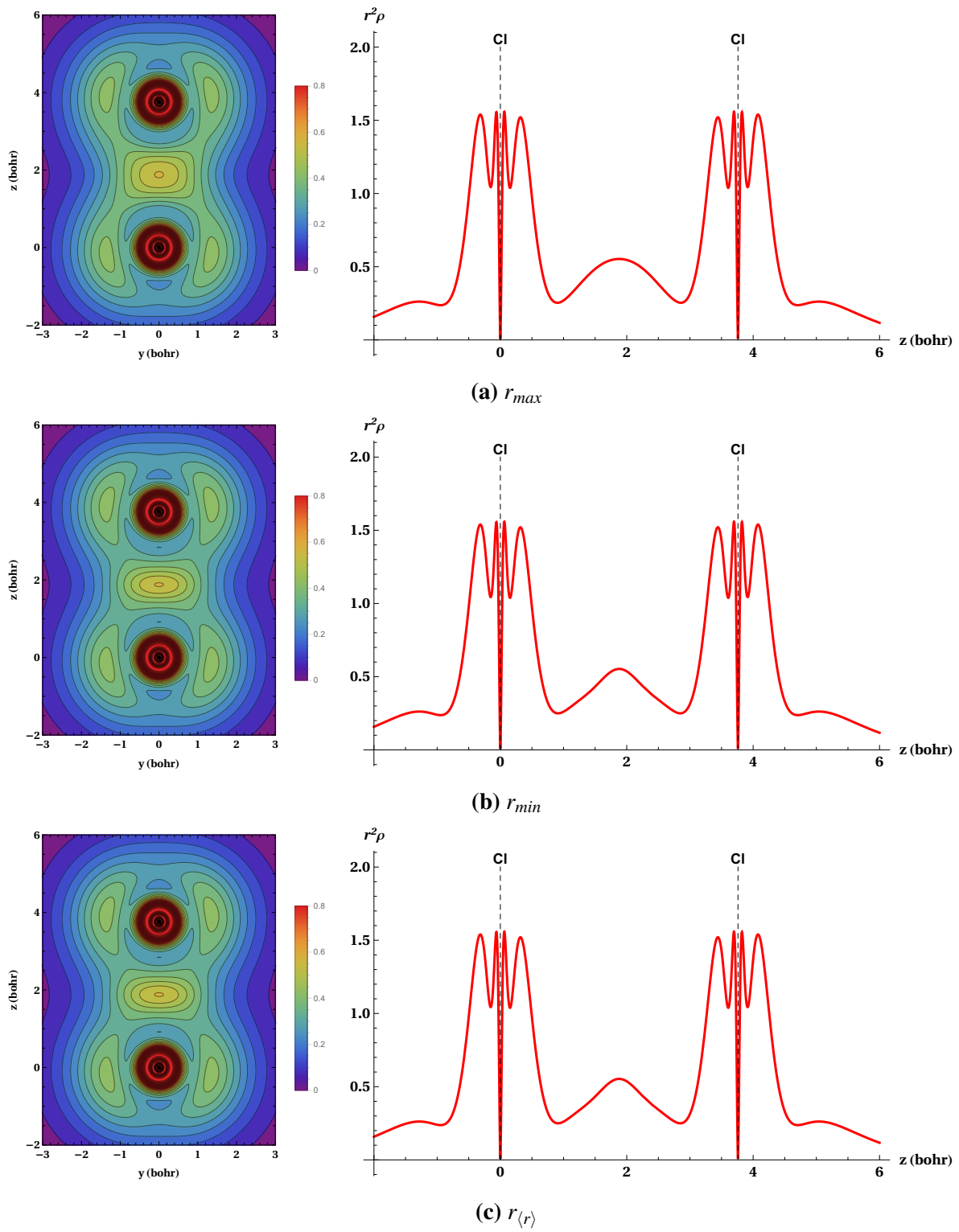
Adjacent to each nucleus along the bond, there are two pairs of maxima and one pair of



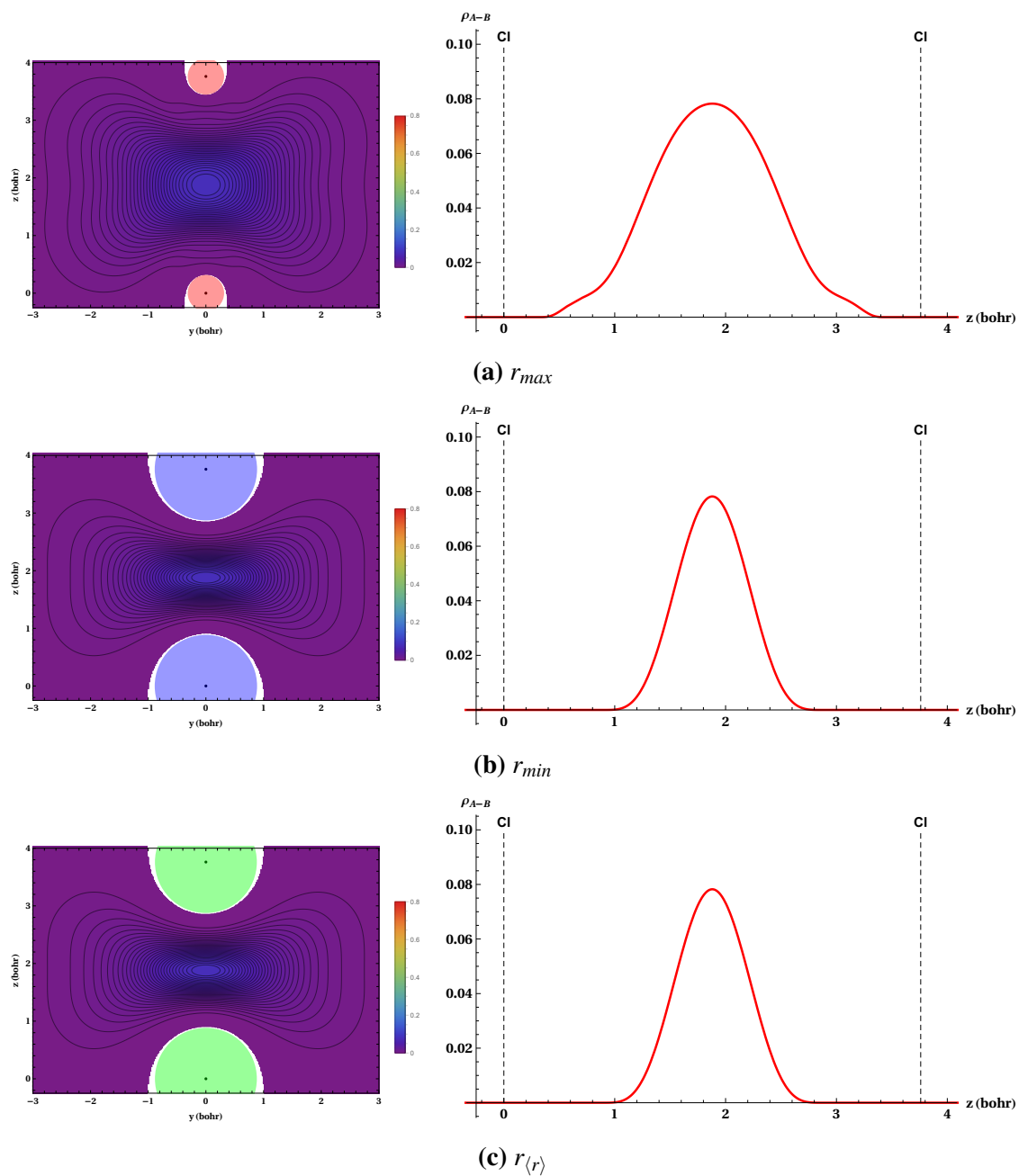
**Figure 3.7:** Bond electron density (BDEN) for  $N_2$  is obtained using the Awad weight at: (a)  $r_{max}$ , (b)  $r_{min}$ , and (c)  $r_{\langle r \rangle}$ . The atomic coordinates of the two N atoms are 0.0 and 2.0229 bohr along  $z$  axis. The red, purple, and green disks are the core regions of  $r_{max}$ ,  $r_{min}$ , and  $r_{\langle r \rangle}$ , respectively.

minima. For all different suggested core sizes, these maxima and minima are associated with the two core shells of a chlorine nucleus. Because the Awad weights within the core region for  $r_{max}$ ,  $r_{min}$ , and  $r_{\langle r \rangle}$  are similar, the maxima and minima are located exactly in same positions within this region. The maxima peaks along the bond within the cores are located at  $z = -0.3168, -0.0627, 0.0628,$  and  $0.3177$  bohr with  $1.5394, 1.5565, 1.5591,$  and  $1.5208$  e/bohr radial density, respectively. The minima peaks along the bond within the cores are located at  $z = -0.1526$  and  $0.1537$  bohr with  $1.0436$  and  $1.0386$  e/bohr radial density, respectively.

Similar to  $N_2$  molecule, the non-bonding region has one maximum point on each atom and the positions and RDEN at these points remain invariant when using  $r_{min}$ ,  $r_{\langle r \rangle}$ , and  $r_{max}$  because the Awad weight is always one or zero in this region. In addition, within the bonding region there is only one maximum located exactly in the middle of the bond with  $0.5530$  e/bohr radial density with  $r_{max}$ ,  $r_{min}$ , and  $r_{\langle r \rangle}$ . Unlike positions of minima located at the non-bonding region, the positions of the two minima are located between the nuclei and hence their respective RDEN vary by changing the core size. For  $r_{max}$ , the minimum points are located at  $0.9034$  and  $2.8566$  bohr with  $0.2538$  e/bohr radial density and for both  $r_{min}$  and  $r_{\langle r \rangle}$ , they are located at  $0.9122$  and  $2.8478$  bohr with  $0.2500$  e/bohr radial density. Figure 3.9 shows the BDEN for  $Cl_2$  molecule using the Awad weight at  $r_{min}$ ,  $r_{\langle r \rangle}$ , and  $r_{max}$ . In this figure, the BDEN along the bond using  $r_{max}$  has minor shoulders. As  $r_{max}$  is so small to be used as a core, these shoulders may come from the electron density of the core. In addition, BDEN contour plots of  $r_{min}$ , and  $r_{\langle r \rangle}$  (Figures 3.9b and c) give smoother contour lines comparing to that of  $r_{max}$  (Figure 3.9a). For these reasons  $r_{min}$  and  $r_{\langle r \rangle}$  give better representations of the BDEN than  $r_{max}$ .



**Figure 3.8:** Molecular radial electron density (RDEN) for  $\text{Cl}_2$  obtained using Awad weight at: (a)  $r_{\max}$ , (b)  $r_{\min}$ , and (c)  $r_{\langle r \rangle}$ . The atomic coordinates of the two Cl atoms are 0.0 and 3.7600 bohr along  $z$  axis.



**Figure 3.9:** Bond electron density (BDEN) for  $\text{Cl}_2$  obtained using Awad weight at: (a)  $r_{\max}$ , (b)  $r_{\min}$ , and (c)  $r_{\langle r \rangle}$ . The atomic coordinates of the two Cl atoms are 0.0 and 3.7600 bohr along  $z$  axis.

**Heteronuclear Diatomic Molecules:** We consider the following heteronuclear diatomic molecules: CO, FCl, HF, HCl, LiCl, LiF, LiH and NaH. Although the RDENs and BDENs of the heteronuclear molecules are less uniform than those of the previously discussed homonuclear molecules ( $H_2$ ,  $N_2$  and  $Cl_2$ ), some very similar features are observed. The similarities and differences between these homonuclear and heteronuclear diatomics can be observed by comparing the contour plots and 1D-plots (along the internuclear axis) of these diatomic molecules.

The topology of the RDEN and BDEN were studied along the bond. It was found that the minimum and maximum peaks within the core and non-bonding regions are the same (i.e., have the same locations and values) for  $r_{max}$ ,  $r_{min}$ , and  $r_{\langle r \rangle}$ . This result is expected because the weight is one in the core and non-bonding regions for the target atom and zero for other atoms. In contrast, the locations and values of the maximum and minimum peaks in the bonding region depend on the core size used.

For example, Figure 3.10 shows RDEN for CO obtained using the Awad weight at  $r_{min}$ ,  $r_{\langle r \rangle}$ , and  $r_{max}$ . In this figure, the C atom is located at 0.0 bohr and the O atom is located at 2.0871 bohr along  $z$  axis. Each core region of the CO molecule contains a pair of maximum peaks adjacent to each nucleus. Using  $r_{min}$ ,  $r_{\langle r \rangle}$ , and  $r_{max}$ , the maximum peaks adjacent to the C nucleus are located at -0.1722 and 0.1759 bohr with 0.4948 and 0.5006 e/bohr radial density respectively, whereas the maximum peaks adjacent to O nucleus are located at -0.1313 and 0.1301 bohr (relative to O nucleus) with 0.6811 and 0.6745 e/bohr radial density respectively. The non-bonding region for each nucleus contains one maximum peak that has the same location and RDEN using  $r_{min}$ ,  $r_{\langle r \rangle}$ , and  $r_{max}$ . The maximum peaks of the non-bonding regions are located at  $z = -1.1871$  bohr with RDEN of 0.2802 e/bohr for C atom and at 2.9212 bohr with RDEN of 0.4390 e/bohr for O atom. Similar to the homonu-

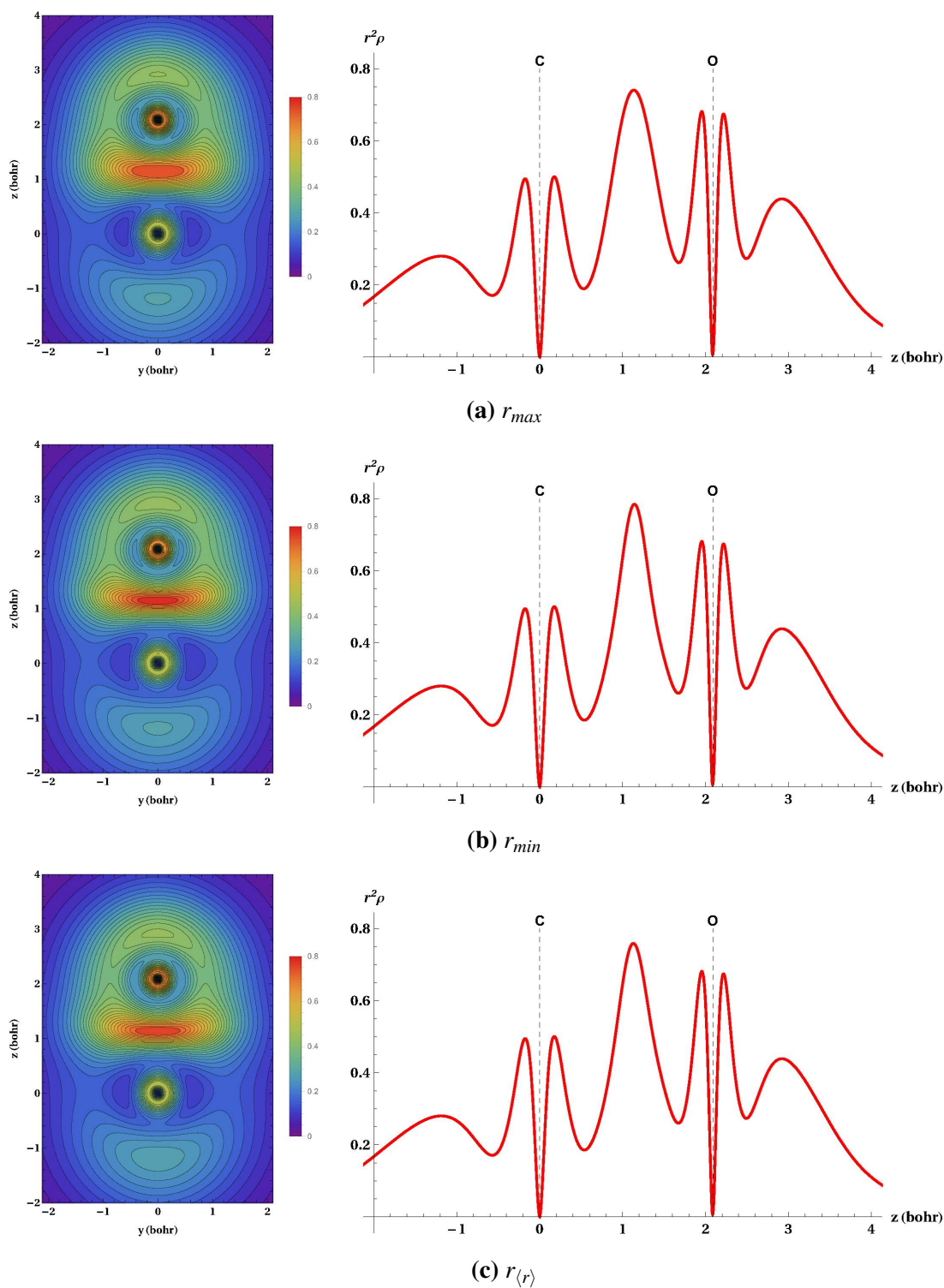


clear molecules, the minima that separate the core and the non-bonding regions have the same location and RDEN for the two atoms using  $r_{max}$ ,  $r_{min}$ , and  $r_{\langle r \rangle}$ . These minima are located at  $-0.5710$  bohr with RDEN of  $0.1710$  e/bohr for C atom and at  $0.4055$  bohr (relative to O nucleus) with RDEN of  $0.2733$  e/bohr for O atom.

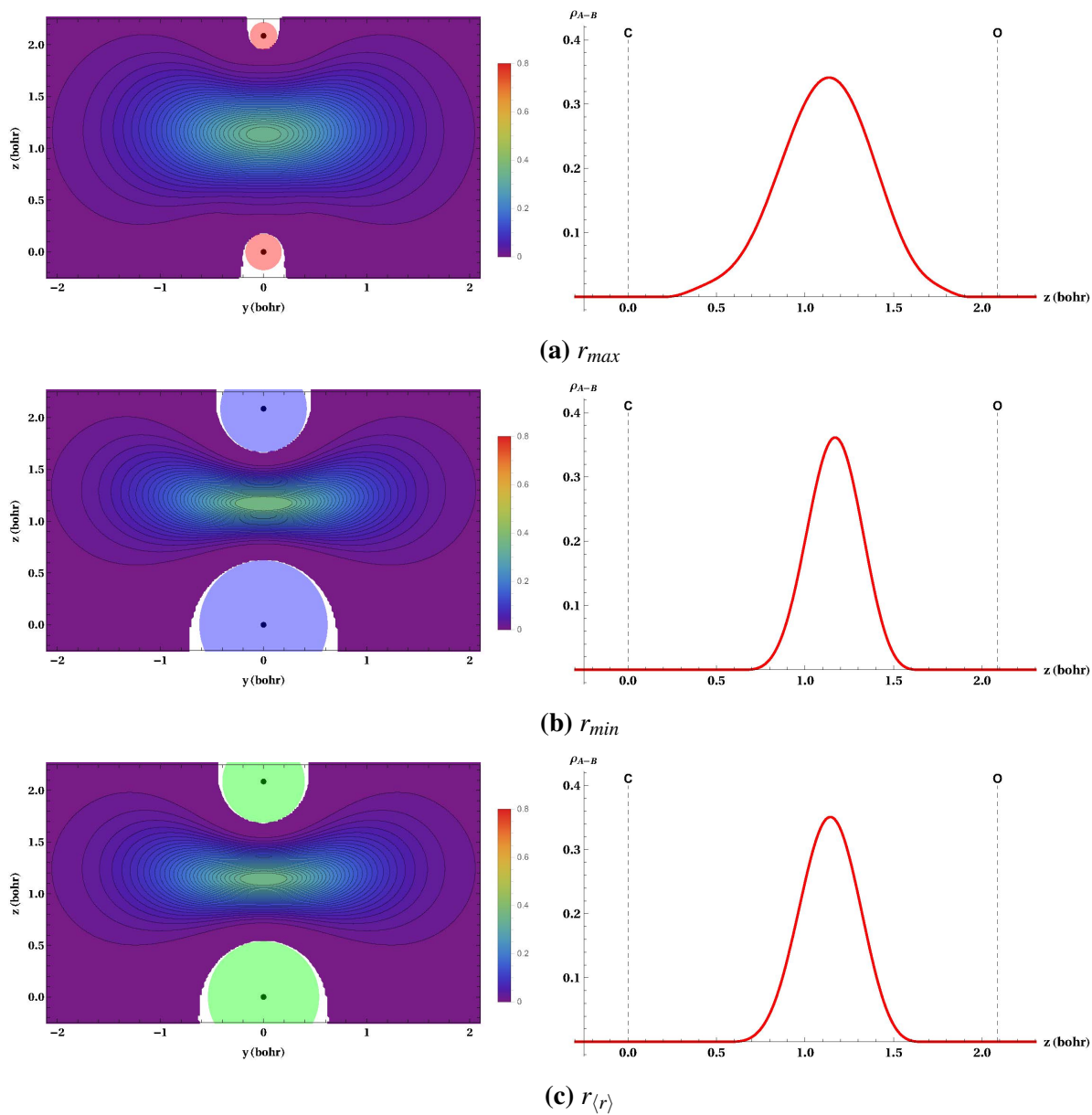
Finally, there is one maximum peak in the bonding region of the CO molecule. The location and value of this peak depend on choosing the core. For example, the maximum peaks are located at  $z = 1.1301, 1.1359, \text{ and } 1.1431$  bohr with  $0.7589, 0.7408, \text{ and } 0.7848$  e/bohr radial densities for  $r_{\langle r \rangle}$ ,  $r_{max}$ , and  $r_{min}$ , respectively. Also, the positions of the two minima that separate the core and bonding regions and hence their respective RDEN vary by changing the core sizes. For  $r_{max}$ , the minimum points are located at  $z = 0.5300$  and  $1.6771$  with RDENs of  $0.1896$  and  $0.2621$  respectively, and for both  $r_{min}$  and  $r_{\langle r \rangle}$ , they are located at  $z = 0.5388$  and  $1.6736$  bohr with RDENs of  $0.1857$  and  $0.2601$  e/bohr respectively.

Figure 3.11a, b, and c show contour plots and 1D-plots (along the internuclear axis) of the BDEN for CO molecule obtained using the Awad weight at  $r_{max}$  (red disk),  $r_{min}$  (blue disk), and  $r_{\langle r \rangle}$  (green disk), respectively. Similar to the results of RDEN, the positions of maximum peaks of the BDEN for CO depends on choosing the core size. The positions of these maximum peaks are located at  $1.16985, 1.14291, \text{ and } 1.13562$  bohr with  $0.36153, 0.35101, \text{ and } 0.34142$  e/bohr electron densities for  $r_{min}$ ,  $r_{\langle r \rangle}$  and  $r_{max}$ , respectively. The resulting curves for 1D-plots of BDEN along the internuclear axis of CO molecule for  $r_{min}$ ,  $r_{\langle r \rangle}$  and  $r_{max}$  are smooth. In contrast, the contour plot lines of  $r_{max}$  core in Figure 3.11a are less smooth compared to the contour plot lines of  $r_{min}$  and  $r_{\langle r \rangle}$  core sizes. Once again, the use of the smallest core size ( $r_{max}$ ) shifts these minima slightly toward nuclei resulting in a wider bonding region than those obtained with  $r_{min}$  and  $r_{\langle r \rangle}$ .

Figures 3.12 and 3.13 show RDENs for LiH and NaH molecules using the Awad weight



**Figure 3.10:** Molecular radial electron density (RDEN) for CO is obtained using the Awad weight at: (a)  $r_{max}$ , (b)  $r_{min}$ , and (c)  $r_{\langle r \rangle}$ . The atomic coordinates of C at 0.0 bohr and O at 2.0871 bohr along  $z$  axis.



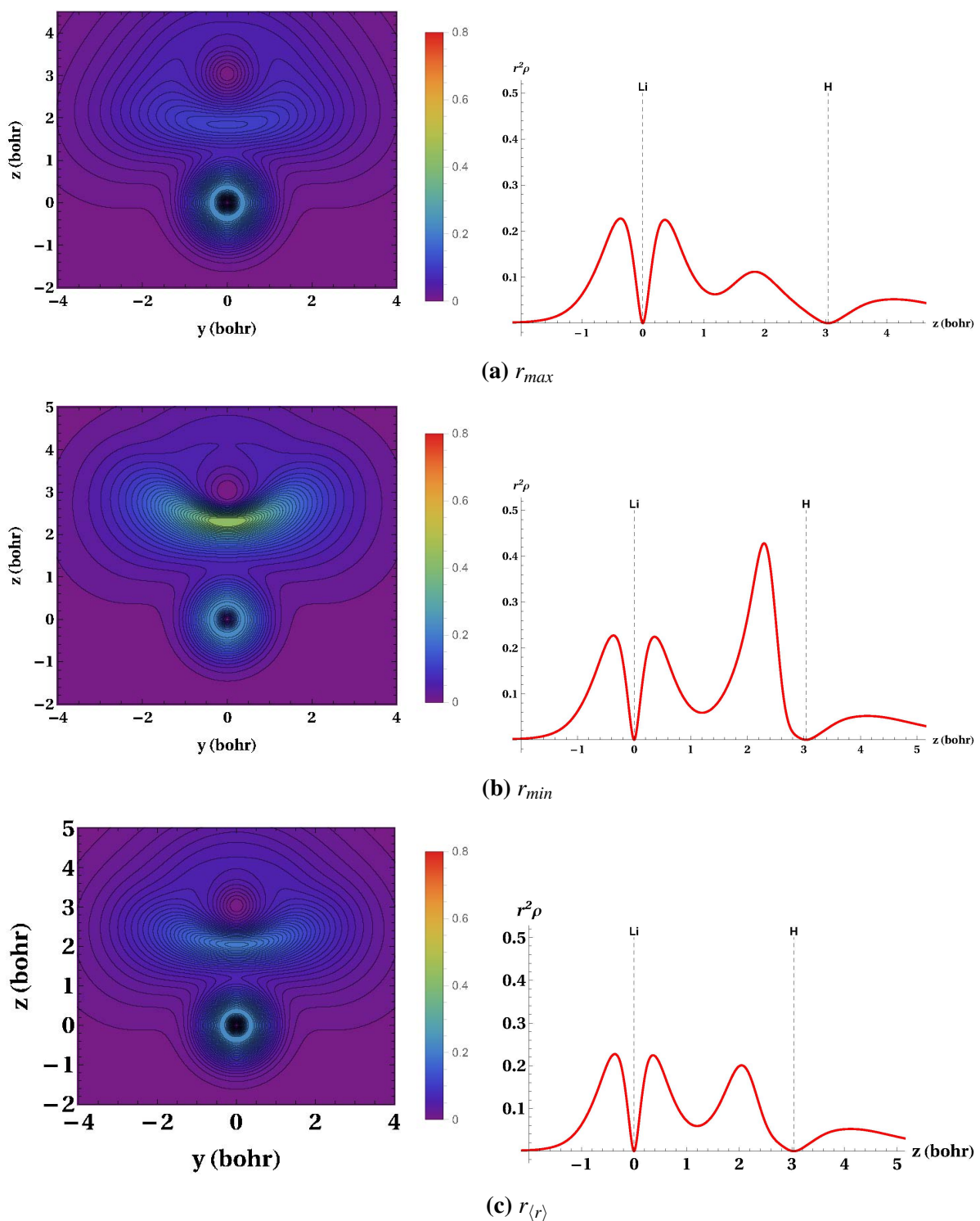
**Figure 3.11:** Bond electron density (BDEN) for CO is obtained using the Awad weight at: (a)  $r_{max}$ , (b)  $r_{min}$ , and (c)  $r_{(r)}$ . The atomic coordinates of C at 0.0 bohr and O at 2.0871 bohr along  $z$  axis. The red, purple, and green disks are the core regions of  $r_{max}$ ,  $r_{min}$ , and  $r_{(r)}$ , respectively.

at different core sizes respectively. In these figures, positions and values of minimum and maximum peaks in the bonding region of the RDEN depend remarkably on choosing of the core. For NaH molecule, positions of these maximum peaks are located at 3.4634, 2.5417, and 2.2375 bohr with 3.0347, 0.2507, and 0.1156 e/bohr electron densities for  $r_{min}$ ,  $r_{\langle r \rangle}$  and  $r_{max}$ , respectively, and for LiH molecule they are located at 2.2964, 2.0446, and 1.8367 bohr with 0.4281, 0.2009, and 0.1166 e/bohr electron densities for  $r_{min}$ ,  $r_{\langle r \rangle}$  and  $r_{max}$ , respectively. Figures 3.14 and 3.15 depict BDENs for NaH and LiH molecules using the Awad weight at different cores respectively. These figures show clearly the significance of choosing the core size as the position, shape, and value of BDENs change remarkably in both NaH and LiH molecules. These changes are more pronounced in molecules containing metal atoms such as Li and Na than molecules contain only non-metallic atoms. Generally for heteronuclear molecules, values of maximum peaks for RDENs are affected by changing the core size, in which the  $r_{min}$  has the highest RDEN values and  $r_{max}$  has the smallest RDEN values. In addition, the curves of BDEN using  $r_{max}$  are wider in width and less smooth than using other core sizes.

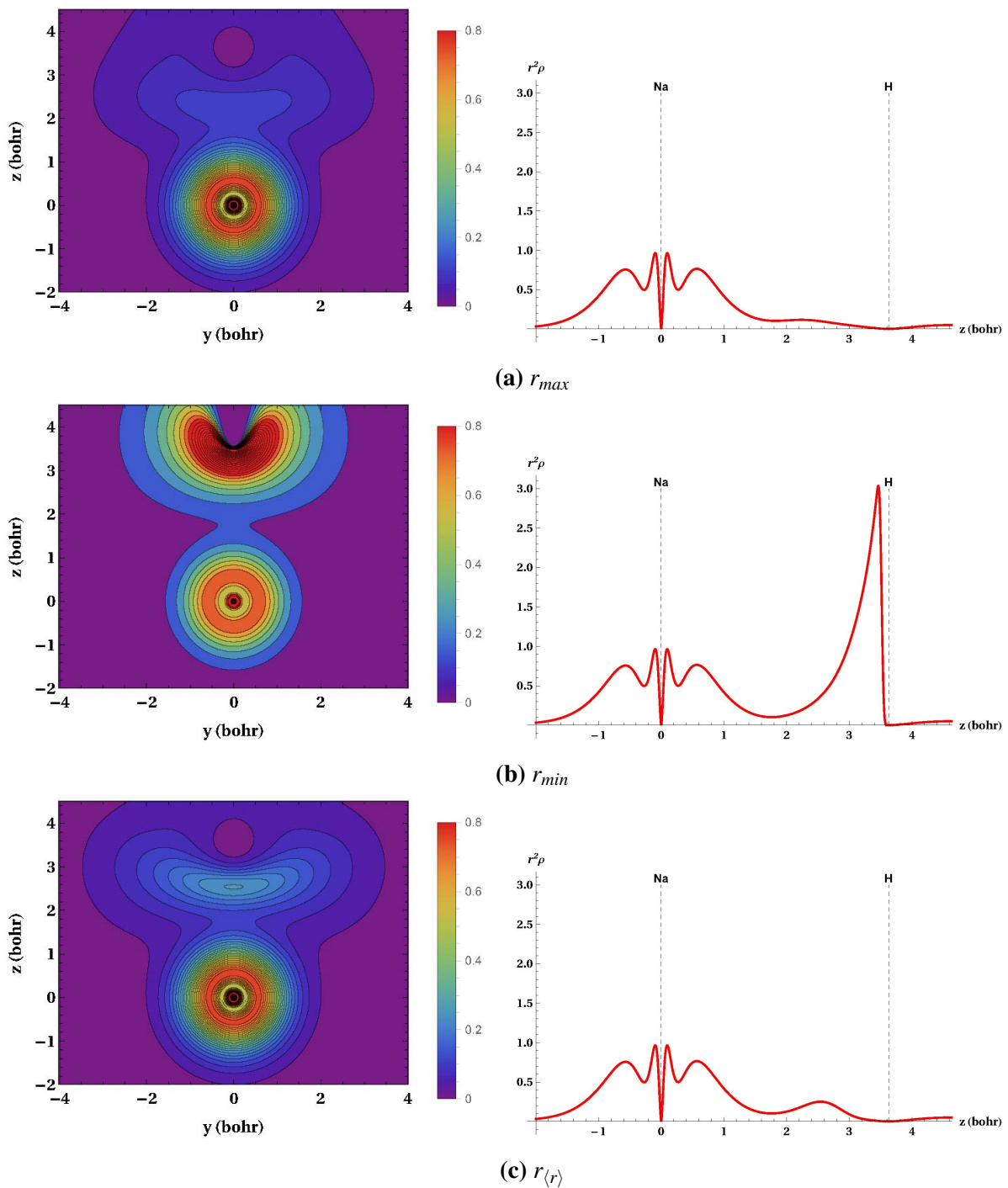
So far we discussed two homonuclear molecules ( $N_2$  and  $Cl_2$ ) and three heteronuclear molecules (CO, LiH, and NaH). The results of the remaining molecules ( $H_2$ , FCl, LiCl, LiF, HCl, and HF) are given in Appendix A.

### 3.2.4 Comparing the Awad and Becke Weights

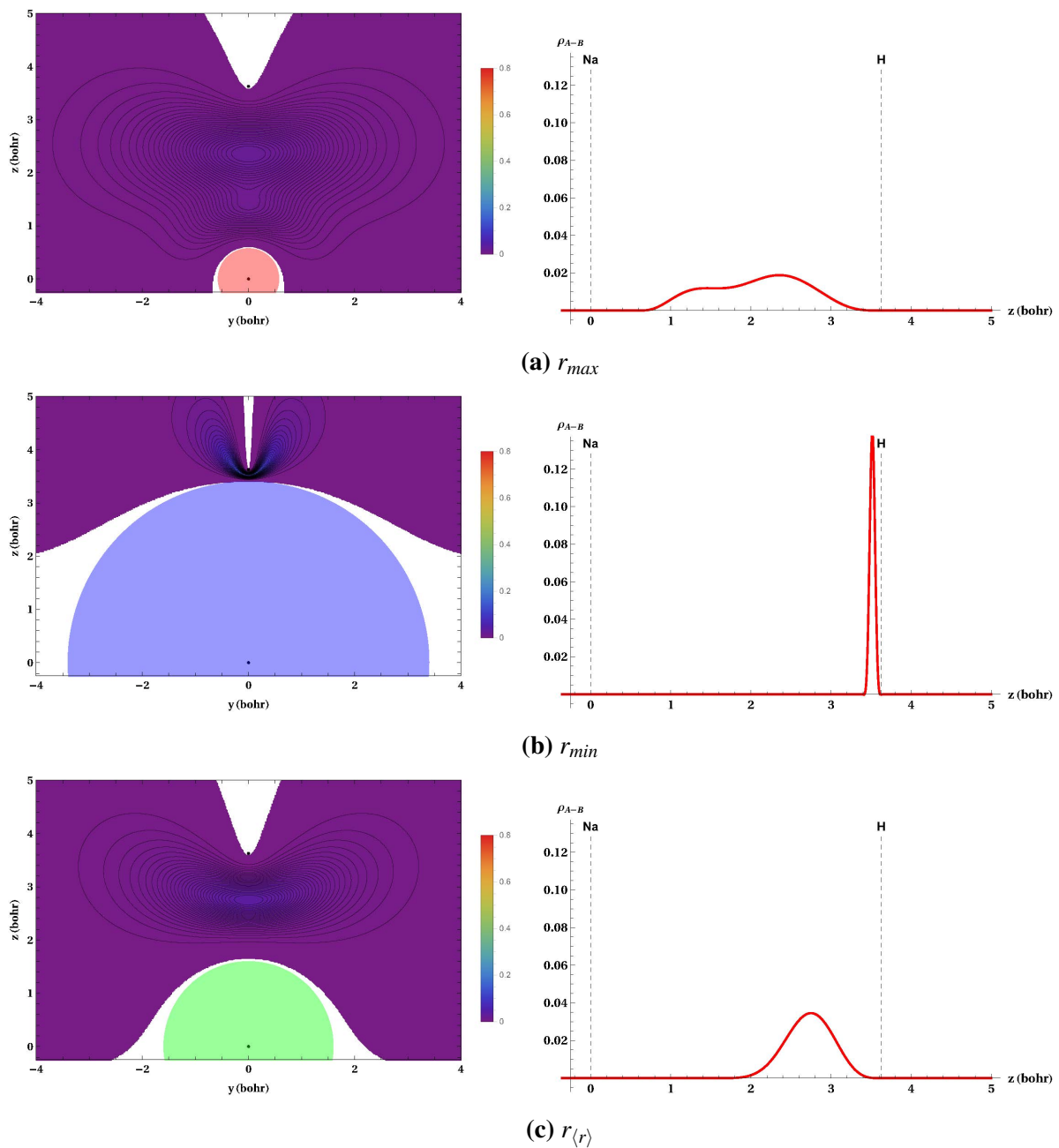
In this section, we compare the molecular RDENs and BDENs obtained using the Awad weight with those obtained using Becke weight. The RDEN and BDEN for LiF molecule using both Becke and Awad weights are shown in Figures 3.16 and 3.17 respectively. We



**Figure 3.12:** Molecular radial electron density (RDEN) for LiH is obtained using the Awad weight at: (a)  $r_{max}$ , (b)  $r_{min}$ , and (c)  $r_{(r)}$ . The atomic coordinates of Li at 0.0 bohr and H at 3.0390 bohr along  $z$  axis.

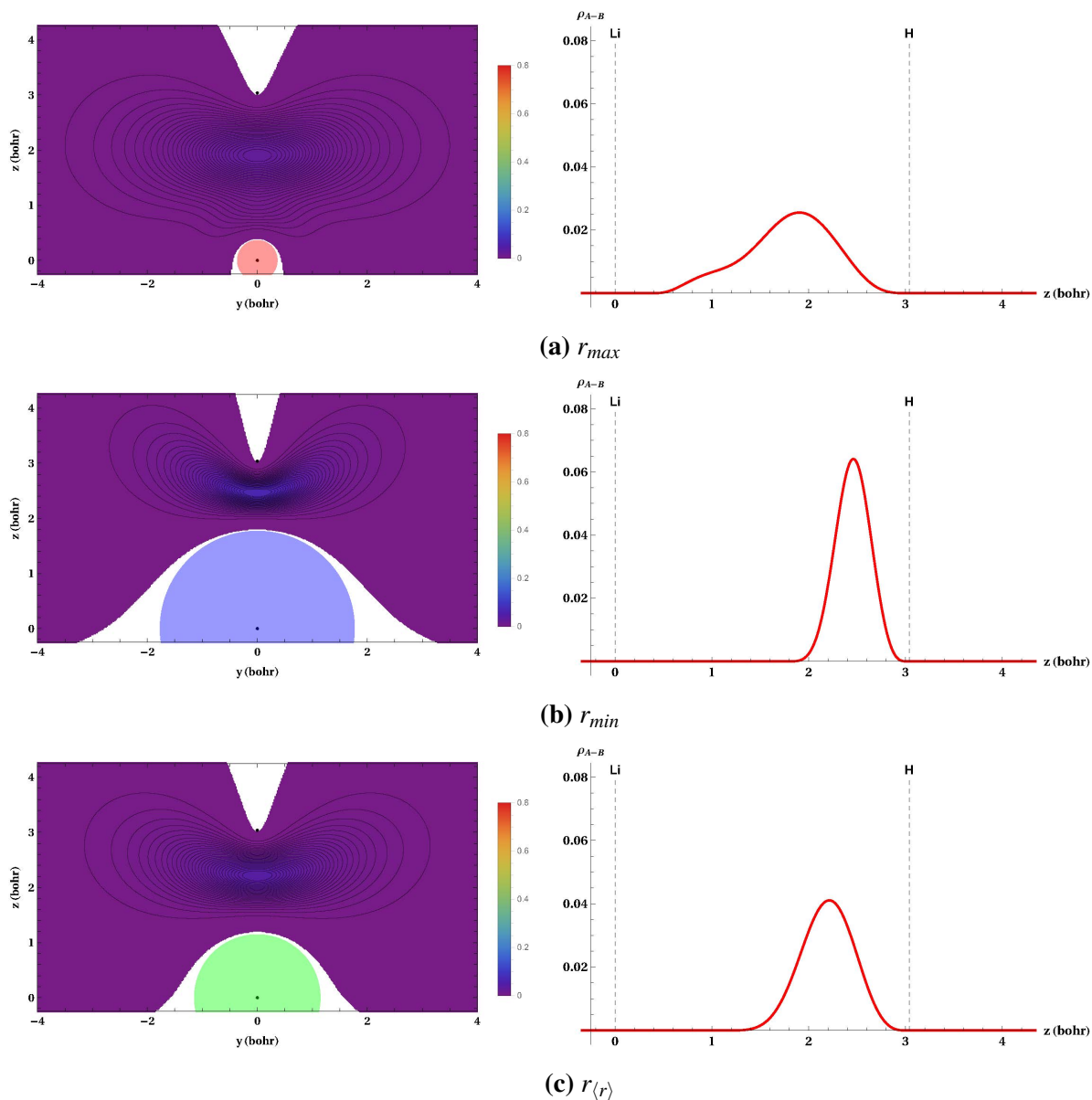


**Figure 3.13:** Molecular radial electron density (RDEN) for NaH is obtained using the Awad weight at: (a)  $r_{max}$ , (b)  $r_{min}$ , and (c)  $r_{(r)}$ . The atomic coordinates of Na at 0.0 bohr and H at 3.6261 bohr along  $z$  axis.



**Figure 3.14:** Bond electron density (BDEN) for NaH is obtained using the Awad weight at: (a)  $r_{max}$ , (b)  $r_{min}$ , and (c)  $r_{(r)}$ . The atomic coordinates of Na at 0.0 bohr and H at 3.6261 bohr along  $z$  axis. The red, purple, and green disks are the core regions of  $r_{max}$ ,  $r_{min}$ , and  $r_{(r)}$ , respectively.





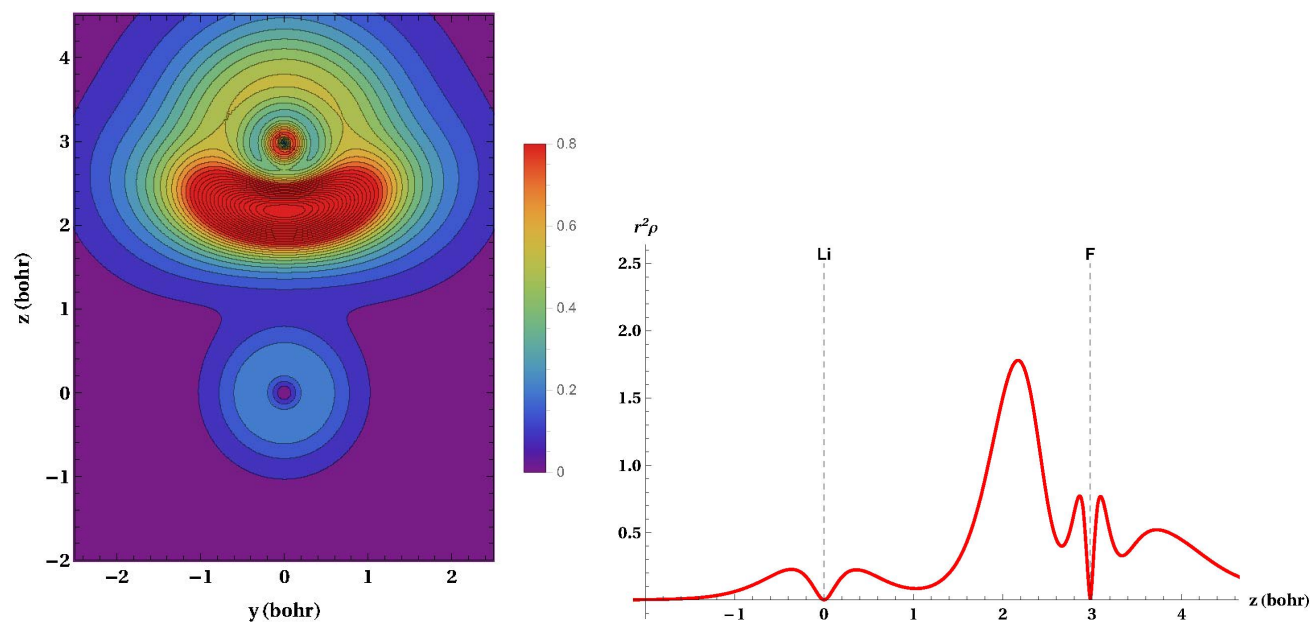
**Figure 3.15:** Bond electron density (BDEN) for LiH is obtained using the Awad weight at: (a)  $r_{max}$ , (b)  $r_{min}$ , and (c)  $r_{(r)}$ . The atomic coordinates of Li at 0.0 bohr and H at 3.0390 bohr along  $z$  axis. The red, purple, and green disks are the core regions of  $r_{max}$ ,  $r_{min}$ , and  $r_{(r)}$ , respectively.



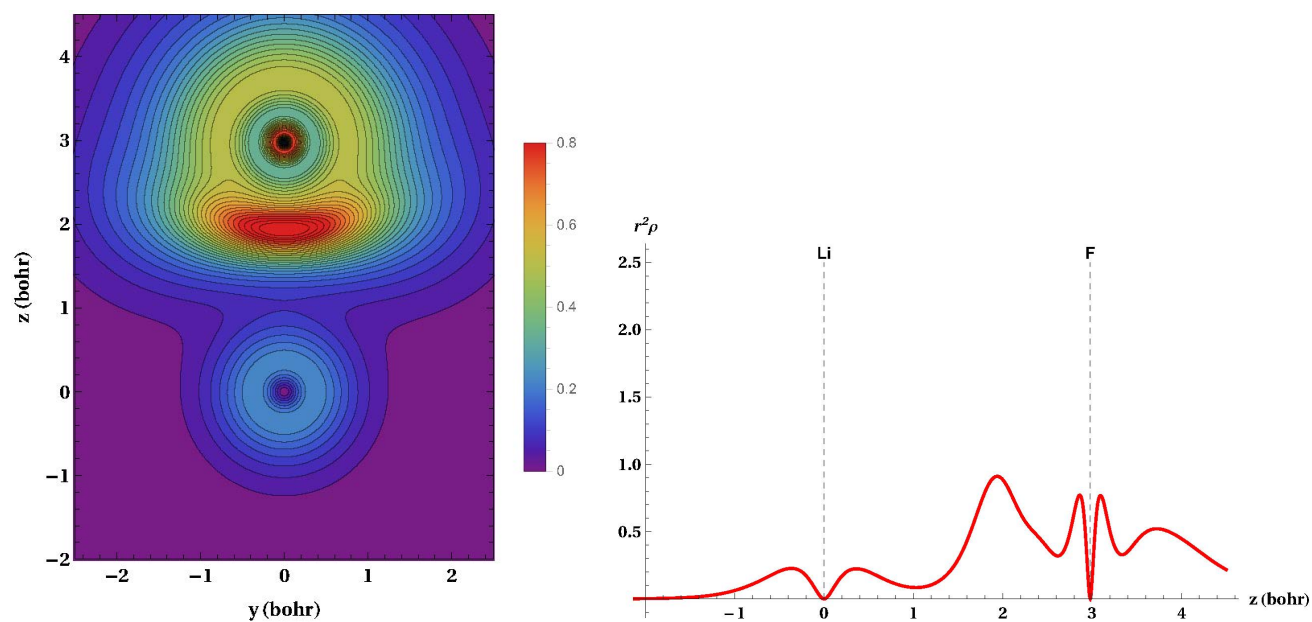
use  $r_{\langle r \rangle}$  as a core size for calculations involving the Awad weight. From these figures, the Awad weight gives better bonding-region representation with both RDEN and BDEN than the Becke weight. As shown in Figure 3.16, it is clear from the RDEN in bonding region that LiF exhibits more ionic properties, in consistence with the chemical sense, when using the Awad weight. When using the Becke weight, the electron density is concentrated in the bonding region indicating more covalent-bond characteristics. Figure 3.17 shows that the electron density in the bonding region of LiF using Becke weight is contaminated from the electron density of the core. In contrast, using the Awad weight the core is assigned to the atom not to the bond or the cores of other atoms. The other studied molecules ( $\text{H}_2$ ,  $\text{N}_2$ ,  $\text{Cl}_2$ ,  $\text{CO}$ ,  $\text{FCl}$ ,  $\text{LiCl}$ ,  $\text{HCl}$ ,  $\text{HF}$ ,  $\text{LiH}$ , and  $\text{NaH}$ ) have the same features for RDEN and BDEN. For brevity, the RDEN and BDEN plots of these molecules are moved to section A.3.

In general, the Awad weight has many favorable features over the Becke weight:

- The Awad weight has flexibility of choosing the core boundary of atoms within the molecule. For example the atomic core sizes can be stored in a database according to the geometry of the molecule.
- The ability to assign the core to the atom not to the bond or the cores of other atoms, giving us the ability to compute the molecular properties in the bonding region (such as the bond order).
- There is no special condition for the type of atom within the molecules. For example, in case of Becke the  $\chi_{AB} = \frac{R_A}{R_B}$  should be adjusted between 0.200 and 0.417, but in our case the Awad weight is not restricted.

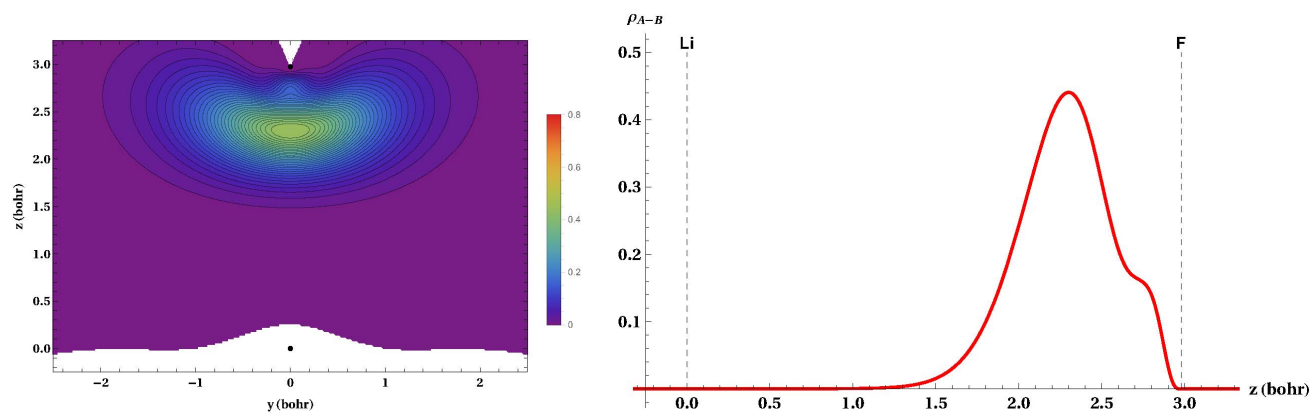


(a) The core size is defined as Becke weight.

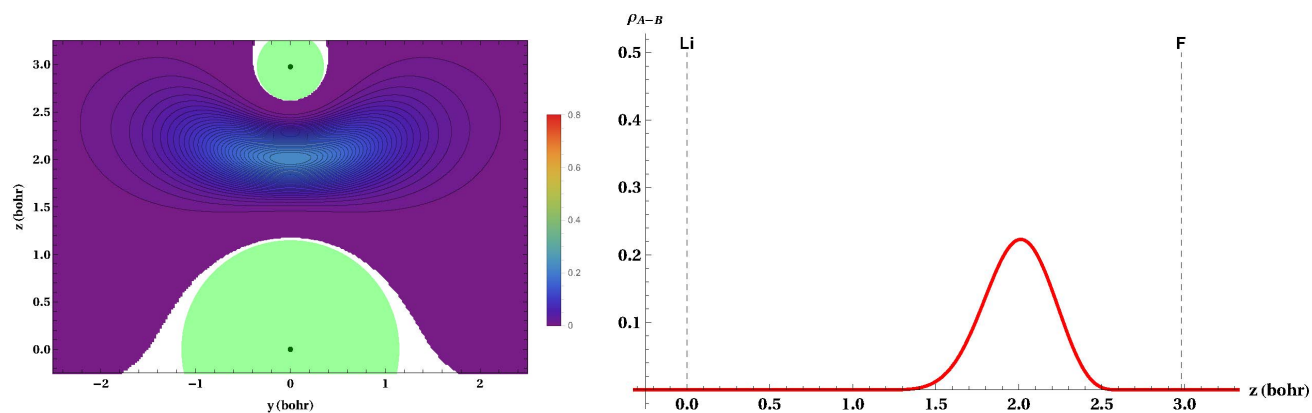


(b) Awad weight at  $r_{\langle r \rangle}$ .

**Figure 3.16:** Molecular radial electron density (RDEN) for LiF is obtained using the Becke and Awad weights. The atomic coordinates of Li at 0.0 bohr and F at 2.9777 bohr along  $z$  axis.



(a) The core size is defined as Becke weight.



(b) Awad weight at  $r_{(r)}$ .

**Figure 3.17:** Bond electron density (BDEN) for LiF is obtained using the Becke and Awad weights. The atomic coordinates of Li at 0.0 bohr and F at 2.9777 bohr along  $z$  axis.

### 3.2.5 Numerical Integration Results

Using numerical integration techniques has become routine in quantum chemistry, especially in DFT. However, as mentioned in subsection 1.6.3, numerical integration is not straightforward in molecular calculations. The popular solution is by partitioning the molecular space into cells or discrete regions. One of the most common partitioning schemes is the Becke weight [4]. In this section the Awad weight is used in the numerical quadrature methods to integrate properties for which the exact analytical results are known, namely, the HF wavefunction. The calculations are performed using the different suggested core sizes  $r_{max}$ ,  $r_{min}$ , and  $r_{\langle r \rangle}$ , then the results were compared with Becke weight. In general, the accuracy of numerical integration methods are evaluated by integrating the electron density for the total number of electrons, for the total DFT energy, or explicitly, for the exchange-correlation energy [5]. In this section the calculated molecular properties include the total number of electrons  $N_e$ , the electron-nuclear potential energy  $V_{ne}$ , and Coulomb potential energy  $V_{ee}$ . Therefore, the values of  $N_e$ ,  $V_{ne}$ , and  $V_{ee}$  calculated using the numerical integration are compared with those calculated by HF wavefunction (the exact values).

The mean absolute error (MAE) of the integration is computed as,

$$\text{MAE} = \frac{1}{N} \sum_i \left| P_{\text{NI}}^i - P_{\text{HF}}^i \right| \quad (3.3)$$

where  $P_{\text{NI}}^i$  is the value of the molecular property,  $N_e$ ,  $V_{ne}$ , or  $V_{ee}$  computed using numerical integration for a molecule  $i$ , and  $P_{\text{HF}}^i$  is the corresponding molecular property calculated using HF, and  $N$  is the number of molecules. In addition for each property the accuracy of

the integration is calculated using the formula given by Gill and Chien [6]

$$\text{accuracy} = -\log_{10} \left| \frac{P_{\text{NI}}}{P_{\text{HF}}} - 1 \right| \quad (3.4)$$

where,  $P_{\text{NI}}$  is the value of a molecular property calculated using numerical integration, and  $P_{\text{HF}}$  is the HF value of the respective property. Table 3.4 shows the error  $P_{\text{NI}}^i - P_{\text{HF}}^i$  and mean absolute error (MAE) values of the total number of electrons  $N_e$  (in  $\mu\text{e}$ ), electron-nuclear potential energy  $V_{ne}$  (in  $\mu\text{hartree}$ ), and Coulomb potential energy  $V_{ee}$  (in  $\mu\text{hartree}$ ) using the Awad weight (with  $r_{\langle r \rangle}$ ,  $r_{\text{max}}$ , and  $r_{\text{min}}$ ) and the Becke weight.

### Number of electrons $N_e$

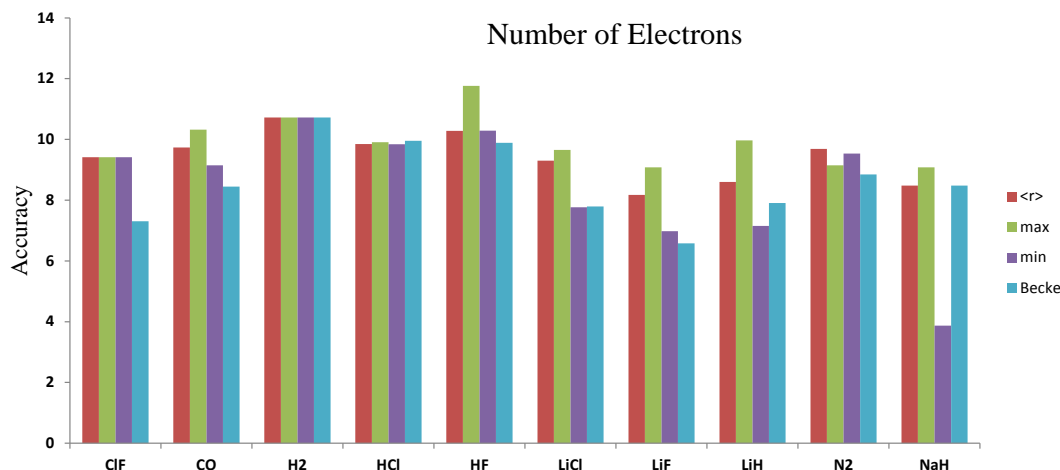
The total number of electrons is calculated by integrating the HF electron density using,

$$N_e = \int \rho(\mathbf{r}) d\mathbf{r} \quad (3.5)$$

where  $N_e$  is number of electrons and  $\rho(\mathbf{r})$  is the molecular electron density at position  $\mathbf{r}$ . The MAE for calculated total number of electrons using the Awad weight with  $r_{\langle r \rangle}$  and  $r_{\text{max}}$  are 0.01524 and 0.003665  $\mu\text{e}$  respectively, and it is 0.4916  $\mu\text{e}$  using Becke weight. Clearly that the performance of Awad weight (with  $r_{\langle r \rangle}$  and  $r_{\text{max}}$ ) is much better than Becke weight. Using  $r_{\text{min}}$  with the Awad weight gives the worst result compared to other core sizes where MAE equals 0.2116  $\mu\text{e}$  excluding the NaH molecule. Including NaH changes MAE significantly to 162.2  $\mu\text{e}$  using  $r_{\text{min}}$ .

Figure 3.18 compares the accuracy (Equation 3.4) of numerical integration results for calculating number of electrons  $N_e$  using the Awad weight (with  $r_{\text{max}}$ ,  $r_{\text{min}}$ , and  $r_{\langle r \rangle}$ ) and Becke

weight. The accuracy results in Figure 3.18 are in agreement with the previous MAE results as the Awad weight (with  $r_{\langle r \rangle}$  or  $r_{max}$ ) gives better results than Becke weight especially for ionic compounds such as LiCl and LiF.



**Figure 3.18:** Comparing the accuracy (Equation 3.4) of numerical integration results for calculating number of electrons  $N_e$  using the Awad weight (with  $r_{max}$ ,  $r_{min}$ , and  $r_{\langle r \rangle}$ ) and Becke weight.

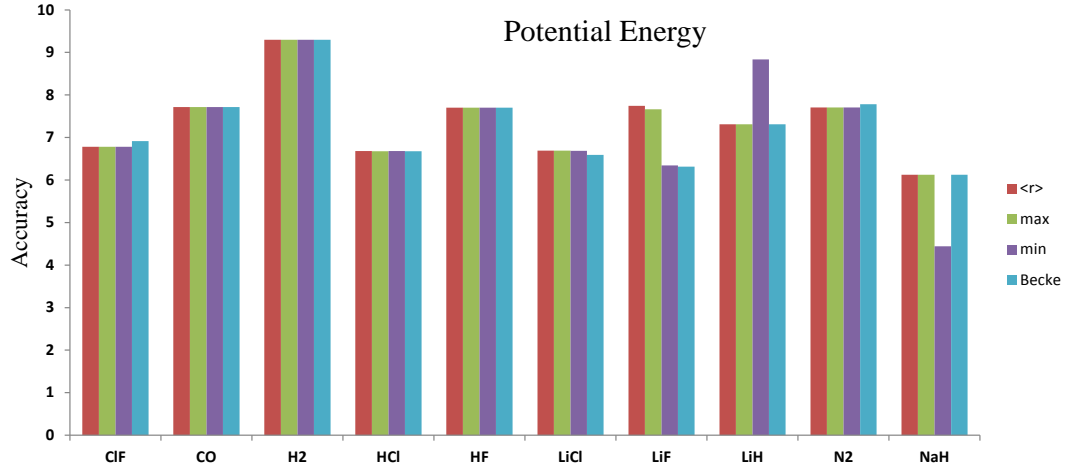
## Potential Energy $V_{ne}$

The electron-nuclear potential energy  $V_{ne}$  is calculated using

$$V_{ne} = - \sum_A Z_A \int \frac{\rho(\mathbf{r})}{|\mathbf{r} - \mathbf{R}_A|} d\mathbf{r} \quad (3.6)$$

where  $\mathbf{R}_A$  is the position of atom  $A$  with atomic charge  $Z_A$  and  $\rho(\mathbf{r})$  is the molecular electron density at position  $\mathbf{r}$ . The errors and MAE values for potential energy are shown in Table 3.4 and the accuracy (Equation 3.4) of numerically calculated potential energy are shown in Figure 3.19. The MAE using the Awad weight with  $r_{\langle r \rangle}$  and  $r_{max}$  are 102.45 and 102.48  $\mu\text{hartree}$  respectively which are slightly better than MAE of 115.01  $\mu\text{hartree}$  ob-

tained using Becke weight. Figure 3.19 shows that using the Awad weight (with  $r_{\langle r \rangle}$  and  $r_{max}$ ) and Becke weight almost have the same accuracy in computing the potential energy for all molecules except for LiF, in which the Becke weight is less accurate.



**Figure 3.19:** Comparing the accuracy (Equation 3.4) of numerical integration results for calculating the electron-nuclear potential energy  $V_{ne}$  using the Awad weight at different core sizes ( $r_{max}$ ,  $r_{min}$ , and  $r_{\langle r \rangle}$ ) and with Becke weight.

### Coulomb energy $V_{ee}$

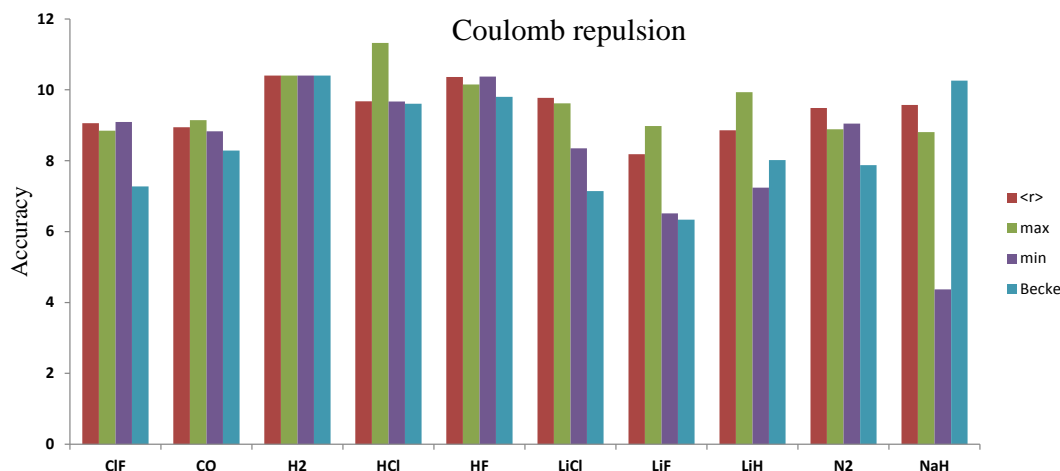
The Coulomb potential energy  $V_{ee}$  is calculated numerically as,

$$V_{ee} = \int \rho(\mathbf{r}_2) \left[ \sum_{\mu\nu} P_{\mu\nu} \int \frac{\phi_{\mu}^*(\mathbf{r}_1) \phi_{\nu}(\mathbf{r}_1)}{|\mathbf{r}_1 - \mathbf{r}_2|} d\mathbf{r}_1 \right] d\mathbf{r}_2 \quad (3.7)$$

$P_{\mu\nu}$  is the density matrix element,  $\rho(\mathbf{r}_2)$  is the molecular electron density at position  $\mathbf{r}_2$ ,  $\phi_{\mu}(\mathbf{r}_1)$  and  $\phi_{\nu}(\mathbf{r}_1)$  are the  $\mu^{th}$  and  $\nu^{th}$  basis functions at position  $\mathbf{r}_1$  respectively, and  $|\mathbf{r}_1 - \mathbf{r}_2|$  is the distance between  $\mathbf{r}_1$  and  $\mathbf{r}_2$ . The first integral ( $d\mathbf{r}_1$ ) in the brackets is computed analytically, while the second integral ( $d\mathbf{r}_2$ ) is computed numerically.

Again, the Coulomb potential energy results show that using the Awad weight with  $r_{\langle r \rangle}$  and  $r_{max}$  gives better results than Becke weight where the MAE equal 0.09160, 0.08454, and 6.295  $\mu\text{hartree}$  for  $r_{\langle r \rangle}$ ,  $r_{max}$ , and Becke weight, respectively. In addition the accuracies of calculated Coulomb potential energies using the Awad weight (with  $r_{\langle r \rangle}$  and  $r_{max}$ ) are better than those obtained using Becke weight for all molecules except NaH as shown in Figure 3.20.

As a conclusion, even though Becke weight was originally designed to perform numerical integration efficiently for calculating molecular properties, the Awad weight gives better results in computing the molecular properties.



**Figure 3.20:** Comparing the accuracy (Equation 3.4) of numerical integration results for calculating the Coulomb potential energy  $V_{ee}$  using the Awad weight at different cores ( $r_{max}$ ,  $r_{min}$ , and  $r_{\langle r \rangle}$ ) and with Becke weight.



**Table 3.4:** The error  $P_{NI}^i - P_{HF}^i$  and MAE values of the total number of electrons  $N_e$  in  $\mu_e$ , the electron-nuclear potential energy  $V_{ne}$  in  $\mu\text{hartree}$ , and Coulomb potential energy  $V_{ee}$  in  $\mu\text{hartree}$  using the Awad weight (with  $r_{(r)}$ ,  $r_{max}$ , and  $r_{min}$ ) and the Becke weight.

|              | Total number of electrons $N_e$ |           |                       |           | Coulomb potential energy $V_{ee}$ |           |                       |           | Electron-nuclear potential energy $V_{ne}$ |           |                       |           |
|--------------|---------------------------------|-----------|-----------------------|-----------|-----------------------------------|-----------|-----------------------|-----------|--|-----------|-----------------------|-----------|
|              | $r_{(r)}$                       | $r_{max}$ | $r_{min}$             | Becke     | $r_{(r)}$                         | $r_{max}$ | $r_{min}$             | Becke     | $r_{(r)}$                                  | $r_{max}$ | $r_{min}$             | Becke     |
| <b>ClF</b>   | 0.008130                        | -0.008290 | 0.010000              | -1.290000 | -0.262400                         | -0.432800 | -0.245200             | -16.10000 | -236.9000                                  | -236.6000 | -236.9000             | -174.2000 |
| <b>CO</b>    | -0.002590                       | 0.000664  | -0.005890             | -0.049900 | -0.086770                         | -0.055280 | -0.113200             | -0.395200 | -5.850000                                  | -5.977000 | -5.762000             | -5.846000 |
| <b>H2</b>    | -0.000038                       | -0.000038 | -0.000038             | -0.000038 | -0.000052                         | -0.000052 | -0.000052             | -0.000052 | -0.001845                                  | -0.001845 | -0.001845             | -0.001845 |
| <b>HCl</b>   | -0.002540                       | 0.002210  | -0.002590             | -0.001990 | -0.044720                         | 0.001009  | -0.044930             | -0.051700 | -232.3000                                  | -232.4000 | -232.3000             | -232.3000 |
| <b>HF</b>    | -0.000522                       | 0.000017  | -0.000516             | -0.001290 | -0.002420                         | 0.003924  | -0.002357             | -0.008764 | -4.557000                                  | -4.602000 | -4.557000             | -4.536000 |
| <b>LiCl</b>  | 0.008630                        | 0.004440  | 0.339000              | 0.321000  | -0.037450                         | 0.053880  | 1.280000              | 15.79000  | -233.1000                                  | -233.4000 | -235.0000             | -293.5000 |
| <b>LiF</b>   | -0.084700                       | -0.005310 | -1.260000             | 3.150000  | -0.427500                         | -0.068940 | 19.59000              | 30.31000  | -4.842000                                  | -5.142000 | -125.0000             | -133.8000 |
| <b>LiH</b>   | -0.006580                       | 0.000432  | -0.282000             | -0.046500 | -0.007829                         | 0.000659  | -0.323000             | -0.053670 | -0.714700                                  | -0.745700 | 0.029860              | -0.636800 |
| <b>N2</b>    | 0.002890                        | -0.007660 | -0.004100             | -0.015000 | -0.024510                         | -0.097470 | -0.067620             | -0.230800 | -5.835000                                  | -5.611000 | -5.723000             | -4.992000 |
| <b>NaH</b>   | 0.035800                        | -0.007590 | -1620.000             | 0.040300  | -0.022320                         | -0.131400 | -3561.000             | -0.004582 | -300.4000                                  | -300.3000 | 14370.00              | -300.3000 |
| <b>MAE =</b> | 0.015242                        | 0.003665  | 0.211570 <sup>a</sup> | 0.491602  | 0.091597                          | 0.084541  | 2.407373 <sup>a</sup> | 6.294477  | 102.4501                                   | 102.4780  | 93.91930 <sup>a</sup> | 115.0113  |

<sup>a</sup>excluding NaH

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## Chapter 4

# Rigid-body Transformations

*“As far as the laws of mathematics refer to reality, they are not certain; and as far as they are certain, they do not refer to reality.”*

— Albert Einstein

A rigid-body transformation preserves distances between every pair of points, in which the shape and size of the object do not change [1]. The rigid-body transformations include translations, rotations, reflections, or their combination. In this chapter we will give a brief introduction about rigid-body transformations.

## 4.1 Rotation and Translation Matrices

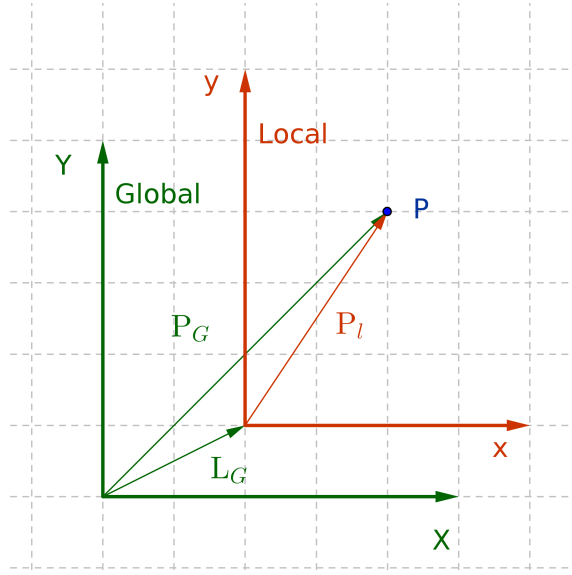
In this section, we will analyze the two dimensional (2D) and 3D transformations between two coordinate systems (one called global (fixed) and the other called local (movable)). We will also derive the translation and rotation matrices. Then we will use both of these matrices to translate, rotate and reflect a set of points.

### Translation

In Figure 4.1 the translation of point P in 2D space between two coordinate systems (global and local) can be described as follow

$$\mathbf{P}_G = \mathbf{L}_G + \mathbf{P}_l \quad ; \quad \mathbf{P}_l = \mathbf{P}_G - \mathbf{L}_G \quad (4.1)$$

where  $\mathbf{P}_G$  and  $\mathbf{P}_l$  represent the vectors of point P in global and local coordinate systems respectively, and  $\mathbf{L}_G$  is the vector representing the amount and direction to translate the local coordinate system to the global coordinate system. Each component of  $\mathbf{P}_G$ ,  $\mathbf{P}_l$ , and



**Figure 4.1:** 2D translation of a coordinate system (local) to another coordinate system (global).

$\mathbf{L}_G$  can be written as the following 2D (Equation 4.2) and 3D (Equation 4.3) matrices,

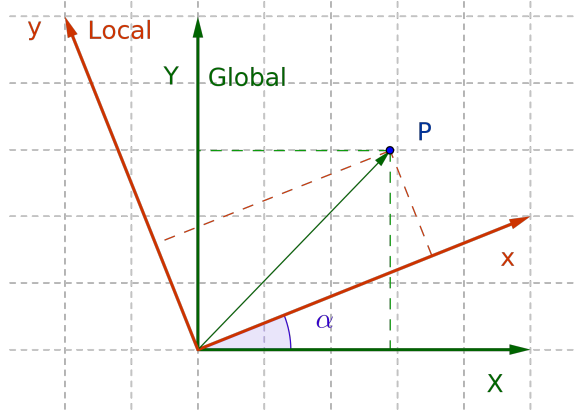
$$\begin{bmatrix} \mathbf{P}_X \\ \mathbf{P}_Y \end{bmatrix} = \begin{bmatrix} \mathbf{L}_X \\ \mathbf{L}_Y \end{bmatrix} + \begin{bmatrix} \mathbf{P}_x \\ \mathbf{P}_y \end{bmatrix} \quad ; \quad \begin{bmatrix} \mathbf{P}_x \\ \mathbf{P}_y \end{bmatrix} = \begin{bmatrix} \mathbf{P}_X \\ \mathbf{P}_Y \end{bmatrix} - \begin{bmatrix} \mathbf{L}_X \\ \mathbf{L}_Y \end{bmatrix} \quad (4.2)$$

$$\begin{bmatrix} \mathbf{P}_X \\ \mathbf{P}_Y \\ \mathbf{P}_Z \end{bmatrix} = \begin{bmatrix} \mathbf{L}_X \\ \mathbf{L}_Y \\ \mathbf{L}_Z \end{bmatrix} + \begin{bmatrix} \mathbf{P}_x \\ \mathbf{P}_y \\ \mathbf{P}_z \end{bmatrix} \quad ; \quad \begin{bmatrix} \mathbf{P}_x \\ \mathbf{P}_y \\ \mathbf{P}_z \end{bmatrix} = \begin{bmatrix} \mathbf{P}_X \\ \mathbf{P}_Y \\ \mathbf{P}_Z \end{bmatrix} - \begin{bmatrix} \mathbf{L}_X \\ \mathbf{L}_Y \\ \mathbf{L}_Z \end{bmatrix} \quad (4.3)$$

where  $\mathbf{P}_X$ ,  $\mathbf{P}_Y$ , and  $\mathbf{P}_Z$  are the components of the vector  $\mathbf{P}_G$ ;  $\mathbf{P}_x$ ,  $\mathbf{P}_y$ , and  $\mathbf{P}_z$  are the components of the vector  $\mathbf{P}_l$ ; and  $\mathbf{L}_X$ ,  $\mathbf{L}_Y$ , and  $\mathbf{L}_Z$  are the components the vector of  $\mathbf{L}_G$ .

## Rotation

The 2D rotation of a coordinate system (local) to another coordinate system (global) is shown in Figure 4.2. Our aim is to analyze the rotation of P from local to global coordi-



**Figure 4.2:** 2D rotation of a coordinate system (local) to other coordinate system (global).

nate systems and vice versa. To achieve this, a trigonometry method is used as shown in Figure 4.3. From Figure 4.3a, one can express the P components in the local coordinate system in terms of the global coordinate system as,

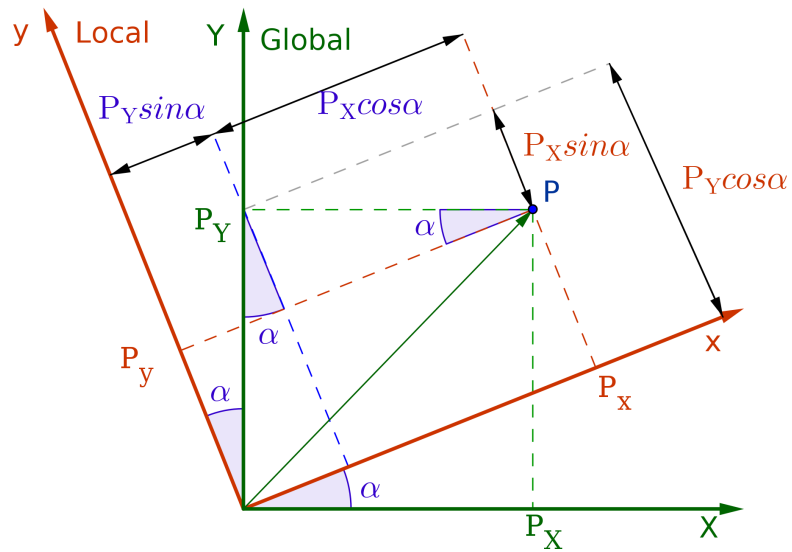
$$\begin{aligned} \mathbf{P}_x &= \mathbf{P}_X \cos \alpha + \mathbf{P}_Y \sin \alpha \\ \mathbf{P}_y &= -\mathbf{P}_X \sin \alpha + \mathbf{P}_Y \cos \alpha \end{aligned} \quad (4.4)$$

the above equations can be written in a matrix form as,

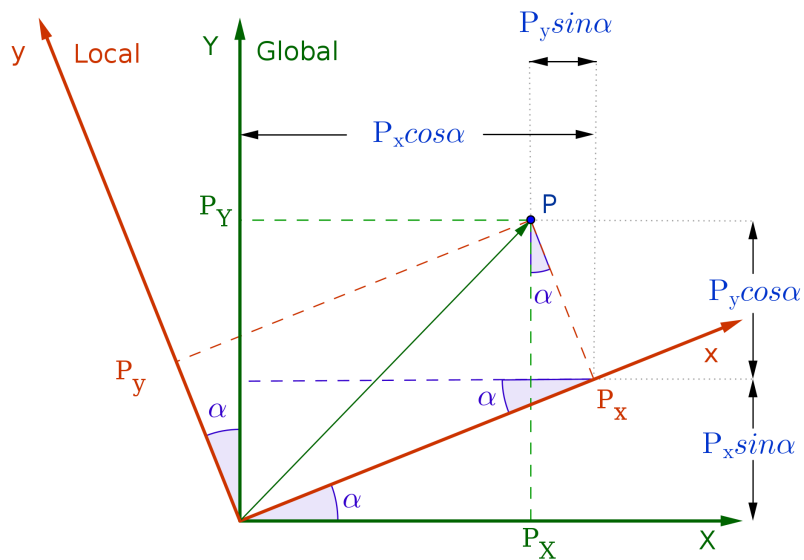
$$\begin{bmatrix} \mathbf{P}_x \\ \mathbf{P}_y \end{bmatrix} = \begin{bmatrix} \cos \alpha & \sin \alpha \\ -\sin \alpha & \cos \alpha \end{bmatrix} \begin{bmatrix} \mathbf{P}_X \\ \mathbf{P}_Y \end{bmatrix} \quad (4.5)$$

or simply as,

$$\mathbf{P}_l = R_{lG} \mathbf{P}_G \quad \text{where,} \quad R_{lG} = \begin{bmatrix} \cos \alpha & \sin \alpha \\ -\sin \alpha & \cos \alpha \end{bmatrix} \quad (4.6)$$



(a) Point  $P$  coordinate at the local coordinate system in terms of global coordinate system



(b) Point  $P$  coordinate at the global coordinate system in terms of local coordinate system

**Figure 4.3:** Express the point  $P$  in different coordinates (global and local) only knowing one coordinate system and the angle of rotation.

where,  $R_{lG}$  is a 2D rotation matrix to rotate  $\mathbf{P}$  from global to local coordinate system. Likewise, using Figure 4.3b one can express the  $\mathbf{P}$  components in the global coordinate system in terms of the local coordinate system as,

$$\begin{aligned}\mathbf{P}_X &= \mathbf{P}_x \cos \alpha - \mathbf{P}_y \sin \alpha \\ \mathbf{P}_Y &= \mathbf{P}_x \sin \alpha + \mathbf{P}_y \cos \alpha\end{aligned}\quad (4.7)$$

where the matrix form is,

$$\begin{bmatrix} \mathbf{P}_X \\ \mathbf{P}_Y \end{bmatrix} = \begin{bmatrix} \cos \alpha & -\sin \alpha \\ \sin \alpha & \cos \alpha \end{bmatrix} \begin{bmatrix} \mathbf{P}_x \\ \mathbf{P}_y \end{bmatrix}\quad (4.8)$$

The last equation can be written as,

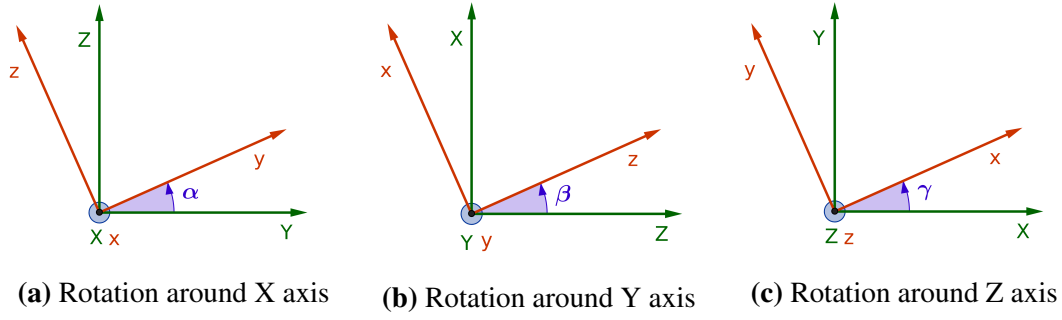
$$\mathbf{P}_G = R_{Gl} \mathbf{P}_l \quad \text{where,} \quad R_{Gl} = \begin{bmatrix} \cos \alpha & -\sin \alpha \\ \sin \alpha & \cos \alpha \end{bmatrix}\quad (4.9)$$

where  $R_{Gl}$  is a 2D rotation matrix to rotate the point  $\mathbf{P}$  from local to global coordinate systems. A nice property of the rotation matrix is that the products of  $R_{Gl}$  and  $R_{lG}$  gives the identity matrix.

$$R_{Gl} R_{lG} = \begin{bmatrix} \cos \alpha & -\sin \alpha \\ \sin \alpha & \cos \alpha \end{bmatrix} \begin{bmatrix} \cos \alpha & \sin \alpha \\ -\sin \alpha & \cos \alpha \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = I \quad (4.10)$$

To derive the rotation matrix in the 3D coordinate system, the rotation can be divided into rotations around X, Y, and Z axes as shown in Figure 4.4.





**Figure 4.4:** Rotation of 3D coordinate system around X, Y and Z axes

One can express the point P components in 3D coordinate system as follows,

$$\begin{aligned}\mathbf{P}_X &= R_{Gl}^X \mathbf{P}_x \\ \mathbf{P}_Y &= R_{Gl}^Y \mathbf{P}_y \\ \mathbf{P}_Z &= R_{Gl}^Z \mathbf{P}_z\end{aligned}\tag{4.11}$$

where  $R_{Gl}^X$ ,  $R_{Gl}^Y$ , and  $R_{Gl}^Z$  are the rotation matrices around X, Y, and Z axes, respectively,

that are defined as,

$$R_{Gl}^X = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos \alpha & -\sin \alpha \\ 0 & \sin \alpha & \cos \alpha \end{bmatrix} \quad (4.12)$$

$$R_{Gl}^Y = \begin{bmatrix} \cos \beta & 0 & -\sin \beta \\ 0 & 1 & 0 \\ \sin \beta & 0 & \cos \beta \end{bmatrix} \quad (4.13)$$

$$R_{Gl}^Z = \begin{bmatrix} \cos \gamma & -\sin \gamma & 0 \\ \sin \gamma & \cos \gamma & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad (4.14)$$

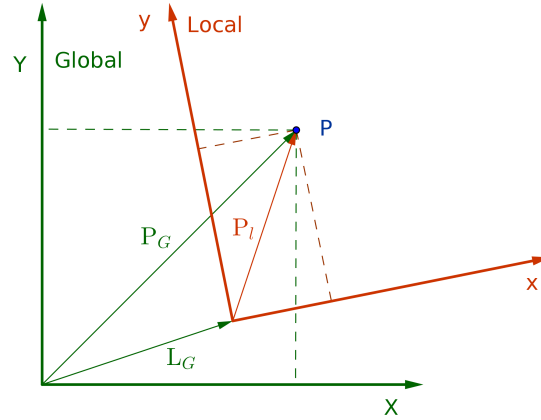
The 3D rotation matrix  $R_{Gl}^{XYZ}$  is the product of  $R_{Gl}^X$ ,  $R_{Gl}^Y$ , and  $R_{Gl}^Z$ .

$$\begin{aligned} R_{Gl}^{XYZ} &= R_{Gl}^X R_{Gl}^Y R_{Gl}^Z \\ &= \begin{bmatrix} \cos \beta \cos \gamma & \sin \alpha \sin \beta \cos \gamma - \cos \alpha \sin \gamma & \cos \alpha \sin \beta \cos \gamma + \sin \alpha \sin \gamma \\ \cos \beta \sin \gamma & \sin \alpha \sin \beta \sin \gamma + \cos \alpha \cos \gamma & \cos \alpha \sin \beta \sin \gamma - \sin \alpha \cos \gamma \\ -\sin \beta & \sin \alpha \cos \beta & \cos \alpha \cos \beta \end{bmatrix} \end{aligned} \quad (4.15)$$

## Rotation and Translation

The rotation and translation can be done with two steps: first rotating then translating the coordinate system. Figure 4.5 shows the rotation and translation of 2D coordinate system

from local to global coordinate system. Mathematically the rotation and translation are



**Figure 4.5:** 2D translation and rotation of a coordinate system (local) to other coordinate system (global).

written as follows,

$$\mathbf{P}_G = R_{Gl}\mathbf{P}_l + \mathbf{L}_G \quad (4.16)$$

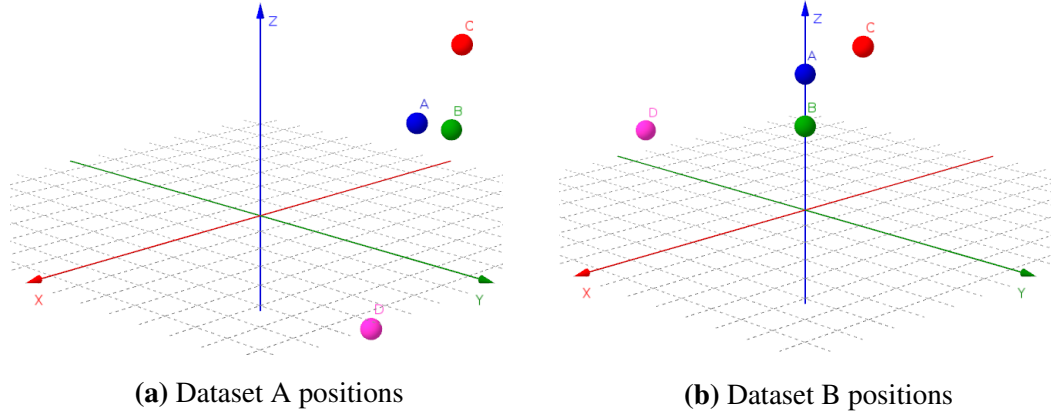
However, if the global coordinate system is rotated and translated to local coordinate system, then we should do the translation firstly and then do the rotation of the coordinate system.

$$\mathbf{P}_l = R_{lG}(\mathbf{P}_G - \mathbf{L}_G) \quad (4.17)$$

## 4.2 Optimal Rotation and Translation

Points matching, also known as a point set registration, is the process of finding the best transformation that aligns two sets of points. A large number of algorithms were developed to compute the rigid transformations between two sets of data [2–6]. These algorithms are trying to find rotation and translation matrices that align two sets of points. In this section,

we will find the best rotation and translation matrices to superimpose dataset A to dataset B (Figure 4.6) using the singular value decomposition (SVD) algorithm [4].



**Figure 4.6:** Two data sets A and B with same number of points.

Assume that we have  $N$  points in a rigid body and let  $\mathbf{A} = \{\mathbf{A}_1, \mathbf{A}_2, \dots, \mathbf{A}_n\}$  be the 3D positions of these points before the movement and  $\mathbf{B} = \{\mathbf{B}_1, \mathbf{B}_2, \dots, \mathbf{B}_n\}$  be the positions after the movement. One can use the following equation to transform  $\mathbf{A}$  into  $\mathbf{B}$ ,

$$\mathbf{B} = \mathbf{L} + \mathbf{R}\mathbf{A} \quad (4.18)$$

where  $\mathbf{R}$  and  $\mathbf{L}$  are the rotation and translation matrices respectively. The steps to compute the optimal  $\mathbf{R}$  and  $\mathbf{L}$  matrices are:

- Computing the centered vectors of  $\mathbf{A}$  and  $\mathbf{B}$  datasets.
- Finding the optimal rotation matrix  $\mathbf{R}$ .
- Finding the translation matrix  $\mathbf{L}$ .

## Computing the centered vectors

Calculating the centered vectors of **A** and **B** datasets is an easy task, and can be done by averaging over all points in the two datasets.

$$\mathbf{A}_c = \frac{1}{N} \sum_{i=1}^N \mathbf{A}_i \quad ; \quad \mathbf{B}_c = \frac{1}{N} \sum_{i=1}^N \mathbf{B}_i \quad (4.19)$$

Where  $\mathbf{A}_c$  and  $\mathbf{B}_c$  are the centered vectors of **A** and **B** datasets respectively, and  $N$  is number of points in the dataset.

## Finding the optimal rotation matrix **R**

In order to find the optimal **R** matrix, we consider the SVD algorithm. SVD is the factorization or decomposition of matrix **M** into the product of three other matrices,

$$\mathbf{M} = \mathbf{U}\mathbf{S}\mathbf{V}^T \quad (4.20)$$

where **M** is an  $m \times n$  matrix, **U** is an  $m \times m$  unitary matrix, **S** is a diagonal  $m \times n$  matrix with non-negative real numbers on the diagonal, **V** is an  $n \times n$  unitary matrix, and  $\mathbf{V}^T$  is the transpose of matrix **V**. To find the optimal rotation we first need to set up a new matrix as follows,

$$\mathbf{H} = \sum_{i=1}^N (\mathbf{A}_i - \mathbf{A}_c) (\mathbf{B}_i - \mathbf{B}_c)^T \quad (4.21)$$

**H** is always a  $3 \times 3$  matrix. The factorization of the **H** matrix using SVD method gives,

$$\mathbf{H} = \mathbf{U}\mathbf{S}\mathbf{V}^T. \quad (4.22)$$

The rotation matrix  $\mathbf{R}$  can be calculated as follows,

$$\mathbf{R} = \mathbf{V}\mathbf{U}^T \quad (4.23)$$

In some cases the reflection matrix is included within the  $\mathbf{R}$  matrix.

### Finding the translation matrix $\mathbf{L}$

The translation matrix  $\mathbf{L}$  can be computed as follows,

$$\mathbf{L} = -\mathbf{R}\mathbf{A}_c + \mathbf{B}_c \quad (4.24)$$

$\mathbf{L}$  is always a  $3 \times 1$  matrix. In calculating  $\mathbf{L}$ , we first rotate  $\mathbf{A}_c$  using  $\mathbf{R}$  then translate by  $\mathbf{B}_c$ .

## 4.3 Standard Cartesian coordinates

In this section, we consider the transformation of the original Cartesian coordinates  $\mathbf{P}^0 = \{\mathbf{P}_1^0, \mathbf{P}_2^0, \mathbf{P}_3^0, \mathbf{P}_4^0\}$  of a dataset into new suggested standard Cartesian coordinates  $\mathbf{P} = \{\mathbf{P}_1, \mathbf{P}_2, \mathbf{P}_3, \mathbf{P}_4\}$ .

The new coordinates are explained in Table 4.1

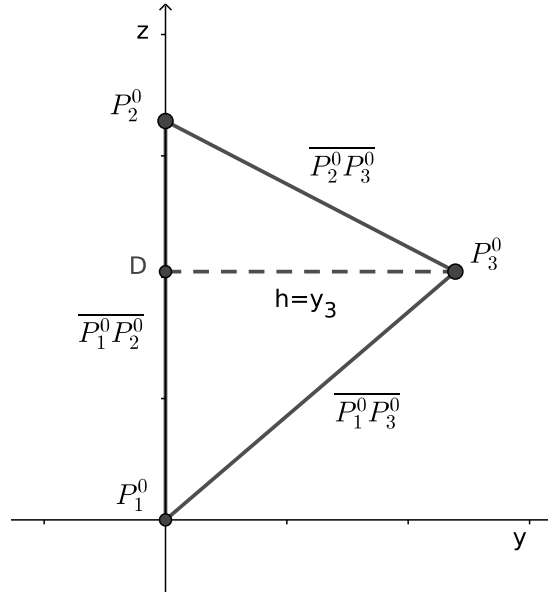
**Table 4.1:** Suggested standard Cartesian coordinates

| Point number   | $x$    | $y$    | $z$    |
|----------------|--------|--------|--------|
| $\mathbf{P}_1$ | 0      | 0      | 0      |
| $\mathbf{P}_2$ | 0      | 0      | $+z_2$ |
| $\mathbf{P}_3$ | 0      | $+y_3$ | $z_3$  |
| $\mathbf{P}_4$ | $+x_4$ | $y_4$  | $z_4$  |

In the new coordinates, the points  $\mathbf{P}_1$ ,  $\mathbf{P}_2$ , and  $\mathbf{P}_3$  are located in the origin, on the positive  $z$  axis, and in  $yz$  plane with a positive  $y$  coordinate, respectively, and  $\mathbf{P}_4$  is any point located in the  $xyz$  space with a positive  $x$  coordinate. Clearly,  $z_2$  can be easily computed, which is the distance between the points:  $\mathbf{P}_1^0 = (x_1^0, y_1^0, z_1^0)$  and  $\mathbf{P}_2^0 = (x_2^0, y_2^0, z_2^0)$ ,

$$z_2 = \overline{P_1^0 P_2^0} = \sqrt{(x_2^0 - x_1^0)^2 + (y_2^0 - y_1^0)^2 + (z_2^0 - z_1^0)^2}. \quad (4.25)$$

The three non-collinear points  $P_1^0$ ,  $P_2^0$ , and  $P_3^0$  form the triangle  $P_1^0 P_2^0 P_3^0$  as shown in Figure 4.7. In this figure, the altitude from side  $\overline{P_1^0 P_2^0}$  represents the coordinate  $y_3$ . Since all



**Figure 4.7:** Triangle  $P_1^0 P_2^0 P_3^0$ , with altitude  $h$  from side  $\overline{AB}$ .

sides of this triangle (i.e.,  $\overline{P_1^0 P_2^0}$ ,  $\overline{P_1^0 P_3^0}$ , and  $\overline{P_2^0 P_3^0}$ ) are known and its semiperimeter  $s$  is given by

$$s = \frac{\overline{P_1^0 P_2^0} + \overline{P_1^0 P_3^0} + \overline{P_2^0 P_3^0}}{2}, \quad (4.26)$$

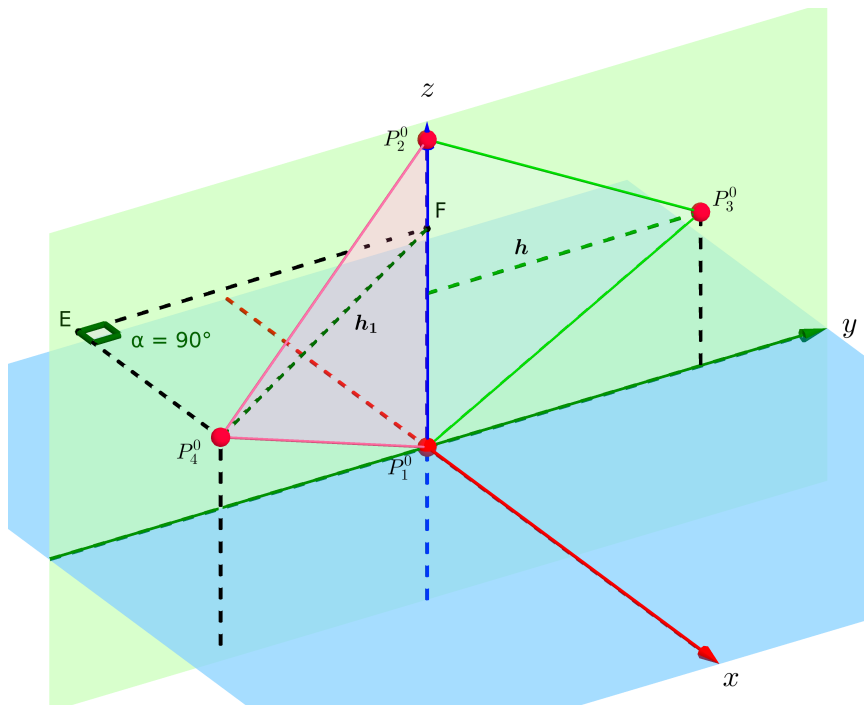
the value of  $y_3$  can be obtained using

$$y_3 = 2 \sqrt{\frac{s(s - \overline{P_1^0 P_2^0})(s - \overline{P_1^0 P_3^0})(s - \overline{P_2^0 P_3^0})}{\overline{P_1^0 P_2^0}}}. \quad (4.27)$$

Because the values of  $y_3$  and  $\overline{P_1^0 P_3^0}$  are known,  $z_3$  can be calculated using Pythagorean triple rule, where

$$z_3 = \sqrt{P_1^0 P_3^{02} - y_3^2} \quad (4.28)$$

Finding the Cartesian coordinates of  $\mathbf{P}_4$  is more tricky. As shown in Figure 4.8, the non-



**Figure 4.8:** The relationship between the points  $P_1^0$ ,  $P_2^0$ ,  $P_3^0$ , and  $P_4^0$  in the origin coordinates.

collinear points  $P_1^0$ ,  $P_2^0$ , and  $P_3^0$  are located on the  $yz$  plane of the origin coordinates. The plane can be described by a point and a normal (perpendicular) vector ( $\vec{n}$ ). The unique



normal vector through  $P_1^0$  and perpendicular to the plane  $yz$  can be obtained by the cross product,

$$\begin{aligned}\vec{n} &= \overrightarrow{P_1^0 P_2^0} \times \overrightarrow{P_1^0 P_3^0} \\ &= \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ x_2^0 - x_1^0 & y_2^0 - y_1^0 & z_2^0 - z_1^0 \\ x_3^0 - x_1^0 & y_3^0 - y_1^0 & z_3^0 - z_1^0 \end{vmatrix} \end{aligned} \quad (4.29)$$

Because  $\vec{n}$  is orthogonal to the plane, it is also orthogonal to any vector in the plane. Now, assume that  $P^0 = (x^0, y^0, z^0)$  is any point in the plane, then the vector  $\overrightarrow{P_1^0 P^0} = \langle x^0 - x_1^0, y^0 - y_1^0, z^0 - z_1^0 \rangle$  is located completely in the plane and it is perpendicular to  $\vec{n}$ . And because the dot product of two orthogonal vectors gives zero, the equation of the plane can be obtained as,

$$\vec{n} \cdot \langle x^0 - x_1^0, y^0 - y_1^0, z^0 - z_1^0 \rangle = 0 \quad (4.30)$$

where Equation 4.30 is called the vector equation of the plane. By solving the above equation one obtains,

$$s_1 x^0 + s_2 y^0 + s_3 z^0 + d = 0 \quad (4.31)$$

where

$$s_1 = (y_2^0 - y_1^0)(z_3^0 - z_1^0) - (z_2^0 - z_1^0)(y_3^0 - y_1^0)$$

$$s_2 = (z_2^0 - z_1^0)(x_3^0 - x_1^0) - (x_2^0 - x_1^0)(z_3^0 - z_1^0)$$

$$s_3 = (x_2^0 - x_1^0)(y_3^0 - y_1^0) - (y_2^0 - y_1^0)(x_3^0 - x_1^0)$$

$$d = -(s_1 x_1^0 + s_2 y_1^0 + s_3 z_1^0)$$

In Figure 4.8, the shortest distance between  $s_1x^0 + s_2y^0 + s_3z^0 + d = 0$  (the plane) and the point  $P_4^0 = (x_4^0, y_4^0, z_4^0)$  can be computed using the law of distance from a point to a plane,

$$\left| \overline{P_4^0 E} \right| = \frac{(s_1x_4^0 + s_2y_4^0 + s_3z_4^0 + d)}{\sqrt{s_1^2 + s_2^2 + s_3^2}} \quad (4.32)$$

It is clear as shown in Figure 4.8 that the value of  $x_4$  equals the distance  $\left| \overline{P_4^0 E} \right|$ . In Figure 4.8,  $h_1$  is the altitude of the triangle  $P_1^0 P_2^0 P_4^0$  from side  $\overline{P_1^0 P_2^0}$ . Similar to the calculation of  $y_3$  in Equation 4.27, one can calculate  $h_1$ . Using  $h_1$ , the values of  $y_4$  and  $z_4$  can be calculated using the Pythagorean triple rule as,

$$y_4 = \sqrt{h_1^2 - x_4^2} \quad (4.33)$$

$$z_4 = \sqrt{\left| \overline{P_1^0 P_4^0} \right|^2 - h_1^2} \quad (4.34)$$

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## Chapter 5

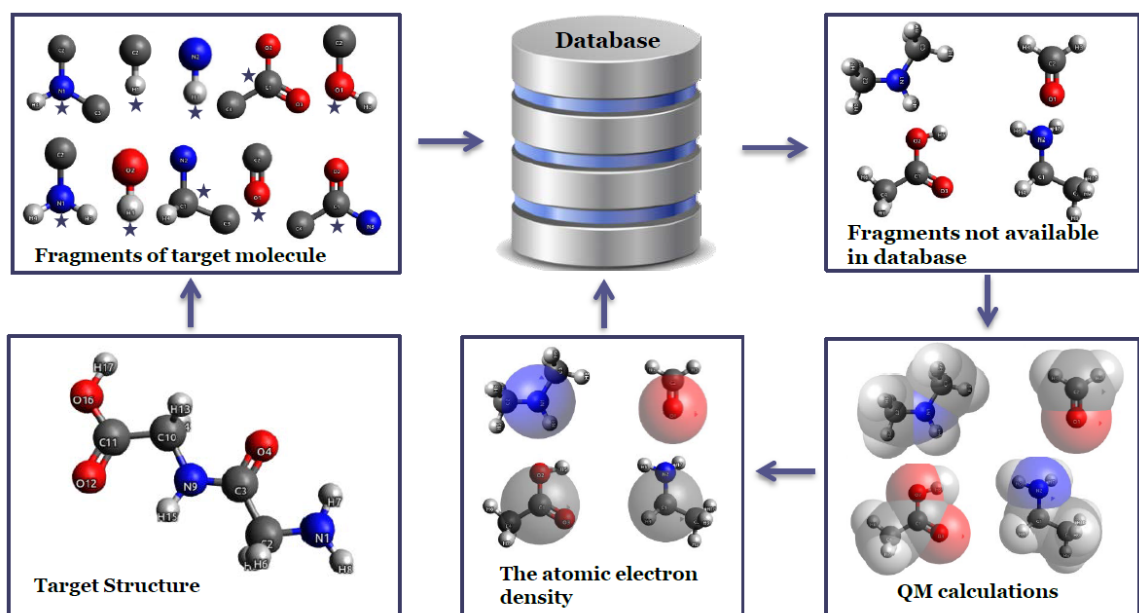
# Molecular Fragmentation

*“It would be possible to describe everything scientifically, but it would make no sense; it would be without meaning, as if you described a Beethoven symphony as a variation of wave pressure.”*

— Albert Einstein

Our method of computing the molecular properties of the target molecules includes three general steps: (i) generating small molecules (fragments) from molecules of interest. Each fragment defined a specific atom in molecule type based on its neighbor (ii) storing some properties of the fragments in the database, (iii) using these properties to compute the molecular properties of the target molecule. Figure 5.1 illustrates the main steps used by code written within the MUNgauss package to generate the fragments, compute their properties, and store these values in the database:

- Generate unique fragments from the molecule of interest. The size of the fragments can be controlled through the code.
- Check the database for the availability of each fragment.
- Generating the missing fragments, and complete the valency with hydrogen atoms.
- Optimize the structure of these fragments (optional).
- Perform QM calculations on these fragments.
- Compute the atomic properties for the target atom in these fragments.
- Store these atomic properties in the database with all other necessary information, such as, basis set, grid type, partitioning weight name, and the coordinates of these fragments.
- Store the grid points with all other necessary information at each point, such as: the coordinates of these grids, electron density, partitioning weights, and angular weights.



**Figure 5.1:** Describing the process of generating the fragments, computing the fragments properties, and storing these properties in the database.

In this chapter, we will use graph theory principles to generate fragments from molecules, illustrate our method to complete the valency of the fragment to build a real closed-shell molecule, explain the method for giving unique symbols and labelling the fragment atoms, and describe the structure of the database.

## 5.1 Fragment Generation Principle

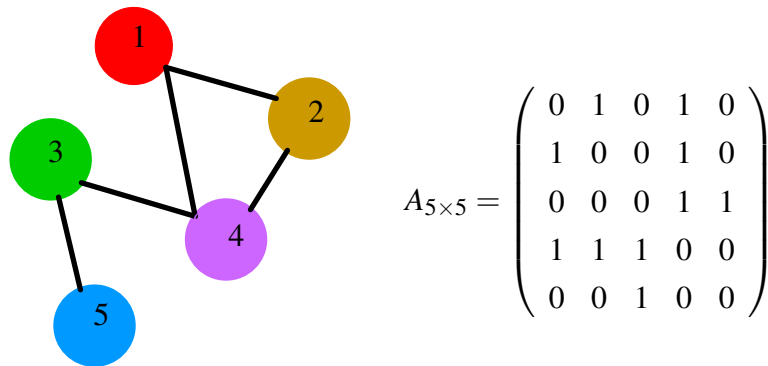
In this section, the principle of generating the fragment structures from the target molecule using graph theory will be discussed. An interesting property of adjacency matrices that allows us generate the fragment structures will be explained below.

### 5.1.1 Adjacency Matrix

In graph theory, an adjacency matrix is a square matrix, in which the elements indicate whether pairs of vertices are connected or not in a finite graph. Given a graph with  $n$  nodes, the adjacency matrix  $A_{n \times n}$  has entries  $A_{ij} = 1$ , if  $i$  is adjacent to  $j$ , and  $A_{ij} = 0$  otherwise. Because the labels of a graph may be selected in different ways, in general there are multiple possible adjacency matrices for a given graph [1].

Interesting things happen when the  $m$  power of the adjacency matrix is taken. The  $m$  power of an adjacency matrix has the property that represents the number of walks of length  $m$  of any pairs of vertices [2].

For example, Figure 5.2 shows a graph with five nodes. The adjacency matrix  $A_{5 \times 5}$  for the graph is also shown in the figure. The matrix is symmetric with dimensions of  $5 \times 5$ , and its elements are binary numbers 0 or 1.



**Figure 5.2:** Five nodes graph and its adjacency matrix ( $A_{5 \times 5}$ ).

The square of adjacency matrix  $A_{5 \times 5}$  gives a new matrix  $A_{5 \times 5}^2$  with the same dimension (in

this case  $5 \times 5$ ) but with edges connecting vertices with path length equal 2.

$$A_{5 \times 5}^2 = \begin{pmatrix} 2 & 1 & 1 & 1 & 0 \\ 1 & 2 & 1 & 1 & 0 \\ 1 & 1 & 2 & 0 & 0 \\ 1 & 1 & 0 & 3 & 1 \\ 0 & 0 & 0 & 1 & 1 \end{pmatrix} \quad (5.1)$$

Now by replacing the non-binary numbers with zeros, and removing all self-path connections by making the diagonal elements equal zeros, one obtains,

$$A_{5 \times 5}^{2'} = \begin{pmatrix} 0 & 1 & 1 & 1 & 0 \\ 1 & 0 & 1 & 1 & 0 \\ 1 & 1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 \end{pmatrix} \quad (5.2)$$

Because of the possibility of rings within the graph, there will be the possibility for different paths between pairs of vertices. To eliminate this possibility, the last matrix  $A_{5 \times 5}^{2'}$  is modified to  $A_{5 \times 5}^{2''}$  by replacing all the ones that shared with  $A_{5 \times 5}$  to zeros.

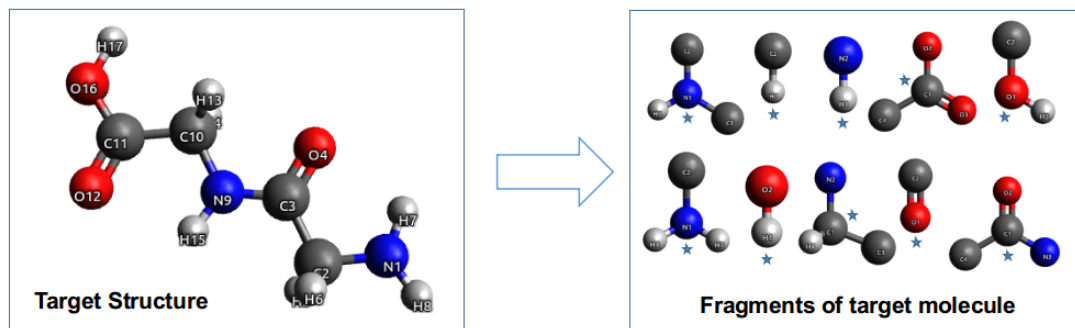
$$A_{5 \times 5}^{2''} = \begin{pmatrix} 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 \end{pmatrix} \quad (5.3)$$

The  $A_{5 \times 5}^{2''}$  matrix shows the pair of vertices that has a distance of exactly 2. The same principle can be done for higher order (i.e., distance of 3 and more).

The adjacency matrix is used to generate atomic fragments from a real molecule to any



desired  $m$  neighbours. For example, Figure 5.3 shows the structures of the unique first-neighbour fragments from the glycine-glycine (GlyGly) dimer. The valency of each fragment should be completed to build a real closed-shell fragment. The method of completing the valencies of the fragment will be discussed in the following section.



**Figure 5.3:** GlyGly dimer unique fragments for the first neighbor atom. The valency of the fragments still need to be completed.

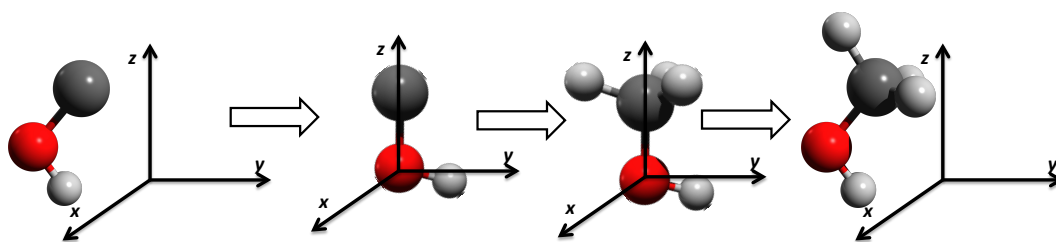
### 5.1.2 Completing the Valency of the Fragment

To complete each missing valency of a fragment of interest, the fragment is translated and rotated until each of its terminal atoms (the atoms that need their valencies to be completed) is on the positive  $z$  axis, and the first neighbour atom has to be located at the origin of the coordinate system. If there is another atom connected to the first neighbour atom, it should be located in the  $yz$  plane. In order to determine the valency of the terminal atom, the order of the bond between the terminal atom and its first neighbour atom is calculated. The type of the bond (single, double, ...) is then determined from the bond order as illustrated in Table 5.1. The classifications shown in this table is used in our entire research, where the bond orders are calculated using the Mayer method [3]. For example, Figure 5.4 shows one

**Table 5.1:** The relation between the type of bond and the range of bond order

| Bond Type            | Bond order range |
|----------------------|------------------|
| Partial single bond  | [0, 0.5)         |
| Single bond          | [0.5, 1.13)      |
| Aromatic double bond | [1.13, 1.6)      |
| Double bond          | [1.6, 2.2)       |
| Aromatic triple bond | [2.2, 2.7)       |
| Triple bond          | [2.7, 3.2)       |

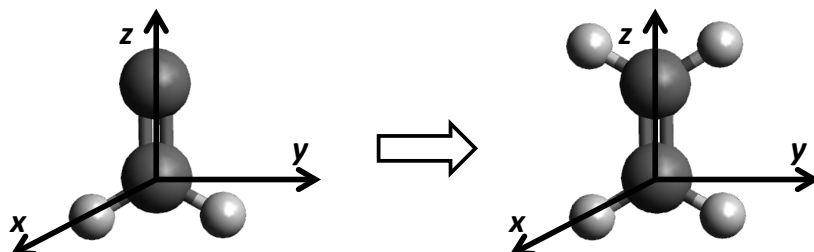
of the GlyGly fragment (HOC) in which the valency of the terminal (C) atom needs to be completed. Firstly, the initial coordinates of the HOC fragment are translated and rotated until the C atom is on the positive  $z$  axis and the first neighbour atom (O) is located at the origin and H atom is located at  $yz$  plane (Figure 5.4). Secondly, the CO bond is determined to be a "single bond" by calculating the CO bond order of molecule of interest. Thus, the carbon atom needs three hydrogen atoms to complete its valency. Because the locations and



**Figure 5.4:** Completing the carbon atom valency by adding three hydrogen atoms. The oxygen is located at the origin, the carbon on the positive  $z$  axis, where CO has a single bond.

bond lengths of other atoms are known, the locations of these hydrogen atoms can easily be determined with minimum steric effect. Figure 5.5 shows another example to complete the valency for the terminal atoms. In this example, the bond between the target C atom and

the other C atom is a double bond, and because locations of the existing H atoms are in yz plane, the two new H atoms should be added in the same plane to form a planar structure.



**Figure 5.5:** Completing the carbon atom valency for ethene by adding two hydrogen atoms, the target carbon is located at the positive  $z$  axis, the other carbon atom at the origin.

Table 5.2 shows the default Cartesian coordinates for the H atoms that have been used in our code to complete the valency of terminal atoms. Not all the bond types (such as aromatic bonds) are included within the code because they need special coding which hopefully will be done in the future. Optionally, the molecular coordinates can be stored in the database before optimization or after they are optimized. To store these molecules in the database, a special symbol (unique symbol for the target atom within the fragment) is needed.

## 5.2 Unique Symbol

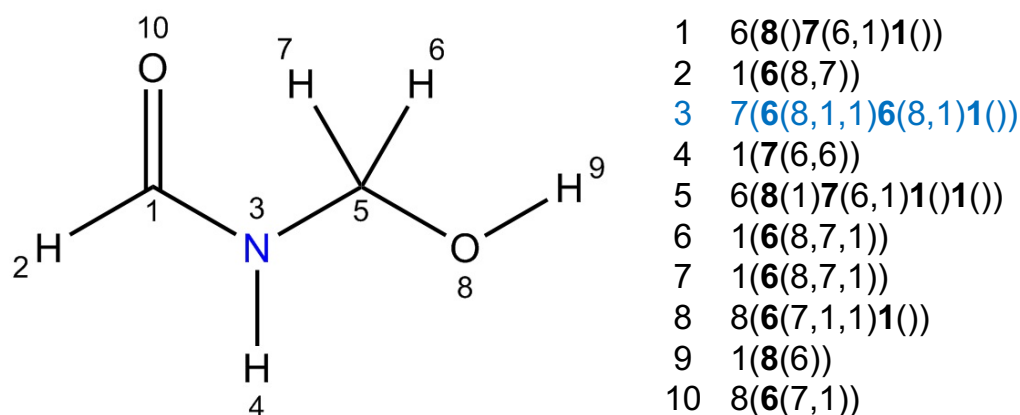
As mentioned in the previous section, a unique symbol (i.e., representation) for an atom within a fragment is very important in order to store the required information about that atom (such as its coordinates, connectivity, ...) in the database and to query this information when it is needed. Many representations of molecules exist in the literature. The Simplified Molecular-Input Line-Entry System (SMILES) [4] and SYBYL Line Notation (SLN) [5, 6]

**Table 5.2:** The default Cartesian coordinates for the hydrogen atoms to complete the valency of terminal atoms

| Bond Type   | Atom                         |    | Valance atoms coordinates (bohr ) |             |                     |
|-------------|------------------------------|----|-----------------------------------|-------------|---------------------|
|             |                              |    | x                                 | y           | z - BL <sup>1</sup> |
| Single bond | Carbon (-CH <sub>3</sub> )   | 1H | 0.00000000                        | -1.93061734 | 0.68284856          |
|             |                              | 2H | 1.67199927                        | 0.96543892  | 0.68284856          |
|             |                              | 3H | -1.67199914                       | 0.96557695  | 0.68284856          |
|             | Nitrogen (-NH <sub>2</sub> ) | 1H | 0.00000000                        | -1.80992957 | 0.55955062          |
|             |                              | 2H | -1.64331915                       | 0.75857027  | 0.55955062          |
|             | Oxygen (-OH)                 | 1H | 0.00000000                        | -1.81015167 | 0.46813705          |
| double bond | Carbon (=CH <sub>2</sub> )   | 1H | 0.00000000                        | 1.73196206  | 1.06759470          |
|             |                              | 2H | 0.00000000                        | -1.73196206 | 1.06759470          |
|             | Nitrogen (=NH)               | 1H | 0.00000000                        | 0.00000000  | 1.89445030          |
| Triple bond | Carbon ( $\equiv$ CH)        | 1H | 0.00000000                        | 0.00000000  | 2.03456410          |

<sup>1</sup> BL: bond length between terminal and first neighbour atoms.

are used for describing the structure of molecules in the form of line notations. Molfile [7] is a file format that is used for the storage of two- or three- dimensional coordinates for molecules. Structure-Data File (SDF) [7] contains a collection of properties related to the molecular structure. In addition, the International Chemical Identifier (InChI) representation, which was recommended by International Union of Pure and Applied Chemistry (IUPAC), provides a canonical representation of the molecule and offers different layers of description for the chemical structure [8]. Since we need a representation describing the target atom within a fragment rather than describing the entire molecule, all previous representations are not suitable for our research. The suggested symbol (representation) for



**Figure 5.6:** The second level fragment symbols within a molecule. The symbols show how the target atoms are connected for the second level, but does not show the coordinates of these atoms.

the atom of interest within its fragment is shown below,

$$Z(z_1, z_2, \dots)$$

where  $Z$  is the atomic number of atom of interest and  $z_i$  is the atomic number of atom  $i$  in the first neighbour level to the atom of interest, where the level is defined as how far through bonds the atom from the target atom. To extend the symbol for the second neighbour level, each atom in the first level is also written with its neighbour atoms. Therefore, the formula for the suggested symbol becomes,

$$Z(z_1(z_1^1, z_2^1, \dots)z_2(z_1^2, z_2^2, \dots) \dots)$$

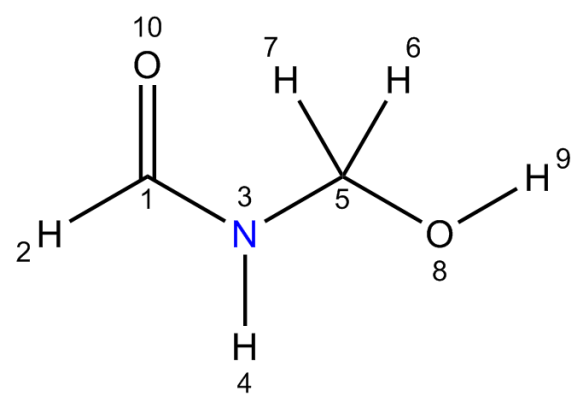
where  $z_i^1$  is the atomic number of the  $i^{th}$  atom in the first neighbour level to the atom of  $z_1$  and  $z_j^2$  is the atomic number of the  $j^{th}$  atom in the first neighbour level to the atom of  $z_2$ . A similar method is applied for higher levels. It is good to mention that each atom should not be included more than once in the symbol and atomic numbers of the atoms in

each neighbour level should be in descending order. For example, Figure 5.6 shows the symbols for ten fragments of a molecule for the second level. As shown in Figure 5.6, two or more fragments can share the same symbol such as fragments 6 and 7. Such fragments are considered to be the same and only one of which is stored in the database.

### 5.3 Labelling Atoms in a Fragment

In order to build a molecule of interest using fragments from the database, the atoms in the molecule of interest and in the database fragment should have the same labelling. The rules used for labelling the atoms are best illustrated with the example that is given in Figure 5.7. Figure 5.7 shows a fragment with numbering of the atoms as given in the input file. Suppose that the target atom is the nitrogen atom (atom number 3):

- Initially, all atoms in the fragment are given the same index: “the number one”.
- Sorting the atoms according to their levels (Criteria 1). The atoms are split into four different levels, two of which are already finalized because they consist of only one atom (atom 3 which is the target atom assigned to level 1, and atom number 9 which is the only atom in level 4).
- Fixing positions of atoms which are unique in a specific level, thus atoms 3 and 9 are fixed to positions 1 and 10, respectively (note that atom 9 takes the position 10 as it is the last atom in last level). So far, each of the second level atoms 1, 4, and 5 has given index 2 and each of the third level atoms 2, 6, 7, 8, and 10 has been given index 5 (note: they have been given index 5 because the four previous atoms have been indexed).



| Atom number | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9  | 10 |
|-------------|---|---|---|---|---|---|---|---|----|----|
| Initiation  | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1  | 1  |
| Criteria 1  | 2 | 5 | 1 | 2 | 2 | 5 | 5 | 5 | 10 | 5  |
| Criteria 2  | 2 | 7 | 1 | 4 | 2 | 7 | 7 | 5 | 10 | 5  |
| Criteria 3  | 3 | 7 | 1 | 4 | 2 | 7 | 7 | 5 | 10 | 6  |

**Figure 5.7:** An example for labeling the fragment's atoms using the suggested rules. In this example the nitrogen atom (atom number 3) is the target atom within the fragment. Not all atoms are sorted (in this case the hydrogen atoms 2, 7, and 8).

- Sorting the atoms according to their atomic number in each level (Criteria 2). The second level atoms are split into two different groups; a group of two C atoms (1 and 5) and a group of one H atom (4), the H group is already finalized and given the position 4. The third level atoms are also split into two different groups; a group consisting of three H atoms (2, 6, and 7), and a group consisting of two O atoms (8 and 10).
- Fixing positions of atoms which are unique in atomic number, thus atom 4 is fixed to position 4.
- Sorting the atoms according to the type of their first neighbour atoms and fixing their

positions (Criteria 3). Each of the two C atoms (1 and 5) has different neighbors; C (5) is connected to two H (6 and 7) and O (8), while C (1) is connected to one H (2) and one O (10), therefore the two carbon atoms are given positions 2 and 3 respectively. Similarly, O (8) is connected to C (5) and H (9), while O (10) is connected only to C (1), therefore they are given positions 5 and 6 respectively.

It is clear that not all terminal atoms are sorted; however, this is not a problem because a few sorted atoms are enough to reach the target of our research. Thus, the already sorted atoms are sufficient to compute the rotation and translation matrices needed to build the molecule of interest.

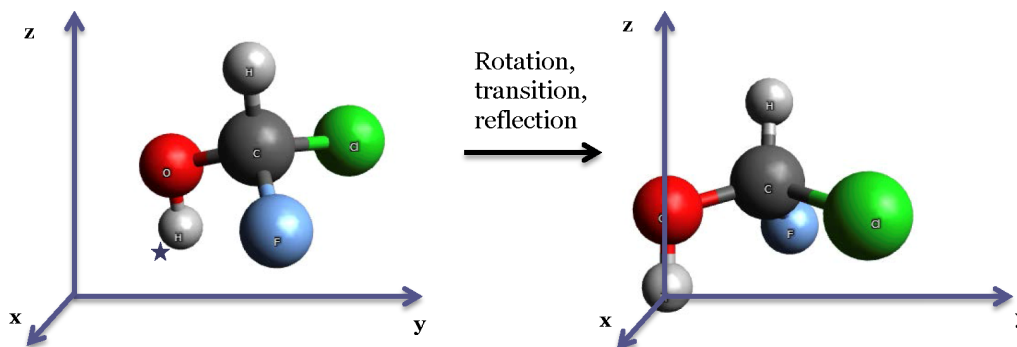
## 5.4 Storing in the Database

In this section, we consider the method for storing fragments in the database. In order to store a fragment in the database, Cartesian coordinates for the target atom and its neighbour atoms, as well as the radial grids of the target atom, should be transformed into the standard form (section 4.3). In section 4.3, the transformation of the original Cartesian coordinates with maximum of four points  $\mathbf{P}^0 = \{\mathbf{P}_1^0, \mathbf{P}_2^0, \mathbf{P}_3^0, \mathbf{P}_4^0\}$  into the standard Cartesian coordinates  $\mathbf{P} = \{\mathbf{P}_1, \mathbf{P}_2, \mathbf{P}_3, \mathbf{P}_4\}$  is discussed. As mentioned in the previous section (section 5.3), the target atom has a label = 1 and the other three atoms in the fragment have labels 2, 3, and 4. In the standard coordinates, the target atom ( $\mathbf{P}_1$ ) is located at the origin, the second ( $\mathbf{P}_2$ ), the third ( $\mathbf{P}_3$ ), and the forth ( $\mathbf{P}_4$ ) atoms are located on the positive  $z$  axis, in the  $yz$  plane with positive  $y$  axis, and in  $xyz$  space with positive  $x$  axis, respectively. In the origin fragment the four atoms have the same labelling (1, 2, 3, and 4) with coordinates of  $\mathbf{P}_1^0$ ,  $\mathbf{P}_2^0$ ,  $\mathbf{P}_3^0$ , and  $\mathbf{P}_4^0$ , respectively. Before starting the storing process, the existence of the target fragment in



the database is checked. If the fragment does not exist in the database, it can be stored as follows:

- Determine the optimal **R** and **L** matrices required to transform  $\mathbf{P}^0$  into **P** (see section 4.2).
- Use **R** and **L** matrices to transfer all original coordinates of all atoms in the fragment to the standard coordinates and store these coordinates in the database.
- Use **R** and **L** matrices to transfer the radial grid coordinates of the target atom to the standard coordinates and store these coordinates in the database with all required information such as the electron density at that position.

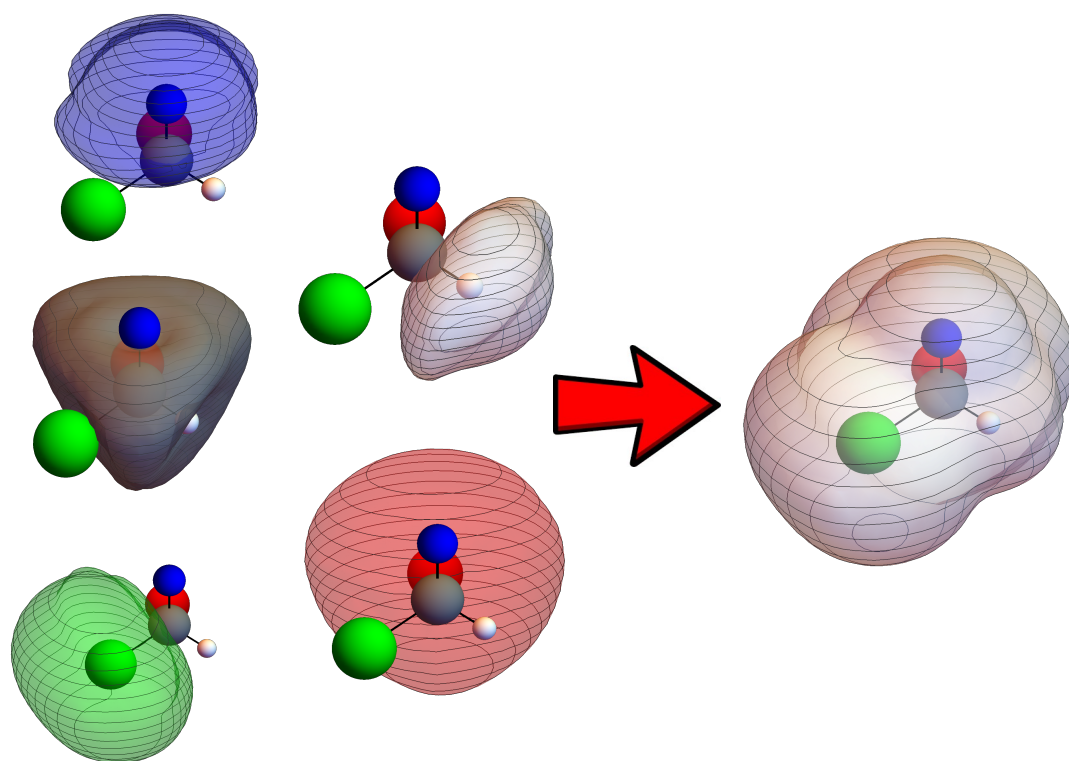


**Figure 5.8:** Rotation, translation and reflection of the target fragment atoms to obtain the new standard Cartesian coordinates.

Figure 5.8 shows an example for rotation, translation and reflection of a target fragment to obtain the standard coordinates.

## 5.5 Building the Atomic Electron Density for Molecule of Interest from the Database

In this section, the method that is used for building the target molecule from its atomic contributions in the database is illustrated (Figure 5.9). In order to build the molecular electron density, Cartesian coordinates for the radial grids of the target atom should be transformed from the database into the coordinates of molecule of interest. The following



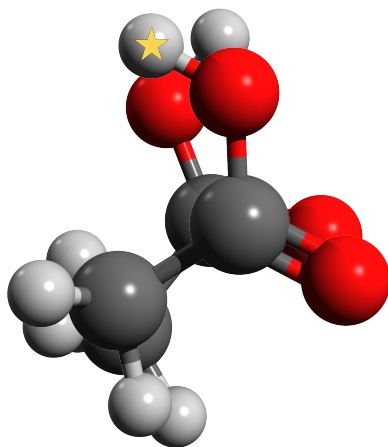
**Figure 5.9:** Building the molecular electron density for CClBrFH molecule from its atomic contributions.

steps have been used to achieve this goal:

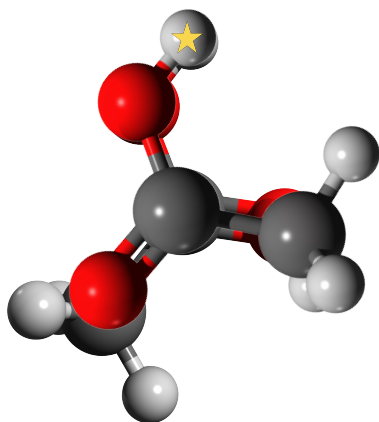
- Search within the database for the availability of each atomic fragment that is needed

in building the molecule of interest. If one of the fragments is not available in the database, the program gives an error message and stops.

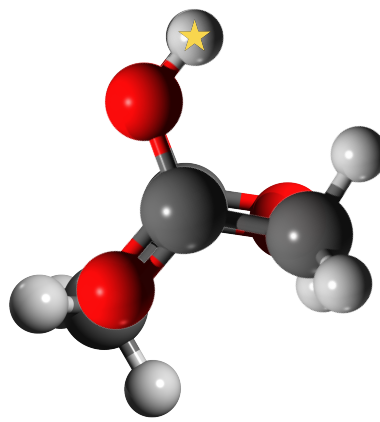
- Extract the coordinates of atoms for all fragments in the molecule (these are the original coordinates for the molecule that are available in the input file).
- Determine the optimal **R** and **L** matrices required to transform fragment coordinates in the database into coordinates for the molecule of interest (see section 4.2). Unfortunately, using **R** and **L** matrices gives approximate results in many cases. These approximations are due to the geometry of the fragment for the target atom in the molecule being different from its fragment in the database. In order to solve this problem, the following steps are used:
  - The target atom is given a large weight (e.g., 2000 times) and atoms located in level one and two are given weights equal to the cube and the square of their atomic number respectively.
  - Determine the optimal **R**<sub>1</sub> and **L**<sub>1</sub> matrices required to transform fragment coordinates in the database into coordinates for the molecule of interest.
  - Another correction is done by translating **L**<sub>2</sub> for the fragment atoms to make the target atoms exactly align (as shown in Figure 5.10).
- These **R**<sub>1</sub>, **L**<sub>1</sub> and **L**<sub>2</sub> matrices are also applied on the radial grid coordinates in the database.
- All the new coordinates for the atoms and radial grids are stored in memory later to perform calculations of the molecular properties.



(a) Before applying the weight



(b) After applying the weight and before the second correction



(c) After applying the weight and the second correction

**Figure 5.10:** Superimpose the electron density for H atom in two different fragments.

To make the process of extracting the information from the database easier to understand, the database structure will be briefly explained in the next section.

## **5.6 The Database Structure**

Mainly, the database consists of four parts: the index table, the table of atomic coordinates in fragments, atomic properties table, and grid points tables.

### **5.6.1 Index Table**

The index table can improve query performance by allowing applications to more quickly locate the data to retrieve from the database. The index fields contain the fragment information (i.e., index numbers, number of atoms, ...), the target atom, all neighbour atoms, unique symbols, and all the information that is required to build the database file such as (type of grid point (SG1), the number of grid points, basis sets, ...)

### **5.6.2 The Table of Atomic Coordinates in Fragments**

The table of atomic coordinates in fragments contains the Cartesians of all atoms of the fragments (the target atom and its adjacent atoms) in standard form (see Figure 5.11).

| index | Atom# | Symbol | Atoms coordinates |             |             |
|-------|-------|--------|-------------------|-------------|-------------|
|       |       |        | x                 | y           | z           |
| 1     | 1     | N      | -0.00000000       | -0.00000000 | -0.00000000 |
| 1     | 2     | C      | -0.00000000       | 0.00000000  | 2.75105551  |
| 1     | 3     | H      | -0.00000000       | 1.77530805  | -0.66135862 |
| 1     | 4     | H      | 1.55817246        | -0.83684605 | -0.67505690 |
| 1     | 5     | C      | -2.44253065       | 1.14781001  | 3.74911504  |
| 1     | 6     | H      | 0.14082685        | -1.94831120 | 3.37532447  |
| 1     | 7     | H      | 1.61112811        | 1.00202446  | 3.56343617  |
| 1     | 8     | H      | -2.49533999       | 1.10132887  | 5.79954205  |
| 1     | 9     | H      | -4.05866791       | 0.11929414  | 3.02374509  |
| 1     | 10    | H      | -2.62572328       | 3.10852077  | 3.16403672  |
| 2     | 1     | C      | -0.00000000       | 0.00000000  | 0.00000000  |
| 2     | 2     | N      | -0.00000000       | -0.00000000 | 2.72673483  |
| 2     | 3     | C      | 0.00000000        | 2.67370798  | -0.98681242 |
| 2     | 4     | H      | 1.66412358        | -0.95687825 | -0.70841454 |
| 2     | 5     | H      | -1.63023945       | -0.91857121 | -0.87412323 |
| 2     | 6     | O      | 0.73389535        | 2.78558377  | -3.38519479 |
| 2     | 7     | O      | -0.65953675       | 4.47859262  | 0.17007957  |
| 2     | 8     | H      | -1.26242488       | 1.23880601  | 3.39503031  |
| 2     | 9     | H      | -0.42062851       | -1.71298806 | 3.40485449  |
| 2     | 10    | H      | 0.61017964        | 4.49551026  | -3.93316048 |
| 3     | 1     | C      | 0.00000000        | -0.00000000 | 0.00000000  |
| 3     | 2     | O      | -0.00000000       | 0.00000000  | 2.51922813  |
| 3     | 3     | O      | 0.00000000        | 1.91584741  | -1.14523169 |
| 3     | 4     | C      | 0.13718953        | -2.58535095 | -1.23415937 |

**Figure 5.11:** Example of the table of atomic coordinates in fragments.

### 5.6.3 Atomic Properties Table

The atomic properties table contains the atomic properties such as number of electrons, potential energy, exchange energy, and kinetic energy.

| #  | Electrons  | Vne_Anal      | Vne_Num       | Vee         | Exchange    | Kinetic     |
|----|------------|---------------|---------------|-------------|-------------|-------------|
| 1  | 7.17023098 | -167.69150164 | -168.94829210 | 44.38568770 | -6.64508877 | 53.99759251 |
| 2  | 6.10412832 | -147.49059857 | -149.54784203 | 48.02137025 | -5.27840352 | 37.98931031 |
| 3  | 6.12393590 | -156.59233845 | -159.43844553 | 48.70876440 | -5.37452974 | 38.69595392 |
| 4  | 8.14254450 | -232.27907844 | -231.68841750 | 66.56720187 | -8.20100789 | 74.21174504 |
| 5  | 0.93848127 | -8.64949861   | -7.87636617   | 4.95036302  | -0.33245444 | 0.59184471  |
| 6  | 0.91258133 | -7.27782717   | -6.56666390   | 3.43856109  | -0.33860744 | 0.68744836  |
| 7  | 8.01305440 | -231.96480014 | -231.44892209 | 60.98631847 | -8.19022679 | 74.10709681 |
| 8  | 0.87780336 | -7.75940322   | -6.92628950   | 3.40707809  | -0.34436075 | 0.79917164  |
| 9  | 7.16507596 | -178.63941845 | -179.96403976 | 49.35095418 | -6.64341230 | 53.99839778 |
| 10 | 6.12406800 | -161.16149114 | -163.66636052 | 53.85300839 | -5.28871878 | 38.02452901 |
| 11 | 6.12272372 | -166.08348069 | -169.03395019 | 52.26454970 | -5.37412357 | 38.70963401 |
| 12 | 0.92491218 | -10.45701213  | -9.40375990   | 6.03059905  | -0.32935389 | 0.58402255  |
| 13 | 6.12901262 | -133.32490844 | -135.08251948 | 39.60376836 | -5.25774169 | 37.85059475 |
| 14 | 0.95506348 | -6.71658982   | -6.19719802   | 3.60926891  | -0.33401225 | 0.59382795  |
| 15 | 6.11254657 | -180.72207797 | -183.47528396 | 64.67883359 | -5.29140246 | 38.05579564 |

**Figure 5.12:** Example of the table of atomic properties.

## 5.6.4 Grid Points Tables

Grid points tables contain the Cartesian coordinates of the radial grid points, with the atomic properties at a specific grid point (such as, electron density, angular weight for numerical integration, partitioning weight, ...). For each atom there is a unique file name containing its properties.

| X           | Y           | Z           | Rho            | BW             | AW             | Vpol        |
|-------------|-------------|-------------|----------------|----------------|----------------|-------------|
| -0.01831060 | 0.01831060  | 0.00000000  | 0.42399444E+00 | 0.10000000E+01 | 0.26508891E-08 | 12.50446618 |
| 0.01831060  | 0.01831060  | 0.00000000  | 0.42405088E+00 | 0.10000000E+01 | 0.26508891E-08 | 12.51811693 |
| -0.01831060 | -0.01831060 | 0.00000000  | 0.42409066E+00 | 0.10000000E+01 | 0.26508891E-08 | 12.49843237 |
| 0.01831060  | -0.01831060 | 0.00000000  | 0.42414790E+00 | 0.10000000E+01 | 0.26508891E-08 | 12.51231617 |
| 0.00000000  | 0.01831060  | 0.01831060  | 0.42798607E+00 | 0.10000000E+01 | 0.26508891E-08 | 12.55707167 |
| 0.00000000  | 0.01831060  | -0.01831060 | 0.42019760E+00 | 0.10000000E+01 | 0.26508891E-08 | 12.46614573 |
| 0.00000000  | -0.01831060 | 0.01831060  | 0.42808615E+00 | 0.10000000E+01 | 0.26508891E-08 | 12.55109776 |
| 0.00000000  | -0.01831060 | -0.01831060 | 0.42029089E+00 | 0.10000000E+01 | 0.26508891E-08 | 12.46028320 |
| -0.01831060 | -0.00000000 | 0.01831060  | 0.42800810E+00 | 0.10000000E+01 | 0.26508891E-08 | 12.54711734 |
| -0.01831060 | -0.00000000 | -0.01831060 | 0.42021760E+00 | 0.10000000E+01 | 0.26508891E-08 | 12.45634761 |
| 0.01831060  | -0.00000000 | 0.01831060  | 0.42806642E+00 | 0.10000000E+01 | 0.26508891E-08 | 12.56098496 |
| 0.01831060  | -0.00000000 | -0.01831060 | 0.42027305E+00 | 0.10000000E+01 | 0.26508891E-08 | 12.47001535 |
| -0.01495054 | 0.01495054  | 0.01495054  | 0.42723408E+00 | 0.10000000E+01 | 0.26301338E-08 | 12.54244926 |
| -0.01495054 | 0.01495054  | -0.01495054 | 0.42087578E+00 | 0.10000000E+01 | 0.26301338E-08 | 12.46828486 |
| 0.01495054  | 0.01495054  | 0.01495054  | 0.42728120E+00 | 0.10000000E+01 | 0.26301338E-08 | 12.55367887 |
| 0.01495054  | 0.01495054  | -0.01495054 | 0.42092100E+00 | 0.10000000E+01 | 0.26301338E-08 | 12.47938222 |
| -0.01495054 | -0.01495054 | 0.01495054  | 0.42731499E+00 | 0.10000000E+01 | 0.26301338E-08 | 12.53750221 |

**Figure 5.13:** Example of a table of grid points.

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# Chapter 6

## Energy Components

*“Not only is the Universe stranger than we think, it is stranger than we can think.”*

— Werner Heisenberg

## 6.1 Theoretical Background

Solving the Schrödinger equation or Kohn-Sham equations for large molecules consisting of several hundreds or thousands of atoms is not possible to deal with. Many strategies for dealing with large systems such as linear scaling methods [1–4] have attracted much attention and have become popular in the chemistry community. The reason why large systems containing many atoms are accessible with these algorithms is their linear scaling with respect to the number of atoms [5]. The divide-and-conquer method is an example of a linear scaling algorithm. This method was originally proposed for density functional theory [6]. The basic idea of the divide-and-conquer method is to break down a large system (entire system) into small subsystems. In this method, the effect of the surrounding environment is included, where a subsystem within the large system is different from the isolated ones. The higher the accuracy desired, the more correction terms are needed, which will greatly increase computational cost. One drawback of the divide-and-conquer method is that it does not provide information on the electronic structure of the entire system since it operates only on the energy of the subsystems [7].

## 6.2 Methodology of Our Theory

In this work, our intention is to obtain the electronic molecular energy components from the summation of their atomic contributions as,

$$T = \sum_{A=1}^M T^A, \quad K = \sum_{A=1}^M K^A, \quad V_{ne} = \sum_{A=1}^M V_{ne}^A, \quad J = \sum_{A=1}^M J^A \quad (6.1)$$

where  $M$  is number of atoms within the molecule.  $T$ ,  $K$ ,  $V_{ne}$ , and  $J$  are the molecular kinetic energy, the molecular exchange energy, the molecular electron-nuclear attraction energy, and the molecular Coulomb repulsion energy, respectively.  $T^A$ ,  $K^A$ ,  $V_{ne}^A$ , and  $J^A$  are the kinetic energy, the exchange energy, the electron-nuclear attraction energy, and the Coulomb repulsion energy of atom  $A$  in the molecule, respectively.

In the next sections, we will discuss the derivation of  $T^A$ ,  $K^A$ ,  $V_{ne}^A$ , and  $J^A$ .

### 6.2.1 Kinetic Energy ( $T$ )

The molecular kinetic energy [8] can be defined as,

$$\begin{aligned} T &= \frac{1}{2} \sum_a^N \int |\nabla \psi_a(\mathbf{r})|^2 d\mathbf{r} \\ &= \frac{1}{2} \sum_a^N \int \nabla \psi_a^*(\mathbf{r}) \nabla \psi_a(\mathbf{r}) d\mathbf{r} \end{aligned} \quad (6.2)$$

where  $\nabla \psi_a(\mathbf{r})$  is the gradient of molecular orbital  $a$  at position  $\mathbf{r}$  and the summation runs over number of the electrons ( $N$ ) of the system. The  $\psi_a(\mathbf{r})$  can be expanded as a linear combination of basis functions  $\{\varphi(\mathbf{r})\}$ ,

$$\psi_a(\mathbf{r}) = \sum_{\mu=1}^k C_{\mu a} \varphi_{\mu}(\mathbf{r}) \quad (6.3)$$

where  $k$  is the number of basis functions, and  $C_{\mu a}$  are the expansion coefficients. Consequently, the kinetic energy (Equation 6.2) becomes,

$$T = \frac{1}{2} \sum_a^N \int \sum_{\mu} C_{\mu a}^* \nabla \varphi_{\mu}^*(\mathbf{r}) \sum_{\nu} C_{\nu a} \nabla \varphi_{\nu}(\mathbf{r}) d\mathbf{r} \quad (6.4)$$

For RHF, Equation 6.4 can be rearranged into,

$$\begin{aligned} T &= \frac{1}{2} \sum_{\mu} \sum_{\nu} \left( 2 \sum_a^{N/2} C_{\mu a}^* C_{\nu a} \right) \int \nabla \phi_{\mu}^*(\mathbf{r}) \nabla \phi_{\nu}(\mathbf{r}) d\mathbf{r} \\ &= \frac{1}{2} \sum_{\mu} \sum_{\nu} P_{\nu\mu} \int dG_{\mu\nu}(\mathbf{r}) d\mathbf{r} \end{aligned} \quad (6.5)$$

where,

$$P_{\nu\mu} = 2 \sum_a^{N/2} C_{\mu a}^* C_{\nu a} \quad (6.6)$$

and

$$dG_{\mu\nu}(\mathbf{r}) = \nabla \phi_{\mu}^*(\mathbf{r}) \nabla \phi_{\nu}(\mathbf{r}) \quad (6.7)$$

The integral term in Equation 6.5 can be computed as,

$$\int dG_{\mu\nu}(\mathbf{r}) d\mathbf{r} = \sum_A \sum_i W_A(\mathbf{r}_i) \omega(\mathbf{r}_i) dG_{\mu\nu}(\mathbf{r}_i) \quad (6.8)$$

where  $\mathbf{r}_i$  is the  $i^{th}$  grid point,  $W_A(\mathbf{r}_i)$  is the partition function of atom  $A$  at  $\mathbf{r}_i$ , and  $\omega(\mathbf{r}_i)$  is the product of angular and radial weights at  $\mathbf{r}_i$ .

By substituting Equation 6.8 into Equation 6.5 one obtains,

$$\begin{aligned} T &= \frac{1}{2} \sum_A \sum_i W_A(\mathbf{r}_i) \omega(\mathbf{r}_i) \left( \sum_{\mu} \sum_{\nu} P_{\nu\mu} dG_{\mu\nu}(\mathbf{r}_i) \right) \\ &= \frac{1}{2} \sum_A \sum_i W_A(\mathbf{r}_i) \omega(\mathbf{r}_i) S(\mathbf{r}_i) \end{aligned} \quad (6.9)$$

where

$$S(\mathbf{r}_i) = \sum_{\mu} \sum_{\nu} P_{\nu\mu} dG_{\mu\nu}(\mathbf{r}_i) \quad (6.10)$$

Using Equation 6.9, the atomic kinetic energy can be defined as,

$$\begin{aligned}
 T^A &= \frac{1}{2} \sum_i W_A(\mathbf{r}_i) \omega(\mathbf{r}_i) S(\mathbf{r}_i) \\
 &= \sum_i \omega(\mathbf{r}_i) \left( \frac{1}{2} W_A(\mathbf{r}_i) S(\mathbf{r}_i) \right) \\
 &= \sum_i \omega(\mathbf{r}_i) T^A(\mathbf{r}_i)
 \end{aligned} \tag{6.11}$$

where

$$T^A(\mathbf{r}_i) = \frac{1}{2} W_A(\mathbf{r}_i) S(\mathbf{r}_i) \tag{6.12}$$

The molecular kinetic energy density at a given grid point  $\mathbf{r}_i$  can be defined as,

$$T(\mathbf{r}_i) = \sum_A T^A(\mathbf{r}_i) \tag{6.13}$$

The radial molecular kinetic energy density at  $\mathbf{r}_i$  is obtained by multiplying the last equation with  $\mathbf{r}_{iA}^2$ ,

$$T_{rad}(\mathbf{r}_i) = \sum_A \mathbf{r}_{iA}^2 T^A(\mathbf{r}_i) \tag{6.14}$$

### 6.2.2 Exchange Energy ( $K$ )

The HF equation of ground state energy for RHF system,  $E_0$ , is defined as,

$$\begin{aligned}
 E_0 &= 2 \sum_a^{N/2} h_{aa} + \sum_{ab}^{N/2} 2J_{ab} - K_{ab} \\
 &= 2 \sum_a^{N/2} h_{aa} + J_{HF} - K_{HF}
 \end{aligned} \tag{6.15}$$

where  $h_{aa}$  is the kinetic and attraction energy to all nuclei of an electron.  $J_{ab}$  and  $K_{ab}$  are called the Coulomb and exchange integrals respectively.  $J_{ab}$  and  $K_{ab}$  always have positive values. The term  $K_{HF}$  in Equation 6.15, which contains the Coulomb repulsion  $J_{aa}$  is defined as,

$$\begin{aligned} K_{HF} &= \sum_b \sum_a K_{ab} \\ &= 2 \sum_b \sum_{b < a} K_{ab} + \sum_a J_{aa} \end{aligned} \quad (6.16)$$

The first term in the last equation is the exchange energy,

$$K = 2 \sum_b \sum_{b < a} K_{ab} \quad (6.17)$$

which represents the interaction energies between electrons with parallel spins. The second term in Equation 6.16 is canceled by an equivalent term in the Coulomb energy in the HF equation (Equation 6.15).

The two-electron integral  $K_{ab}$  does not have a simple classical interpretation.

$$K_{ab} = \int \int \frac{\psi_a^*(\mathbf{r}_1) \psi_b^*(\mathbf{r}_2) \psi_b(\mathbf{r}_1) \psi_a(\mathbf{r}_2)}{r_{12}} d\mathbf{r}_2 d\mathbf{r}_1 \quad (6.18)$$

For simplicity,  $K_{ab}$  can be rearranged and written as,

$$\begin{aligned} K_{ab} &= \int \psi_a^*(\mathbf{r}_1) \psi_b(\mathbf{r}_1) \left[ \int \frac{\psi_b^*(\mathbf{r}_2) \psi_a(\mathbf{r}_2)}{r_{12}} d\mathbf{r}_2 \right] d\mathbf{r}_1 \\ &= \int \psi_a^*(\mathbf{r}_1) \psi_b(\mathbf{r}_1) V^{ba}(\mathbf{r}_1) d\mathbf{r}_1 \end{aligned} \quad (6.19)$$

where

$$V^{ba}(\mathbf{r}_1) = \int \frac{\psi_b^*(\mathbf{r}_2)\psi_a(\mathbf{r}_2)}{\mathbf{r}_{12}} d\mathbf{r}_2 \quad (6.20)$$

Using Equation 6.3, the product  $\psi_a^*(\mathbf{r}_1)\psi_b(\mathbf{r}_1)$  in Equation 6.19 becomes,

$$\begin{aligned} \psi_a^*(\mathbf{r}_1)\psi_b(\mathbf{r}_1) &= \sum_{\lambda} C_{\lambda a}^* \phi_{\lambda}^*(\mathbf{r}_1) \sum_{\sigma} C_{\sigma b} \phi_{\sigma}(\mathbf{r}_1) \\ &= \sum_{\lambda} \sum_{\sigma} C_{\lambda a}^* C_{\sigma b} \phi_{\lambda}^*(\mathbf{r}_1) \phi_{\sigma}(\mathbf{r}_1) \\ &= \sum_{\lambda} \sum_{\sigma} P_{\sigma\lambda}^{ab} G_{\lambda\sigma}(\mathbf{r}_1) \end{aligned} \quad (6.21)$$

where  $P_{\sigma\lambda}^{ab}$  is the product of  $C_{\lambda a}^*$  with the  $C_{\sigma b}$ , and  $G_{\lambda\sigma}(\mathbf{r}_1)$  is the product of  $\phi_{\lambda}^*(\mathbf{r}_1)$  with  $\phi_{\sigma}(\mathbf{r}_1)$ . Here we define  $S^{ab}(\mathbf{r}_1)$  as follows,

$$S^{ab}(\mathbf{r}_1) = \sum_{\lambda} \sum_{\sigma} P_{\lambda\sigma}^{ab} G_{\sigma\lambda}(\mathbf{r}_1) \quad (6.22)$$

The substitution of Equation 6.3 into Equation 6.20 gives,

$$\begin{aligned} V^{ba}(\mathbf{r}_1) &= \int \frac{\psi_b^*(\mathbf{r}_2)\psi_a(\mathbf{r}_2)}{\mathbf{r}_{12}} d\mathbf{r}_2 \\ &= \int \frac{\sum_{\mu} C_{\mu b}^* \phi_{\mu}^*(\mathbf{r}_2) \sum_{\nu} C_{\nu a} \phi_{\nu}(\mathbf{r}_2)}{\mathbf{r}_{12}} d\mathbf{r}_2 \end{aligned} \quad (6.23)$$

The last equation may be rearranged to obtain,

$$\begin{aligned} V^{ba}(\mathbf{r}_1) &= \sum_{\mu} \sum_{\nu} C_{\mu b}^* C_{\nu a} \int \frac{\phi_{\mu}^*(\mathbf{r}_2)\phi_{\nu}(\mathbf{r}_2)}{\mathbf{r}_{12}} d\mathbf{r}_2 \\ &= \sum_{\mu} \sum_{\nu} P_{\nu\mu}^{ba} V_{\mu\nu}(\mathbf{r}_1) \end{aligned} \quad (6.24)$$



where  $P_{\nu\mu}^{ba}$  is the product of  $C_{\mu b}^*$  with  $C_{\nu a}$  and  $V_{\mu\nu}(\mathbf{r}_1)$  is,

$$V_{\mu\nu}(\mathbf{r}_1) = \int \frac{\varphi_{\mu}^*(\mathbf{r}_2)\varphi_{\nu}(\mathbf{r}_2)}{\mathbf{r}_{12}} d\mathbf{r}_2 \quad (6.25)$$

Equation 6.19 can be rewritten as,

$$K_{ab} = \int S^{ab}(\mathbf{r}_1) V^{ba}(\mathbf{r}_1) d\mathbf{r}_1 \quad (6.26)$$

$K_{ab}$  can be calculated numerically as,

$$\begin{aligned} K_{ab} &= \sum_A \sum_i W_A(\mathbf{r}_i) \omega(\mathbf{r}_i) S^{ab}(\mathbf{r}_i) V^{ba}(\mathbf{r}_i) \\ &= \sum_A \sum_i \omega(\mathbf{r}_i) \left( W_A(\mathbf{r}_i) S^{ab}(\mathbf{r}_i) V^{ba}(\mathbf{r}_i) \right) \\ &= \sum_A \sum_i \omega(\mathbf{r}_i) K_{ab}^A(\mathbf{r}_i) \end{aligned} \quad (6.27)$$

where,

$$K_{ab}^A(\mathbf{r}_i) = W_A(\mathbf{r}_i) S^{ab}(\mathbf{r}_i) V^{ba}(\mathbf{r}_i) \quad (6.28)$$

Using Equation 6.27, the atomic two-electron integral  $K_{ab}^A$  can be obtained as,

$$K_{ab}^A = \sum_i \omega(\mathbf{r}_i) K_{ab}^A(\mathbf{r}_i) \quad (6.29)$$

Using Equation 6.27, the exchange energy ( $K$ ) and  $K_{HF}$  term can be calculated by summing over the molecular orbitals to get,

$$K = 2 \sum_A \sum_b \sum_{b < a} \sum_i \omega(\mathbf{r}_i) K_{ab}^A(\mathbf{r}_i) \quad (6.30)$$

and

$$K_{HF} = 2 \sum_A \sum_b \sum_{b < a} \sum_i \omega(\mathbf{r}_i) K_{ab}^A(\mathbf{r}_i) + \sum_A \sum_a \sum_i \omega(\mathbf{r}_i) K_{aa}^A(\mathbf{r}_i) \quad (6.31)$$

Using Equation 6.30, the atomic exchange energy ( $K^A$ ) can be computed as,

$$\begin{aligned} K^A &= 2 \sum_b \sum_{b < a} \sum_i \omega(\mathbf{r}_i) K_{ab}^A(\mathbf{r}_i) \\ &= \sum_i \omega(\mathbf{r}_i) \left( 2 \sum_b \sum_{b < a} K_{ab}^A(\mathbf{r}_i) \right) \\ &= \sum_i \omega(\mathbf{r}_i) K^A(\mathbf{r}_i) \end{aligned} \quad (6.32)$$

where,

$$K^A(\mathbf{r}_i) = 2 \sum_b \sum_{b < a} K_{ab}^A(\mathbf{r}_i) \quad (6.33)$$

And by using Equation 6.31 the  $K_{HF}^A$  term can be computed as,

$$K_{HF}^A = \sum_i \omega(\mathbf{r}_i) K_{HF}^A(\mathbf{r}_i) \quad (6.34)$$

where

$$K_{HF}^A(\mathbf{r}_i) = 2 \sum_b \sum_{b < a} K_{ab}^A(\mathbf{r}_i) + \sum_a K_{aa}^A(\mathbf{r}_i) \quad (6.35)$$

The molecular exchange energy density at the grid point  $\mathbf{r}_i$  can be defined as,

$$K(\mathbf{r}_i) = \sum_A K^A(\mathbf{r}_i) \quad (6.36)$$

and by multiplying the last equation with  $\mathbf{r}_{iA}^2$ , the radial exchange energy density of a molecule can be obtained,

$$K_{rad}(\mathbf{r}_i) = \sum_A \mathbf{r}_{iA}^2 K^A(\mathbf{r}_i) \quad (6.37)$$

### 6.2.3 Coulomb Energy ( $J$ )

The term  $J_{HF}$  in Equation 6.15 is defined as,

$$\begin{aligned} J_{HF} &= 2 \sum_b \sum_a J_{ab} \\ &= 2 \left( 2 \sum_b \sum_{b < a} J_{ab} + \sum_a J_{aa} \right) \\ &= 4 \sum_b \sum_{b < a} J_{ab} + 2 \sum_a J_{aa} \end{aligned} \quad (6.38)$$

Whereas, the Coulomb energy ( $J$ ) is,

$$J = 4 \sum_b \sum_{b < a} J_{ab} + \sum_a J_{aa} \quad (6.39)$$

The two-electron integral  $J_{ab}$  represents the Coulomb repulsion between charge clouds  $|\psi_a(\mathbf{r}_1)|^2$  and  $|\psi_b(\mathbf{r}_2)|^2$ .

$$\begin{aligned} J_{ab} &= \int \int \frac{\psi_a^*(\mathbf{r}_1)\psi_b^*(\mathbf{r}_2)\psi_a(\mathbf{r}_1)\psi_b(\mathbf{r}_2)}{r_{12}} d\mathbf{r}_1 d\mathbf{r}_2 \\ &= \int \int \frac{|\psi_a(\mathbf{r}_1)|^2 |\psi_b(\mathbf{r}_2)|^2}{r_{12}} d\mathbf{r}_1 d\mathbf{r}_2 \end{aligned} \quad (6.40)$$

$J_{ab}$  is called the Coulomb integral and always has positive values. In this section, we will derive the Coulomb energy using the molecular orbital expansion method and the electron density method.

### 6.2.3.1 Molecular Orbital Expansion Method

Using Equation 6.40,  $J_{ab}$  can be rewritten as,

$$\begin{aligned} J_{ab} &= \int |\psi_a(\mathbf{r}_1)|^2 \left[ \int \frac{|\psi_b(\mathbf{r}_2)|^2}{r_{12}} d\mathbf{r}_2 \right] d\mathbf{r}_1 \\ &= \int |\psi_a(\mathbf{r}_1)|^2 V^{bb}(\mathbf{r}_1) d\mathbf{r}_1 \end{aligned} \quad (6.41)$$

where  $V^{bb}(\mathbf{r}_1)$  is defined as,

$$V^{bb}(\mathbf{r}_1) = \int \frac{|\psi_b(\mathbf{r}_2)|^2}{r_{12}} d\mathbf{r}_2 \quad (6.42)$$

Using Equation 6.3 the  $|\psi_a(\mathbf{r}_1)|^2$  term in Equation 6.41 becomes,

$$\begin{aligned}
|\psi_a(\mathbf{r}_1)|^2 &= \psi_a^*(\mathbf{r}_1)\psi_a(\mathbf{r}_1) \\
&= \sum_{\lambda} C_{\lambda a}^* \phi_{\lambda}^*(\mathbf{r}_1) \sum_{\sigma} C_{\sigma a} \phi_{\sigma}(\mathbf{r}_1) \\
&= \sum_{\lambda} \sum_{\sigma} C_{\lambda a}^* C_{\sigma a} \phi_{\lambda}^*(\mathbf{r}_1) \phi_{\sigma}(\mathbf{r}_1) \\
&= \sum_{\lambda} \sum_{\sigma} P_{\sigma\lambda}^{aa} G_{\lambda\sigma}(\mathbf{r}_1)
\end{aligned} \tag{6.43}$$

where  $P_{\sigma\lambda}^{aa}$  is the product of  $C_{\lambda a}^*$  with  $C_{\sigma a}$  and  $G_{\lambda\sigma}(\mathbf{r}_1)$  is the product of  $\phi_{\lambda}^*(\mathbf{r}_1)$  with  $\phi_{\sigma}(\mathbf{r}_1)$ . Furthermore, the substitution of the molecular orbital expansion 6.3 into Equation 6.42 gives,

$$\begin{aligned}
V^{bb}(\mathbf{r}_1) &= \int \frac{|\psi_b(\mathbf{r}_2)|^2}{\mathbf{r}_{12}} d\mathbf{r}_2 \\
&= \int \frac{\sum_{\mu} C_{\mu b}^* \phi_{\mu}^*(\mathbf{r}_2) \sum_{\nu} C_{\nu b} \phi_{\nu}(\mathbf{r}_2)}{\mathbf{r}_{12}} d\mathbf{r}_2 \\
&= \sum_{\mu} \sum_{\nu} C_{\mu b}^* C_{\nu b} \int \frac{\phi_{\mu}^*(\mathbf{r}_2) \phi_{\nu}(\mathbf{r}_2)}{\mathbf{r}_{12}} d\mathbf{r}_2 \\
&= \sum_{\mu} \sum_{\nu} P_{\nu\mu}^{bb} V_{\mu\nu}(\mathbf{r}_1)
\end{aligned} \tag{6.44}$$

where  $P_{\nu\mu}^{bb}$  is the product of  $C_{\mu b}^*$  with  $C_{\nu b}$  and  $V_{\mu\nu}(\mathbf{r}_1)$  is defined in Equation 6.25. Equation 6.41 can be written as,

$$\begin{aligned}
J_{ab} &= \int \sum_{\lambda} \sum_{\sigma} P_{\sigma\lambda}^{aa} G_{\lambda\sigma}(\mathbf{r}_1) \sum_{\mu} \sum_{\nu} P_{\nu\mu}^{bb} V_{\mu\nu}(\mathbf{r}_1) d\mathbf{r}_1 \\
&= \int S^{aa}(\mathbf{r}_1) V^{bb}(\mathbf{r}_1) d\mathbf{r}_1
\end{aligned} \tag{6.45}$$

where  $S^{aa}(\mathbf{r}_1)$  is defined as,

$$S^{aa}(\mathbf{r}_1) = \sum_{\lambda} \sum_{\sigma} P_{\sigma\lambda}^{aa} G_{\lambda\sigma}(\mathbf{r}_1) \quad (6.46)$$

$J_{ab}$  can be calculated numerically as follows,

$$\begin{aligned} J_{ab} &= \sum_A \sum_i W_A(\mathbf{r}_i) \omega(\mathbf{r}_i) S^{aa}(\mathbf{r}_i) V^{bb}(\mathbf{r}_i) \\ &= \sum_A \sum_i \omega(\mathbf{r}_i) \left( W_A(\mathbf{r}_i) S^{aa}(\mathbf{r}_i) V^{bb}(\mathbf{r}_i) \right) \\ &= \sum_A \sum_i \omega(\mathbf{r}_i) J_{ab}^A(\mathbf{r}_i) \end{aligned} \quad (6.47)$$

where,

$$J_{ab}^A(\mathbf{r}_i) = W_A(\mathbf{r}_i) S^{aa}(\mathbf{r}_i) V^{bb}(\mathbf{r}_i) \quad (6.48)$$

Using Equation 6.47, the atomic two-electron integral  $J_{ab}^A$  can be obtained as,

$$J_{ab}^A = \sum_i \omega(\mathbf{r}_i) J_{ab}^A(\mathbf{r}_i) \quad (6.49)$$

Using Equation 6.47, the Coulomb energy ( $J$ ) and  $J_{HF}$  term can be computed by summing over molecular orbitals to get,

$$\begin{aligned} J &= 4 \sum_A \sum_b \sum_{b < a} \sum_i \omega(\mathbf{r}_i) J_{ab}^A(\mathbf{r}_i) + \sum_A \sum_a \sum_i \omega(\mathbf{r}_i) J_{aa}^A(\mathbf{r}_i) \\ &= \sum_A \sum_i \omega(\mathbf{r}_i) \left( 4 \sum_b \sum_{b < a} J_{ab}^A(\mathbf{r}_i) + \sum_a J_{aa}^A(\mathbf{r}_i) \right) \end{aligned} \quad (6.50)$$

and

$$J_{HF} = \sum_A \sum_i \omega(\mathbf{r}_i) \left( 4 \sum_b \sum_{b < a} J_{ab}^A(\mathbf{r}_i) + 2 \sum_a J_{aa}^A(\mathbf{r}_i) \right) \quad (6.51)$$

Using Equation 6.50, the atomic Coulomb energy  $J^A$  can be computed as,

$$\begin{aligned} J^A &= \sum_i \omega(\mathbf{r}_i) \left( 4 \sum_b \sum_{b < a} J_{ab}^A(\mathbf{r}_i) + \sum_a J_{aa}^A(\mathbf{r}_i) \right) \\ &= \sum_i \omega(\mathbf{r}_i) J^A(\mathbf{r}_i) \end{aligned} \quad (6.52)$$

where,

$$J^A(\mathbf{r}_i) = 4 \sum_b \sum_{b < a} J_{ab}^A(\mathbf{r}_i) + \sum_a J_{aa}^A(\mathbf{r}_i) \quad (6.53)$$

Using Equation 6.51, the  $J_{HF}^A$  term can be computed as,

$$J_{HF}^A = \sum_i \omega(\mathbf{r}_i) J_{HF}^A(\mathbf{r}_i) \quad (6.54)$$

where,

$$J_{HF}^A(\mathbf{r}_i) = 4 \sum_b \sum_{b < a} J_{ab}^A(\mathbf{r}_i) + 2 \sum_a J_{aa}^A(\mathbf{r}_i) \quad (6.55)$$

The molecular Coulomb energy density at the grid point  $\mathbf{r}_i$  can be defined as,

$$J(\mathbf{r}_i) = \sum_A J^A(\mathbf{r}_i) \quad (6.56)$$

and by multiplying the last equation with  $\mathbf{r}_{iA}^2$ , the radial Coulomb energy of molecule can be obtained,

$$J_{rad}(\mathbf{r}_i) = \sum_A \mathbf{r}_{iA}^2 J^A(\mathbf{r}_i) \quad (6.57)$$

### 6.2.3.2 Electron Density Method

Starting from molecular electron density  $\rho(\mathbf{r})$ , the classical Coulomb energy can be written as,

$$J[\rho] = \frac{1}{2} \int \int \frac{\rho(\mathbf{r}_1)\rho(\mathbf{r}_2)}{\mathbf{r}_{12}} d\mathbf{r}_1 d\mathbf{r}_2 \quad (6.58)$$

The above equation can be rearranged as below,

$$\begin{aligned} J[\rho] &= \frac{1}{2} \int \rho(\mathbf{r}_1) \left[ \int \frac{\rho(\mathbf{r}_2)}{\mathbf{r}_{12}} d\mathbf{r}_2 \right] d\mathbf{r}_1 \\ &= \frac{1}{2} \int \rho(\mathbf{r}_1) V(\mathbf{r}_1) d\mathbf{r}_1 \\ &= \frac{1}{2} \int \sum_A W_A(\mathbf{r}_1) \rho(\mathbf{r}_1) V(\mathbf{r}_1) d\mathbf{r}_1 \\ &= \frac{1}{2} \int \sum_A \rho_A(\mathbf{r}_1) V(\mathbf{r}_1) d\mathbf{r}_1 \end{aligned} \quad (6.59)$$

where  $\rho_A(\mathbf{r})$  is the atomic electron density of atom A and

$$V(\mathbf{r}_1) = \int \frac{\rho(\mathbf{r}_2)}{\mathbf{r}_{12}} d\mathbf{r}_2 \quad (6.60)$$



$V(\mathbf{r}_1)$  can be written as,

$$\begin{aligned}
V(\mathbf{r}_1) &= \int \frac{\rho(\mathbf{r}_2)}{r_{12}} d\mathbf{r}_2 \\
&= \sum_B \int \frac{W_B(\mathbf{r}_2)\rho(\mathbf{r}_2)}{r_{12}} d\mathbf{r}_2 \\
&= \sum_B \int \frac{\rho_B(\mathbf{r}_2)}{r_{12}} d\mathbf{r}_2
\end{aligned} \tag{6.61}$$

$V(\mathbf{r}_1)$  can be computed numerically as follows,

$$V(\mathbf{r}_1) = \sum_B \sum_j \frac{W_B(\mathbf{r}_j)\omega(\mathbf{r}_j)\rho(\mathbf{r}_j)}{r_{1j}} \tag{6.62}$$

At grid  $\mathbf{r}_i$  the equation becomes,

$$V(\mathbf{r}_i) = \sum_B \sum_j \frac{W_B(\mathbf{r}_j)\omega(\mathbf{r}_j)\rho(\mathbf{r}_j)}{r_{ij}} \tag{6.63}$$

Starting from Equation 6.59, the Coulomb energy can be calculated numerically as follows,

$$J[\rho] = \frac{1}{2} \sum_A \sum_i W_A(\mathbf{r}_i)\omega(\mathbf{r}_i)\rho(\mathbf{r}_i)V(\mathbf{r}_i) \tag{6.64}$$

and the atomic Coulomb energy density  $J^A[\rho]$  can be computed as,

$$\begin{aligned}
J^A[\rho] &= \frac{1}{2} \sum_i W_A(\mathbf{r}_i)\omega(\mathbf{r}_i)\rho(\mathbf{r}_i)V(\mathbf{r}_i) \\
&= \sum_i \omega(\mathbf{r}_i) \left( \frac{1}{2} W_A(\mathbf{r}_i)\rho(\mathbf{r}_i)V(\mathbf{r}_i) \right) \\
&= \sum_i \omega(\mathbf{r}_i) J^A[\rho(\mathbf{r}_i)]
\end{aligned} \tag{6.65}$$

where

$$J^A[\rho(\mathbf{r}_i)] = \frac{1}{2}W_A(\mathbf{r}_i)\rho(\mathbf{r}_i)V(\mathbf{r}_i) \quad (6.66)$$

The molecular Coulomb energy density at the grid point  $\mathbf{r}_i$  can be defined as,

$$J[\rho(\mathbf{r}_i)] = \sum_A J^A[\rho(\mathbf{r}_i)] \quad (6.67)$$

and by multiplying the last equation with  $\mathbf{r}_{iA}^2$ , the radial Coulomb energy density of a molecule can be obtained,

$$J_{rad}[\rho(\mathbf{r}_i)] = \sum_A \mathbf{r}_{iA}^2 J^A[\rho(\mathbf{r}_i)] \quad (6.68)$$

With substitution of Equation 6.63 in Equation 6.64,  $J$  becomes,

$$\begin{aligned} J[\rho] &= \frac{1}{2} \sum_A \sum_i W_A(\mathbf{r}_i) \omega(\mathbf{r}_i) \rho(\mathbf{r}_i) \left[ \sum_B \sum_j \frac{W_B(\mathbf{r}_j) \omega(\mathbf{r}_j) \rho(\mathbf{r}_j)}{\mathbf{r}_{ij}} \right] \\ &= \frac{1}{2} \sum_A \sum_i \omega(\mathbf{r}_i) \rho_A(\mathbf{r}_i) \left[ \sum_B \sum_j \frac{\omega(\mathbf{r}_j) \rho_B(\mathbf{r}_j)}{\mathbf{r}_{ij}} \right] \\ &= \frac{1}{2} \sum_A \sum_B \sum_i \sum_j \frac{Q_A(\mathbf{r}_i) Q_B(\mathbf{r}_j)}{\mathbf{r}_{ij}} \end{aligned} \quad (6.69)$$

where

$$Q_A(\mathbf{r}_k) = W_A(\mathbf{r}_k) \omega(\mathbf{r}_k) \rho(\mathbf{r}_k) \quad (6.70)$$

Note that the values  $W_A(\mathbf{r}_k)$ ,  $\omega(\mathbf{r}_k)$ , and  $\rho(\mathbf{r}_k)$  can be stored in the database, or for simplicity  $Q_A(\mathbf{r}_k)$  can be stored instead of these values. Later these values can be used for calculating  $J$  of the target molecule.

### 6.2.4 Potential Energy, Nuclear-Electron Attraction Energy ( $V_{ne}$ )

The electron-nuclear potential energy is defined as,

$$V_{ne}[\rho] = - \sum_A Z_A \int \frac{\rho(\mathbf{r})}{|\mathbf{r} - \mathbf{R}_A|} d\mathbf{r} \quad (6.71)$$

where,  $\mathbf{R}_A$  is the position vector of the atom  $A$  with atomic charge  $Z_A$ . The above equation can be written as follows,

$$V_{ne}[\rho] = - \sum_A Z_A \sum_B \int \frac{W_B(\mathbf{r})\rho(\mathbf{r})}{|\mathbf{r} - \mathbf{R}_A|} d\mathbf{r} \quad (6.72)$$

$V_{ne}[\rho]$  can be computed numerically as,

$$\begin{aligned} V_{ne}[\rho] &= - \sum_A Z_A \sum_B \sum_j \frac{W_B(\mathbf{r}_j)\omega(\mathbf{r}_j)\rho(\mathbf{r}_j)}{|\mathbf{r}_j - \mathbf{R}_A|} \\ &= - \sum_A Z_A \sum_B \sum_j \frac{\omega(\mathbf{r}_j)\rho_B(\mathbf{r}_j)}{|\mathbf{r}_j - \mathbf{R}_A|} \\ &= - \sum_A Z_A \sum_B \sum_j \frac{Q_B(\mathbf{r}_j)}{|\mathbf{r}_j - \mathbf{R}_A|} \end{aligned} \quad (6.73)$$

Using the above equation, the atomic electron-nuclear potential energy  $V_{ne}^A$  can be obtained as,

$$V_{ne}^A[\rho] = -Z_A \sum_B \sum_j \frac{Q_B(\mathbf{r}_j)}{|\mathbf{r}_j - \mathbf{R}_A|} \quad (6.74)$$

The molecular potential energy density at the grid point  $\mathbf{r}_j$  can be defined as,

$$\begin{aligned} V_{ne}[\rho(\mathbf{r}_j)] &= -\sum_A Z_A \sum_B \frac{W_B(\mathbf{r}_j)\rho(\mathbf{r}_j)}{\mathbf{r}_{jA}} \\ &= -\sum_A Z_A \sum_B \frac{\rho_B(\mathbf{r}_j)}{\mathbf{r}_{jA}} \end{aligned} \quad (6.75)$$

and by multiplying the last equation with  $\mathbf{r}_{jB}^2$ , the radial molecular potential energy can be obtained,

$$V_{ne,rad}[\rho(\mathbf{r}_j)] = -\sum_A Z_A \sum_B \frac{\mathbf{r}_{jB}^2 \rho_B(\mathbf{r}_j)}{\mathbf{r}_{jA}} \quad (6.76)$$

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## Chapter 7

# AIMD Calculations and Results

*“Everything we call real is made of things that cannot be regarded as real.”*

— Niels Bohr

## 7.1 Computational Method

All the calculations were performed in MUNgauss package [1]. The molecular radial kinetic, exchange, Coulomb, and potential energy densities are computed at HF/6-31G(d) and employing a mesh size of 0.05 bohr. The results of numerical integration were calculated using the 6-31G(d) basis sets on HF/6-31G(d) optimized structures, and the SG1 grid is used in the calculations. All of the visual aids, including contour, relief, and the 3D plots were created using Mathematica Version 11.2 graphing package [2].



## 7.2 Molecular Properties Results and Discussions

In this chapter, we will discuss the results of computing the molecular properties from the corresponding fragment (atomic) properties. As mentioned before, each fragment represents an atom in a specific molecular environment. The calculated molecular properties are compared with those calculated by the HF wavefunction (the exact values). The ”%error” is computed as,

$$\%error = \frac{P_{HF} - P_{AIM}}{P_{HF}} \times 100\% \quad (7.1)$$

where  $P_{AIM}$  and  $P_{HF}$  are the values of a molecular property computed using atoms in molecules density (AIMD) method and HF, respectively. In addition for each molecular property the mean absolute percentage error (MAPE) is computed as,

$$MAPE = \frac{100\%}{M} \sum_i \left| \frac{P_{HF}^i - P_{AIM}^i}{P_{HF}^i} \right| \quad (7.2)$$

where  $P_{AIM}^i$  is the value of the molecular property computed using AIMD for a molecule  $i$ , and  $P_{HF}^i$  is the corresponding molecular property calculated using HF, and  $M$  is the number of molecules.

Table 7.1 shows some examples of atomic properties that have been stored in the database. In these examples the atomic properties are computed using Awad weight, and second neighbour of non-optimized fragments.

In Figure 7.1 we introduce a notation to represent the fragment for the target atom in the molecule of interest (i.e., the target molecule). To write the notation, the size of fragment (i.e., the number of neighbour level) goes above the symbol whereas the atomic symbol and the atom number of the atom in the molecule of interest are written as a subscript. For

**Table 7.1:** Some of the atomic properties  $N_e^A$ ,  $K^A$ ,  $K_{HF}^A$ ,  $T^A$ ,  $V_{ne}^A$ ,  $J^A$ , and  $J_{HF}^A$ , calculated using SG1 grid, Awad weight, second neighbour of non-optimized fragments at HF/6-31G(d).

| Symbol                      | $N_e^A$  | $K^A$     | $K_{HF}^A$ | $T^A$     | $V_{ne}^A$  | $J^A$     | $J_{HF}^A$ |
|-----------------------------|----------|-----------|------------|-----------|-------------|-----------|------------|
| 1(6(6,1,1))                 | 1.071925 | -0.188903 | -0.374779  | 0.594538  | -6.729386   | 4.075700  | 4.261576   |
| 1(6(7,6,1))                 | 1.069721 | -0.195488 | -0.377186  | 0.590994  | -8.639958   | 5.456195  | 5.637894   |
| 1(6(7,6,6))                 | 1.074858 | -0.201258 | -0.379658  | 0.582953  | -10.438871  | 6.766430  | 6.944830   |
| 1(6(8,6,6))                 | 1.065122 | -0.193933 | -0.379249  | 0.588868  | -10.564649  | 6.731344  | 6.916660   |
| 1(7(6,1))                   | 1.020490 | -0.175350 | -0.379122  | 0.687671  | -7.302757   | 3.710862  | 3.914635   |
| 1(7(6,6))                   | 1.004696 | -0.189775 | -0.373623  | 0.663992  | -9.078792   | 4.777218  | 4.961065   |
| 1(8(6))                     | 0.957115 | -0.162286 | -0.375158  | 0.792281  | -7.781516   | 3.554150  | 3.767022   |
| 6(6(7,6,1)1(0)1(0))         | 5.797046 | -0.946114 | -5.140066  | 37.832324 | -133.014107 | 33.046856 | 37.240808  |
| 6(6(8,6,1)1(0)1(0))         | 5.793904 | -0.938579 | -5.142095  | 37.834155 | -133.263508 | 34.221182 | 38.424698  |
| 6(7(1,1)6(8,6,1)6(8,7)1(0)) | 5.874279 | -1.058858 | -5.196807  | 38.012005 | -179.724239 | 57.117083 | 61.255032  |
| 6(7(1,1)6(8,6,1)6(8,8)1(0)) | 5.865734 | -1.035455 | -5.195416  | 38.018367 | -180.023043 | 57.457843 | 61.617804  |
| 6(7(1,1)6(8,7)1(0)1(0))     | 5.820390 | -0.941964 | -5.171103  | 37.979300 | -146.945699 | 41.124838 | 45.353977  |
| 6(7(1,1)6(8,7)6(1,1,1)1(0)) | 5.867287 | -0.967023 | -5.193536  | 38.022886 | -160.737219 | 46.995839 | 51.222352  |
| 6(7(1,1)6(8,8)1(0)1(0))     | 5.810699 | -0.879369 | -5.167286  | 37.968746 | -147.175448 | 41.347383 | 45.635301  |
| 6(7(1,1)6(8,8)6(1,1,1)1(0)) | 5.858824 | -0.947523 | -5.189427  | 38.003355 | -160.849703 | 47.117248 | 51.359152  |
| 6(7(6,1)6(8,6,1)6(8,8)1(0)) | 5.856394 | -1.086767 | -5.186843  | 37.956271 | -189.041650 | 60.672298 | 64.772374  |
| 6(7(6,1)6(8,7)1(0)1(0))     | 5.808371 | -0.992022 | -5.160104  | 37.919678 | -155.923481 | 44.174403 | 48.342486  |
| 6(7(6,1)6(8,8)1(0)1(0))     | 5.804767 | -0.969884 | -5.168492  | 37.954400 | -156.995171 | 45.013871 | 49.212480  |
| 6(7(6,1)6(8,8)6(1,1,1)1(0)) | 5.852320 | -1.014654 | -5.188123  | 37.976373 | -170.491347 | 50.688196 | 54.861665  |
| 6(8(0)7(6,1)6(7,1,1))       | 5.876251 | -0.957532 | -5.245189  | 38.500984 | -164.810017 | 45.769518 | 50.057175  |
| 6(8(0)7(6,1)6(7,6,1))       | 5.872220 | -0.971928 | -5.243648  | 38.509980 | -174.386728 | 49.133169 | 53.404889  |
| 6(8(1)6(7,6,1)6(1,1,1)1(0)) | 5.833517 | -0.985297 | -5.184959  | 38.166212 | -163.262139 | 44.362219 | 48.561881  |
| 6(8(1)8(6(7,1,1))           | 5.850333 | -0.896454 | -5.248576  | 38.697097 | -156.560471 | 42.169020 | 46.521142  |
| 6(8(1)8(6(7,6,1))           | 5.854054 | -0.920244 | -5.250513  | 38.710557 | -166.075004 | 45.504417 | 49.834686  |
| 7(6(6,1,1)1(0)1(0))         | 6.995128 | -1.207685 | -6.588524  | 54.009604 | -167.953973 | 38.223335 | 43.604174  |
| 7(6(6,6,1)1(0)1(0))         | 6.997995 | -1.296852 | -6.589653  | 54.011239 | -178.900238 | 43.216108 | 48.508909  |
| 7(6(6,6,1)6(8,6)1(0))       | 6.987776 | -1.439793 | -6.617214  | 54.047767 | -217.879964 | 60.669501 | 65.846921  |
| 7(6(8,6)6(6,1,1)1(0))       | 6.977344 | -1.404146 | -6.614876  | 54.045749 | -206.902577 | 55.784664 | 60.995394  |
| 8(6(6,6,1)1(0))             | 8.137391 | -1.986194 | -8.244030  | 74.165832 | -233.187879 | 56.538266 | 62.796103  |
| 8(6(7,6))                   | 8.275259 | -1.942467 | -8.258353  | 74.218555 | -231.329357 | 59.417003 | 65.732889  |
| 8(6(8,6))                   | 8.237650 | -1.888308 | -8.252445  | 74.225501 | -232.183586 | 61.040105 | 67.404242  |
| 8(6(8,6)1(0))               | 8.044935 | -2.036277 | -8.213370  | 74.121111 | -231.986033 | 55.098164 | 61.275256  |



**Figure 7.1:** The suggested notation to represent the fragment for the target atom in the molecule of interest. **S**, **A**, and **n** are the size of fragment, the atomic symbol of target atom, and the atom number in the molecule of interest, respectively.

example, in Figure 7.3 the  $\mathbf{F}_{\text{C11}}^1$  (molecule A),  $\mathbf{F}_{\text{C11}}^2$  (molecule B), and  $\mathbf{F}_{\text{C11}}^3$  (molecule C) are three different carbon atom fragments obtained from the molecule of interest (molecule D) as first, second, and third neighbour atoms, respectively.

Also, we suggest notations to represent the target atom within its fragment and within the molecule of interest. These notations are shown in Figures 7.2a and 7.2b respectively. For example, in Figure 7.3 the  ${}^{\text{C1}}\mathbf{F}_{\text{C11}}^1$ ,  ${}^{\text{C1}}\mathbf{F}_{\text{C11}}^2$ ,  ${}^{\text{C1}}\mathbf{F}_{\text{C11}}^3$ , and  $\mathbf{T}_{\text{C11}}$  are the target carbon atoms in  $\mathbf{F}_{\text{C11}}^1$ ,  $\mathbf{F}_{\text{C11}}^2$ ,  $\mathbf{F}_{\text{C11}}^3$ , and the molecule of interest, respectively.

The effect of geometry on the atomic properties was studied by calculating some of atomic properties (e.g.,  $T^A$ ,  $K^A$ ,  $J^A$ , and  $V_{ne}^A$ ) of the carbon atom within four different geometries:  $\mathbf{F}_{\text{C11}}^1$ ,  $\mathbf{F}_{\text{C11}}^2$ ,  $\mathbf{F}_{\text{C11}}^3$ , and the molecule of interest (D molecule). As can be seen from Figure 7.3, the fragments  $\mathbf{F}_{\text{C11}}^1$ ,  $\mathbf{F}_{\text{C11}}^2$ , and  $\mathbf{F}_{\text{C11}}^3$  are completed with hydrogen atoms to get real molecules. In addition to the properties of the carbon atoms,  ${}^{\text{C1}}\mathbf{F}_{\text{C11}}^1$ ,  ${}^{\text{C1}}\mathbf{F}_{\text{C11}}^2$ ,  ${}^{\text{C1}}\mathbf{F}_{\text{C11}}^3$ , and  $\mathbf{T}_{\text{C11}}$ , their molecular properties (e.g.,  $T$ ,  $K$ ,  $J$ , and  $V_{ne}$ ) are also studied. In the following sections, each of the studied properties will be discussed separately.

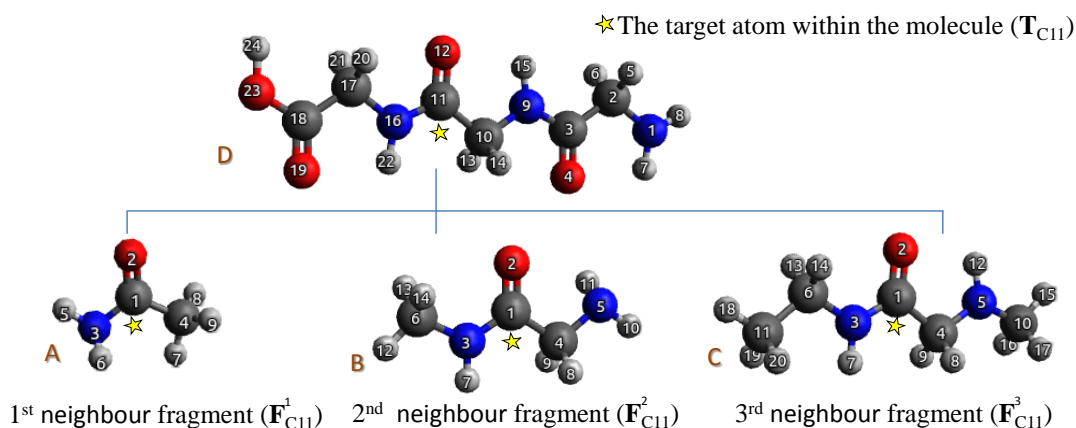


(a) The target atom within the fragment. **S**, **A**, **n**, **T**, and **m** are the size of the fragment, the atomic symbol of the target atom in the molecule of interest, the atom number in the molecule of interest, the atomic symbol of target atom within the fragment, and the atom number within the fragment, respectively.



(b) The target atom within the molecule of interest (target molecule). **A** and **n** are the atomic symbol and its atom number of the target atom in the molecule of interest respectively.

**Figure 7.2:** Suggested notations to represent the target atom within its fragment and the molecule of the interest.

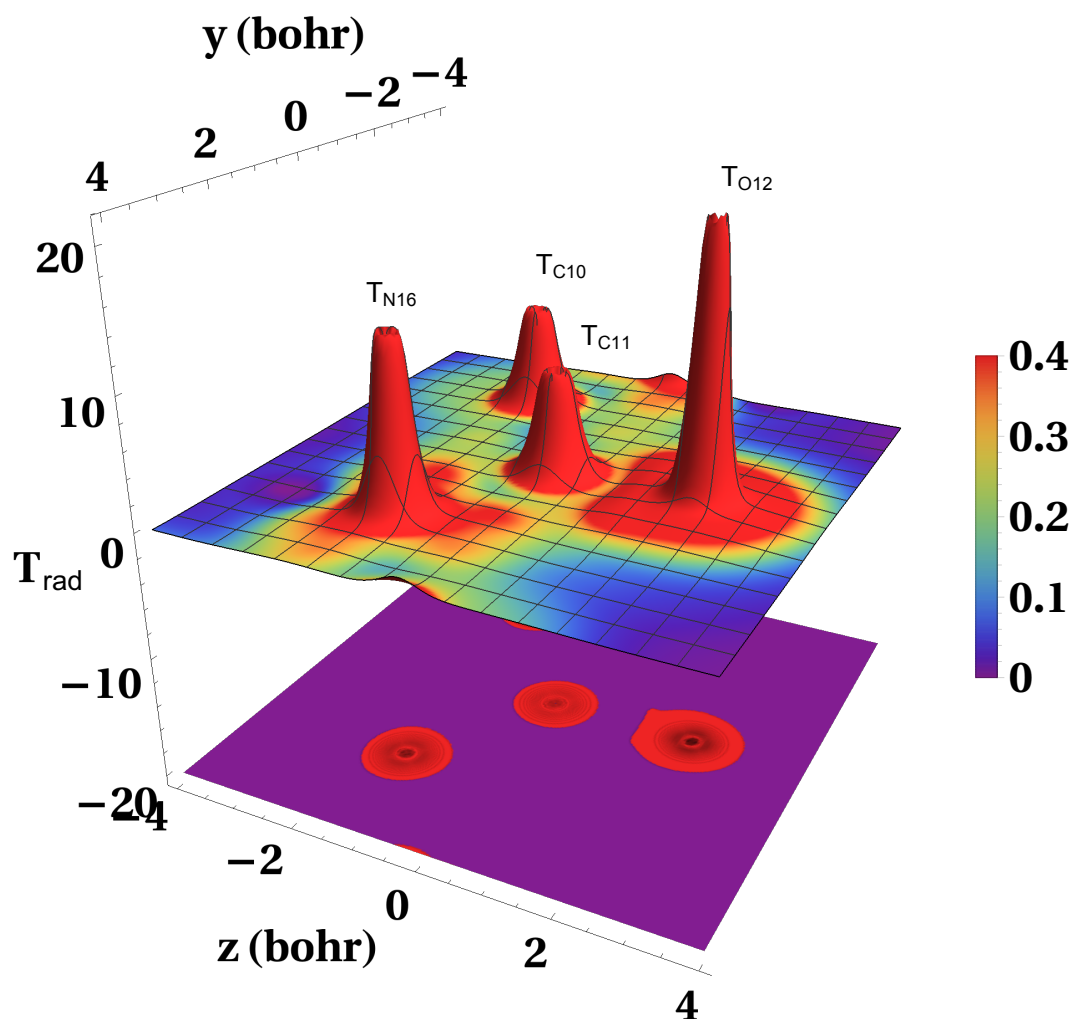


**Figure 7.3:** The target carbon atom within the four structures  $F_{C11}^1$ ,  $F_{C11}^2$ ,  $F_{C11}^3$ , and  $T_{C11}$ , to study the effect of geometry on the atomic energies components.

### 7.3 Kinetic Energy ( $T$ )

Figure 7.4 shows a 3D-contour plot of the molecular radial kinetic energy density (Equation 6.14) of the target carbon atom ( $T_{C11}$ ) in molecule of interest (D molecule in Figure 7.3). As shown in Figure 7.4, the kinetic energies of electrons in the bonding region are very small compared to the non-bonding region, in which the electrons have more kinetic energy when they are close to the nuclei. This is expected because the velocity is inversely proportional to the distance from the nucleus, thus electrons have greater velocities near the nucleus, therefore more kinetic energy. As shown in Figure 7.4, we can visually distinguish the radial kinetic energy density for each atom in the molecule, where the atomic radial kinetic energy densities appear as cone shapes. The atomic radial kinetic energy is proportional to the atomic number of the atom, where oxygen has the largest kinetic energy followed by nitrogen, then carbon.

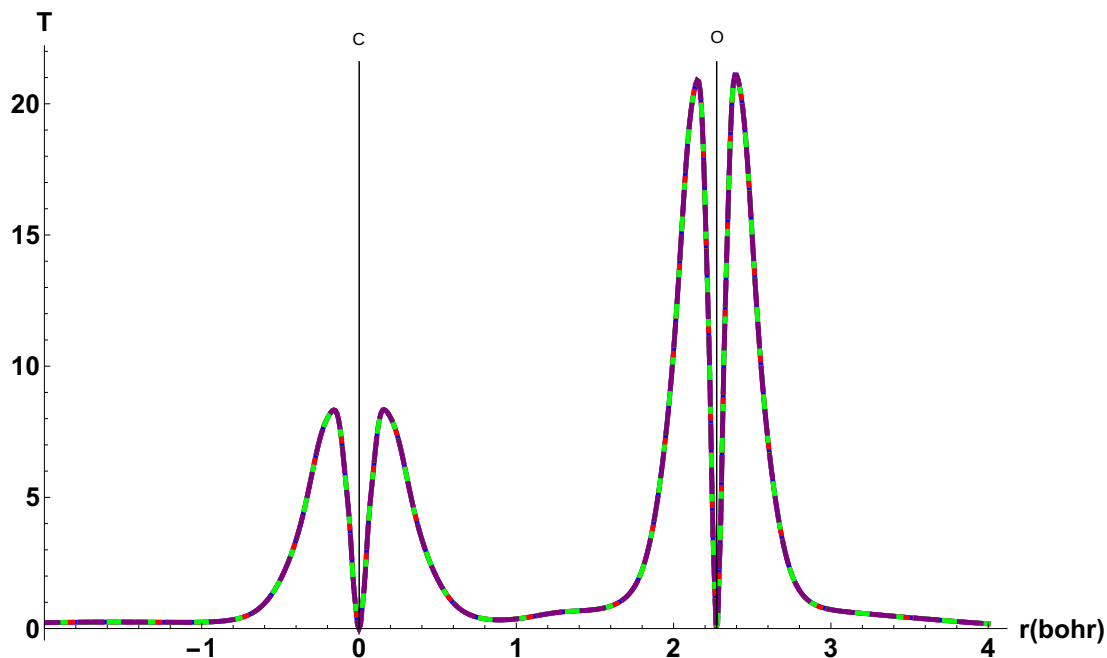
In Figure 7.5, we compare the radial kinetic energy density (Equation 6.14) along the bond between the carbon atom and oxygen atom for  $F_{C11}^1$  (red dash line),  $F_{C11}^2$  (green dash line),



**Figure 7.4:** The radial kinetic energy density for the molecule of interest in Figure 7.3,  $T_{\text{C11}}$  is the target carbon atom.

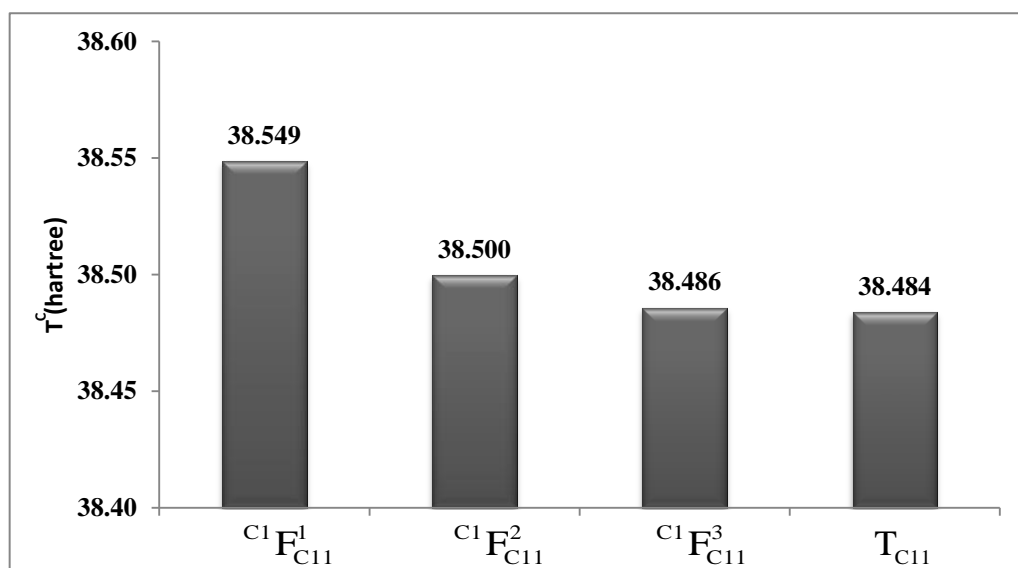
$F_{\text{C11}}^3$  (purple dash line), and the D molecule (blue solid line). As shown in this figure, the radial kinetic energy densities for  $F_{\text{C11}}^1$ ,  $F_{\text{C11}}^2$ ,  $F_{\text{C11}}^3$ , and D molecule are almost identical. Maximum peaks of the radial kinetic energy densities adjacent to the carbon nucleus are located at -0.1586 and 0.1587 bohr with 8.335 and 8.344 radial kinetic energy densities respectively, whereas the maximum peaks adjacent to the oxygen nucleus are located at -0.1205 and 0.1208 bohr (relative to oxygen nucleus) with 20.909 and 21.128 radial kinetic

energy densities respectively. These results show that positions of the peaks in the oxygen atom are closer to its nucleus than the peaks in the carbon atom to its nucleus due to the difference in effective nuclear charges.



**Figure 7.5:** The radial kinetic energy density along the bond between the target carbon atom and the oxygen atom for  $F_{C11}^1$  (red dash line),  $F_{C11}^2$  (green dash line),  $F_{C11}^3$  (purple dash line), and the molecule of interest, D molecule, (blue solid line). The carbon is located at 0.0 bohr and oxygen is located at 2.2739 bohr.

The computed kinetic energy (Equation 6.11) of the target carbon atom for the four molecules  $F_{C11}^1$ ,  $F_{C11}^2$ ,  $F_{C11}^3$ , and D molecule are displayed in Figure 7.6. The results show that the kinetic energies of the carbon atom are inversely proportional to the size of the molecule. As shown in Figure 7.6, there is no significant change in the carbon kinetic energy of  $C1F_{C11}^3$  and  $T_{C11}$ . It is clear from these results that the first and second neighbors of atoms have the largest effect on the computed kinetic energies. Therefore, the kinetic energy is not an atomic property as it is affected by surrounding atoms. For instance, the atomic kinetic



**Figure 7.6:** Effect of size of the fragment on the atomic kinetic energy ( $T^C$ ) of the target carbon atom.

energy of the carbon atom is computed in the following environments: isolated atom,  $CH_4$ ,  $HCOH$ ,  $HCONH_2$ , and  $CF_4$ , and found to be 37.945, 37.706, 38.153, 38.363, and 39.948 hartree, respectively. These results clearly show that the atomic kinetic energy depends on the surrounding environment.

The molecular kinetic energy is calculated by summing over atomic kinetic energies,

$$T^{sum} = \sum_{A=1}^M T^A \quad (7.3)$$

where  $T^A$  is the kinetic energy of atom  $A$  computed using Equation 6.11 and  $M$  is the number of atoms in the target molecule. The atomic kinetic energies are obtained two different ways: from that stored in the database ( $T_{database}^A$ ) and from using the direct method ( $T_{direct}^A$ ). In the direct method, the kinetic energies for each atom are calculated in a specific environment (fragment) by keeping the same Cartesian coordinates of the atoms in the target



**Table 7.2:** Comparing the molecular kinetic energies calculated using the sum over atomic kinetic energies that are stored in the database ( $T_{database}^{sum} = \sum_{A=1} T_{database}^A$ ) and by using the direct method ( $T_{direct}^{sum} = \sum_{A=1} T_{direct}^A$ ) with those calculated numerically (Equation 6.9) for the target molecule (the exact values).  $\mathbf{F}^1$ ,  $\mathbf{F}^2$ , and  $\mathbf{F}^3$  are the size of fragments as first, second, and third neighbor atoms, respectively.

| Target molecule<br>$\mathbf{T}$ |                          | Fragments (Sum over atoms)                   |                   |                   |  |                   |                   |
|---------------------------------|--------------------------|--|-------------------|-------------------|--|-------------------|-------------------|
| Molecule                        | (Analytical) (Numerical) | $T_{database}^{sum} = \sum_A T_{database}^A$ |                   |                   | $T_{direct}^{sum} = \sum_A T_{direct}^A$ |                   |                   |
|                                 |                          | $\mathbf{F}^1$                               | $\mathbf{F}^2$    | $\mathbf{F}^3$    | $\mathbf{F}^1$                           | $\mathbf{F}^2$    | $\mathbf{F}^3$    |
| Gly                             | 282.300                  | 282.295                                      | 282.372 (-0.027%) | 282.310 (-0.005%) | 282.578 (-0.100%)                        | 282.362 (-0.024%) | 282.310 (-0.005%) |
| Ala                             | 321.328                  | 321.312                                      | 321.438 (-0.039%) | 321.364 (-0.016%) | 321.606 (-0.091%)                        | 321.421 (-0.034%) | 321.346 (-0.010%) |
| Thr                             | 435.039                  | 435.008                                      | 435.168 (-0.037%) | 435.113 (-0.024%) | 435.361 (-0.081%)                        | 435.153 (-0.033%) | 435.070 (-0.014%) |
| GlyGly                          | 488.763                  | 488.760                                      | 488.948 (-0.038%) | 488.825 (-0.013%) | 489.375 (-0.126%)                        | 488.935 (-0.036%) | 488.836 (-0.016%) |
| AlaAla                          | 566.816                  | 566.776                                      | 567.073 (-0.053%) | 566.942 (-0.029%) | 567.409 (-0.112%)                        | 567.039 (-0.046%) | 566.890 (-0.020%) |
| ThrThr                          | 794.214                  | 794.193                                      | 794.472 (-0.035%) | 794.399 (-0.026%) | 794.892 (-0.088%)                        | 794.422 (-0.029%) | 794.331 (-0.017%) |
| GlyAla                          | 527.791                  | 527.748                                      | 528.019 (-0.051%) | 527.886 (-0.026%) | 528.379 (-0.119%)                        | 527.987 (-0.045%) | 527.842 (-0.018%) |
| GlyThr                          | 641.493                  | 641.463                                      | 641.697 (-0.037%) | 641.593 (-0.020%) | 642.101 (-0.100%)                        | 641.673 (-0.033%) | 641.568 (-0.016%) |
| AlaThr                          | 680.521                  | 680.487                                      | 680.768 (-0.041%) | 680.685 (-0.029%) | 681.141 (-0.096%)                        | 680.741 (-0.037%) | 680.615 (-0.019%) |
| GlyGlyGly1                      | 695.223                  | 695.181                                      | 695.479 (-0.043%) | 695.103 (0.011%)  | 696.119 (-0.135%)                        | 695.490 (-0.044%) | 695.307 (-0.018%) |
| GlyGlyGly2                      | 695.226                  | 695.191                                      | 695.479 (-0.041%) | 695.103 (0.013%)  | 696.080 (-0.128%)                        | 695.395 (-0.029%) | 695.295 (-0.015%) |
| MAPE =                          |                          | 0.121%                                       | 0.040%            | 0.019%            | 0.107%                                   | 0.036%            | 0.015%            |

molecule.

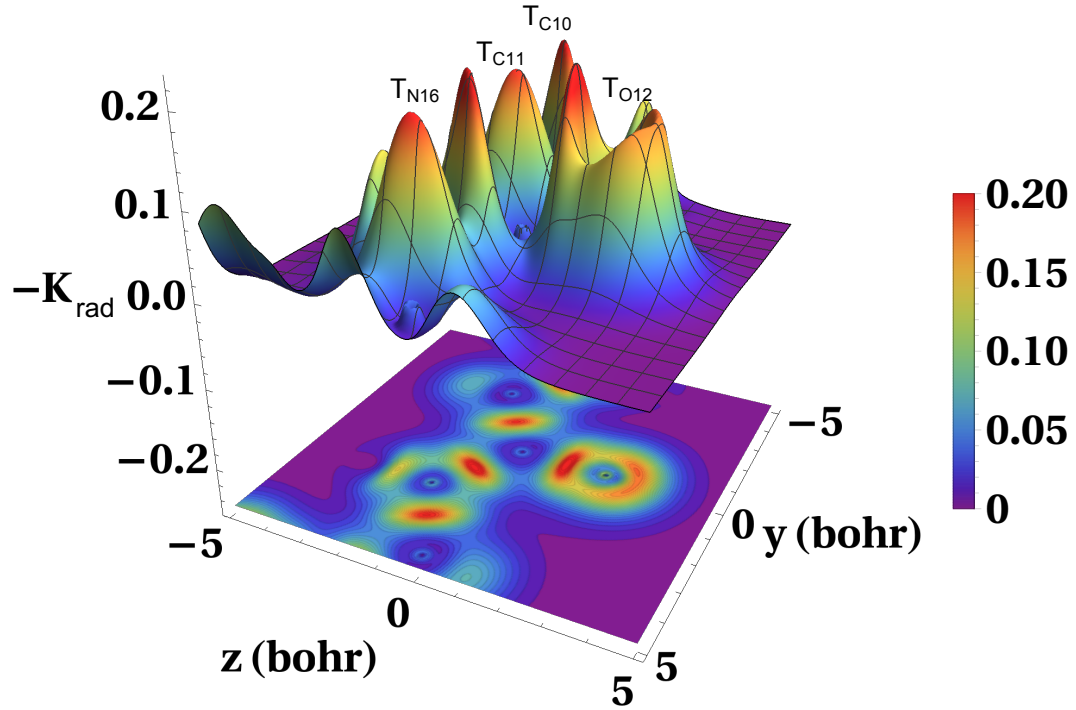
To investigate the validity of our results, we compare the kinetic energies of three mono-amino acids (glycine Gly, alanine Ala, and threonine Thr), six di-amino acids (GlyGly, AlaAla, ThrThr, GlyAla, GlyThr, and AlaThr), and two isomers of tri-amino acid (GlyGlyGly1, GlyGlyGly2) calculated using the sum over atomic kinetic energies stored in the database ( $T_{database}^{sum} = \sum_{A=1} T_{database}^A$ ) and by using the direct method ( $T_{direct}^{sum} = \sum_{A=1} T_{direct}^A$ ) with those calculated numerically (Equation 6.9) for the target molecule (the exact values).

Table 7.2 shows the results of  $T_{database}^{sum}$  and  $T_{direct}^{sum}$  calculated using different size of fragments,  $\mathbf{F}^1$ ,  $\mathbf{F}^2$ , and  $\mathbf{F}^3$  as first, second, and third neighbor atoms, respectively. The MAPE of  $T_{database}^{sum}$  are about 0.121%, 0.040%, and 0.019% for  $\mathbf{F}^1$ ,  $\mathbf{F}^2$ , and  $\mathbf{F}^3$ , respectively. Whereas, The MAPE of  $T_{direct}^{sum}$  are about 0.107%, 0.036%, and 0.015% for  $\mathbf{F}^1$ ,  $\mathbf{F}^2$ , and  $\mathbf{F}^3$ , respectively. These results as tabulated in Table 7.2 are encouraging, where %errors obtained for  $T_{direct}^{sum}$  are slightly better than %errors obtained for  $T_{database}^{sum}$  and the %errors have always negative values (two exceptions are GlyGlyGly1 and GlyGlyGly2 for  $T_{database}^{sum}$  using  $\mathbf{F}^3$ ).

## 7.4 Exchange Energy ( $K$ )

Figure 7.7 shows a 3D-contour plot of the molecular radial exchange energy density (Equation 6.37) of target carbon atom  $\mathbf{T}_{C11}$  in the molecule of interest (D molecule in Figure 7.3). As shown in Figure 7.7, we cannot visually distinguish the exchange energy density for each atom in the molecule, where the atomic radial exchange energy densities are shared in the bonding regions.

The radial exchange energy density along the bond between the target carbon atom and oxygen atom for  $\mathbf{F}_{C11}^1$  (red dash line),  $\mathbf{F}_{C11}^2$  (green dash line),  $\mathbf{F}_{C11}^3$  (purple dash line), and



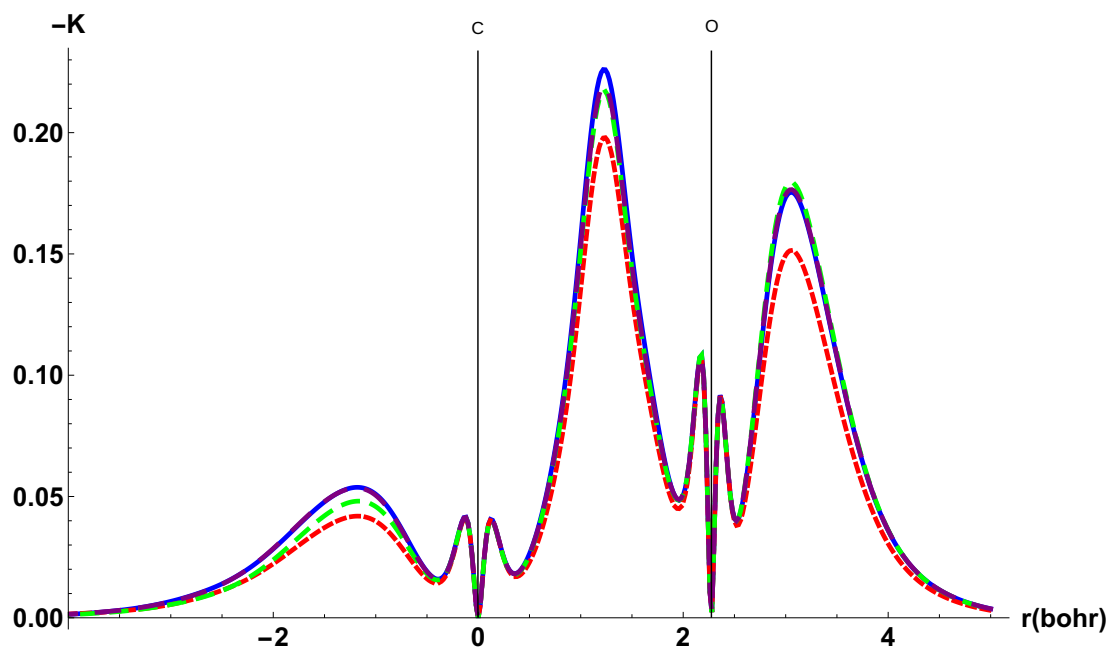
**Figure 7.7:** The radial exchange energy density for D molecule (Figure 7.3),  $T_{C11}$  is the target carbon atom.

the D molecule (blue solid line) are compared in Figure 7.8. As shown in Figure 7.8, the exchange energy density is dependent on the size of the molecule. For example, the molecular radial exchange energy density for the D molecule is larger than  $F_{C11}^1$ .

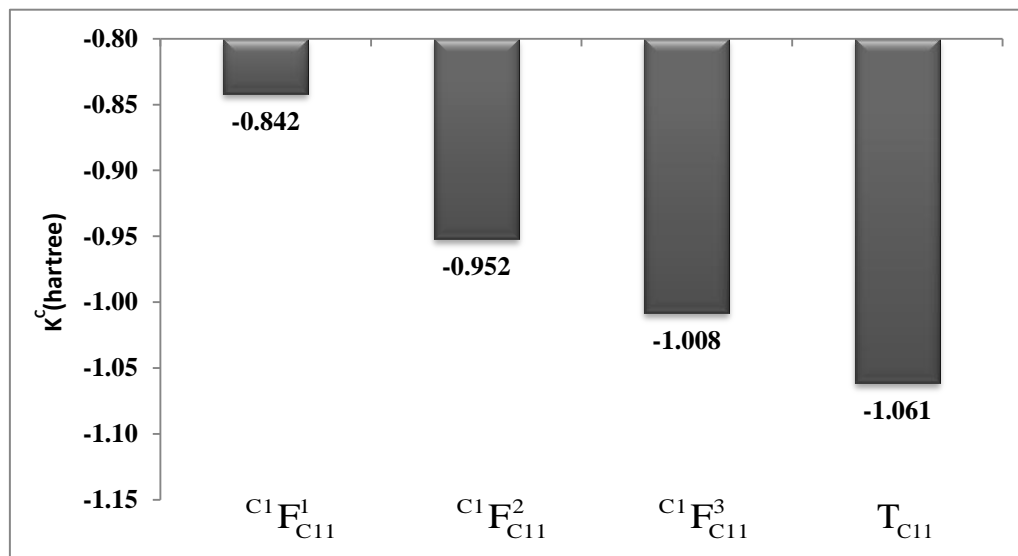
The computed exchange energies (Equation 6.32) of the target carbon atom for the four molecules  $F_{C11}^1$ ,  $F_{C11}^2$ ,  $F_{C11}^3$ , and the molecule of interest (D molecule) are displayed in Figure 7.9. The results show that the atomic exchange energy of the carbon atom is proportional to the size of the molecule.

The molecular exchange energy is calculated by summing over atomic exchange energies,

$$K^{sum} = \sum_{A=1}^M K^A \quad (7.4)$$



**Figure 7.8:** The radial exchange energy density along the bond between the target carbon atom and the oxygen atom for  $F_{C11}^1$  (red dash line),  $F_{C11}^2$  (green dash line),  $F_{C11}^3$  (purple dash line), and the molecule of interest, D molecule, (blue solid line). The carbon is located at 0.0 bohr and oxygen is located at 2.2739 bohr.



**Figure 7.9:** Effect of size of the fragment on the atomic exchange energy density ( $K$ ) of target carbon atom.

where  $K^A$  is the exchange energy of atom  $A$  computed using Equation 6.32 and  $M$  is the number of atoms in the target molecule. Table 7.3 compares the exchange energies of three mono-amino acids (glycine Gly, alanine Ala, and threonine Thr), six di-amino acids (GlyGly, AlaAla, ThrThr, GlyAla, GlyThr, and AlaThr), and two isomers of tri-amino acid (GlyGlyGly1 and GlyGlyGly2) calculated using the sum over atomic exchange energies stored in the database ( $K_{database}^{sum} = \sum_{A=1} K_{database}^A$ ) and by using the direct method ( $K_{direct}^{sum} = \sum_{A=1} K_{direct}^A$ ) of  $\mathbf{F}^1$ ,  $\mathbf{F}^2$ , and  $\mathbf{F}^3$  fragments with those calculated numerically (Equation 6.30) for the target molecule. Unfortunately, the MAPE for these molecules is very large. The MAPE of  $K_{database}^{sum}$  are about 26.72%, 12.15%, and 5.86% for  $\mathbf{F}^1$ ,  $\mathbf{F}^2$ , and  $\mathbf{F}^3$ , respectively, and the MAPE of  $K_{direct}^{sum}$  are about 27.03%, 12.32%, and 6.23% for  $\mathbf{F}^1$ ,  $\mathbf{F}^2$ , and  $\mathbf{F}^3$ , respectively. Thus we cannot consider the exchange as an atomic property as its value depends on the geometry and other atoms of the molecule.

Figure 7.10 compares the  $K_{HF,rad}$  term (i.e., the radial exchange energy containing  $J_{aa}$  term) along the bond between the target carbon atom and the oxygen atom for  $\mathbf{F}_{C11}^1$  (red dash line),  $\mathbf{F}_{C11}^2$  (green dash line),  $\mathbf{F}_{C11}^3$  (purple dash line), and the D molecule (blue solid line). As shown in Figure 7.10, the  $K_{HF,rad}$  for  $\mathbf{F}_{C11}^1$ ,  $\mathbf{F}_{C11}^2$ ,  $\mathbf{F}_{C11}^3$ , and D molecule are almost identical.

In Figure 7.11, the  $K_{HF}^A$  term of the carbon atoms,  $\mathbf{C}^1\mathbf{F}_{C11}^1$ ,  $\mathbf{C}^1\mathbf{F}_{C11}^2$ ,  $\mathbf{C}^1\mathbf{F}_{C11}^3$ , and  $\mathbf{T}_{C11}$ , are calculated by using Equation 6.34. As shown in Figure 7.11, there is no significant change in  $K_{HF}^A$  of the carbon atoms in these molecules. Where the  $K_{HF}^A$  are  $-5.241$ ,  $-5.244$ ,  $-5.240$ , and  $-5.240$  hartree for the target carbon atom of  $\mathbf{F}_{C11}^1$ ,  $\mathbf{F}_{C11}^2$ ,  $\mathbf{F}_{C11}^3$  and D molecule, respectively. However, the  $K_{HF}$  is not an atomic property as it is affected by surrounding atoms. For example, the  $K_{HF}^A$  of the carbon atoms are computed in the following environments: isolated atom,  $\text{CH}_4$ ,  $\text{HCOH}$ ,  $\text{HCONH}_2$ , and  $\text{CF}_4$  as  $-5.564$ ,  $-5.093$ ,  $-5.138$ ,  $-5.195$ ,

**Table 7.3:** Comparing the molecular exchange energy densities calculated using the sum over atomic exchange energy densities that are stored in the database ( $K_{database}^{sum} = \sum_{A=1} K_{database}^A$ ) and by using the direct method ( $K_{direct}^{sum} = \sum_{A=1} K_{direct}^A$ ) with those calculated numerically (Equation 6.30) for the target molecule.  $\mathbf{F}^1$ ,  $\mathbf{F}^2$ , and  $\mathbf{F}^3$  are the size of fragments as first, second, and third neighbor atoms, respectively.

| Molecule   | Target molecule<br>K (Numerical) | $\mathbf{K}_{database}^{sum} = \sum_A \mathbf{K}_{database}^A$ (%error) |                  |                  | Fragments (Sum over atoms) |                  |                  | $\mathbf{K}_{direct}^{sum} = \sum_A \mathbf{K}_{direct}^A$ (%error) |                  |                  |
|------------|----------------------------------|---|------------------|------------------|----------------------------|------------------|------------------|---|------------------|------------------|
|            |                                  | $\mathbf{F}^1$  | $\mathbf{F}^2$   | $\mathbf{F}^3$   | $\mathbf{F}^1$             | $\mathbf{F}^2$   | $\mathbf{F}^3$   | $\mathbf{F}^1$  | $\mathbf{F}^2$   | $\mathbf{F}^3$   |
| Gly        | -8.091                           | -6.316 (21.94%)   | -7.812 (3.45%)   | -8.066 (0.32%)   | -6.315 (21.95%)            | -7.811 (3.47%)   | -8.051 (0.51%)   | -6.315 (21.95%)   | -7.811 (3.47%)   | -8.051 (0.51%)   |
| Ala        | -9.805                           | -8.129 (17.09%)   | -9.316 (4.99%)   | -9.734 (0.72%)   | -8.128 (17.11%)            | -9.299 (5.17%)   | -9.718 (0.89%)   | -8.128 (17.11%)   | -9.299 (5.17%)   | -9.718 (0.89%)   |
| Thr        | -14.275                          | -10.856 (23.95%)  | -12.724 (10.86%) | -13.758 (3.62%)  | -10.856 (23.95%)           | -12.711 (10.96%) | -13.708 (3.97%)  | -10.856 (23.95%)  | -12.711 (10.96%) | -13.708 (3.97%)  |
| GlyGly     | -15.582                          | -10.912 (29.97%)  | -13.729 (11.89%) | -14.773 (5.19%)  | -10.907 (30.00%)           | -13.724 (11.92%) | -14.783 (5.13%)  | -10.907 (30.00%)  | -13.724 (11.92%) | -14.783 (5.13%)  |
| AlaAla     | -18.960                          | -14.538 (23.32%)  | -16.608 (12.40%) | -17.705 (6.62%)  | -14.435 (23.87%)           | -16.582 (12.54%) | -17.662 (6.85%)  | -14.435 (23.87%)  | -16.582 (12.54%) | -17.662 (6.85%)  |
| ThrThr     | -27.994                          | -19.992 (28.58%)  | -23.413 (16.36%) | -25.635 (8.43%)  | -19.596 (30.00%)           | -23.380 (16.48%) | -25.529 (8.81%)  | -19.596 (30.00%)  | -23.380 (16.48%) | -25.529 (8.81%)  |
| GlyAla     | -17.104                          | -12.725 (25.60%)  | -15.181 (11.24%) | -16.089 (5.94%)  | -12.943 (24.33%)           | -15.162 (11.35%) | -16.052 (6.15%)  | -12.943 (24.33%)  | -15.162 (11.35%) | -16.052 (6.15%)  |
| GlyThr     | -21.855                          | -15.452 (29.30%)  | -18.549 (15.13%) | -20.364 (6.82%)  | -15.095 (30.93%)           | -18.447 (15.60%) | -19.943 (8.75%)  | -15.095 (30.93%)  | -18.447 (15.60%) | -19.943 (8.75%)  |
| AlaThr     | -23.819                          | -17.265 (27.52%)  | -20.001 (16.03%) | -21.693 (8.93%)  | -17.017 (28.56%)           | -19.938 (16.29%) | -21.502 (9.73%)  | -17.017 (28.56%)  | -19.938 (16.29%) | -21.502 (9.73%)  |
| GlyGlyGly1 | -22.354                          | -15.508 (30.63%)  | -19.606 (12.29%) | -21.173 (5.28%)  | -15.504 (30.65%)           | -19.549 (12.55%) | -20.865 (6.66%)  | -15.504 (30.65%)  | -19.549 (12.55%) | -20.865 (6.66%)  |
| GlyGlyGly2 | -24.218                          | -15.508 (35.97%)  | -19.606 (19.04%) | -21.173 (12.57%) | -15.493 (36.03%)           | -19.558 (19.24%) | -21.527 (11.11%) | -15.493 (36.03%)  | -19.558 (19.24%) | -21.527 (11.11%) |
| MAPE =     |                                  | 26.715%   | 12.154%          | 5.858%           | 27.033%                    | 12.325%          | 6.231%           | 27.033%   | 12.325%          | 6.231%           |

and -5.142 hartree, respectively. These results clearly show that the  $K_{HF}^A$  depends on the surrounding environment and the first neighbour of atoms is enough to compute the  $K_{HF}^A$ .

The  $K_{HF}$  is computed by summing over  $K_{HF}^A$ ,

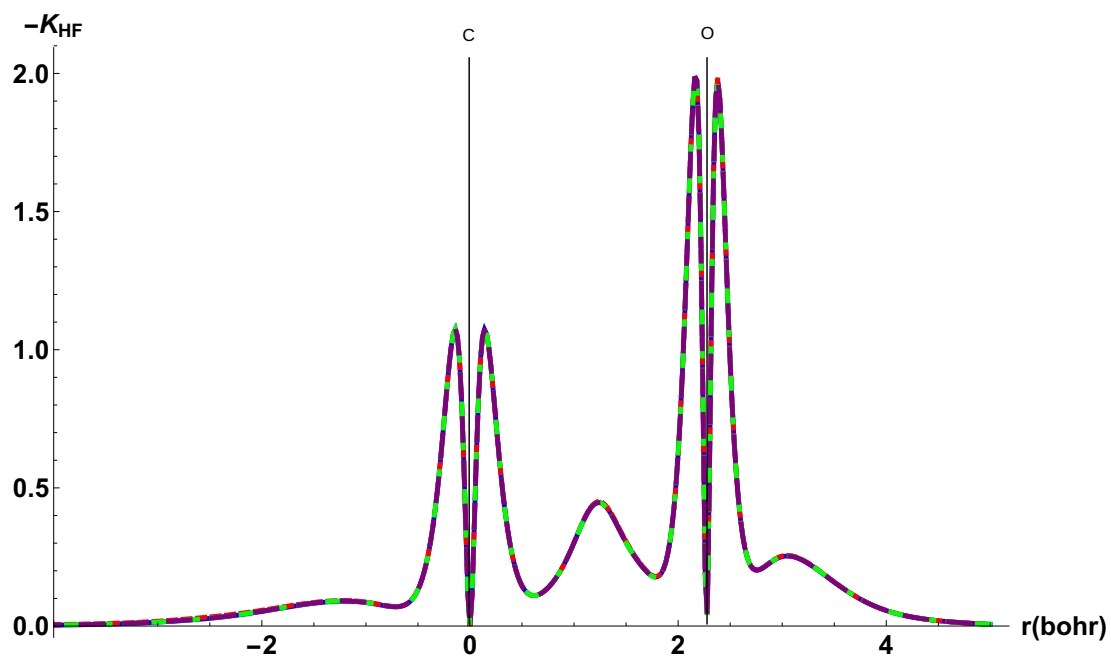
$$K_{HF}^{sum} = \sum_{A=1}^M K_{HF}^A \quad (7.5)$$

where  $M$  is the number of atoms in the target molecule.

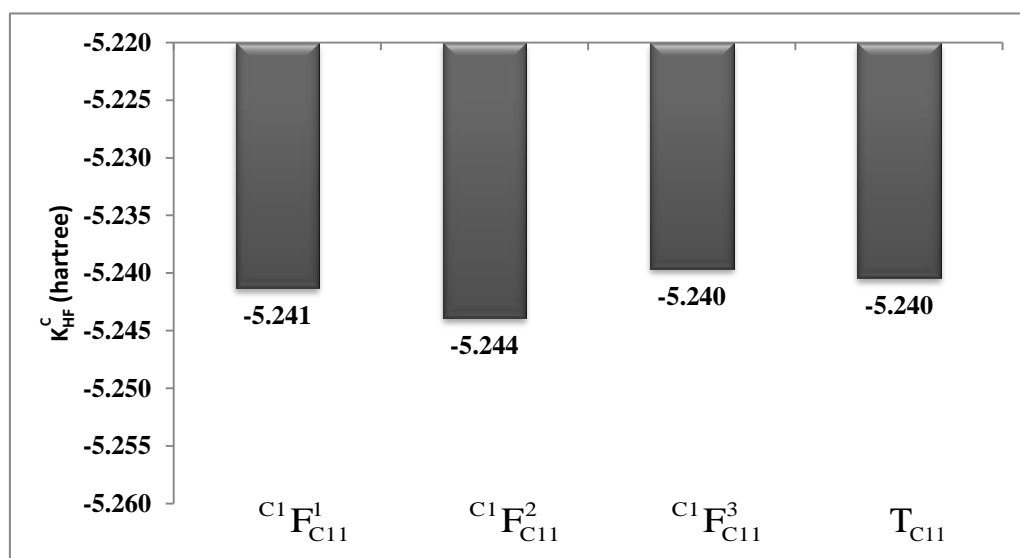
To investigate the validity of our results, we compare the  $K_{HF}$  of three mono-amino acids (glycine Gly, alanine Ala, and threonine Thr), six di-amino acids (GlyGly, AlaAla, ThrThr, GlyAla, GlyThr, and AlaThr), and two isomers of tri-amino acid (GlyGlyGly1, GlyGlyGly2) computed using the sum over  $K_{HF}^A$  that stored in the database ( $K_{HF,database}^{sum} = \sum_{A=1} K_{HF,database}^A$ ) and by using the direct method ( $K_{HF,direct}^{sum} = \sum_{A=1} K_{HF,direct}^A$ ) of  $\mathbf{F}^1$ ,  $\mathbf{F}^2$ , and  $\mathbf{F}^3$  fragments size with those computed numerically (Equation 6.31) for the target molecule. These results as tabulated in Table 7.4 are encouraging. As shown in Table 7.4, the MAPE of  $K_{HF,database}^{sum}$  are about 0.053%, 0.058%, and 0.035% for  $\mathbf{F}^1$ ,  $\mathbf{F}^2$ , and  $\mathbf{F}^3$ , respectively, and the MAPE of  $K_{HF,direct}^{sum}$  are about 0.067%, 0.058%, and 0.032% for  $\mathbf{F}^1$ ,  $\mathbf{F}^2$ , and  $\mathbf{F}^3$ , respectively.

## 7.5 Potential Energy ( $V_{ne}$ )

3D-contour plot of the molecular radial potential energy density (Equation 6.76) of the target carbon atom ( $\mathbf{T}_{C11}$ ) in the D molecule is shown in Figure 7.12. As shown in Figure 7.12, the electrons have more radial potential energy densities when they are close to the nuclei. This is expected because the potential energy density at specific coordinate



**Figure 7.10:** The  $K_{HF,rad}^A$  along the bond between the target carbon atom and the oxygen atom for  $F_{C11}^1$  (red dash line),  $F_{C11}^2$  (green dash line),  $F_{C11}^3$  (purple dash line), and the molecule of interest, D molecule, (blue solid line). The carbon is located at 0.0 bohr and oxygen is located at 2.2739 bohr.

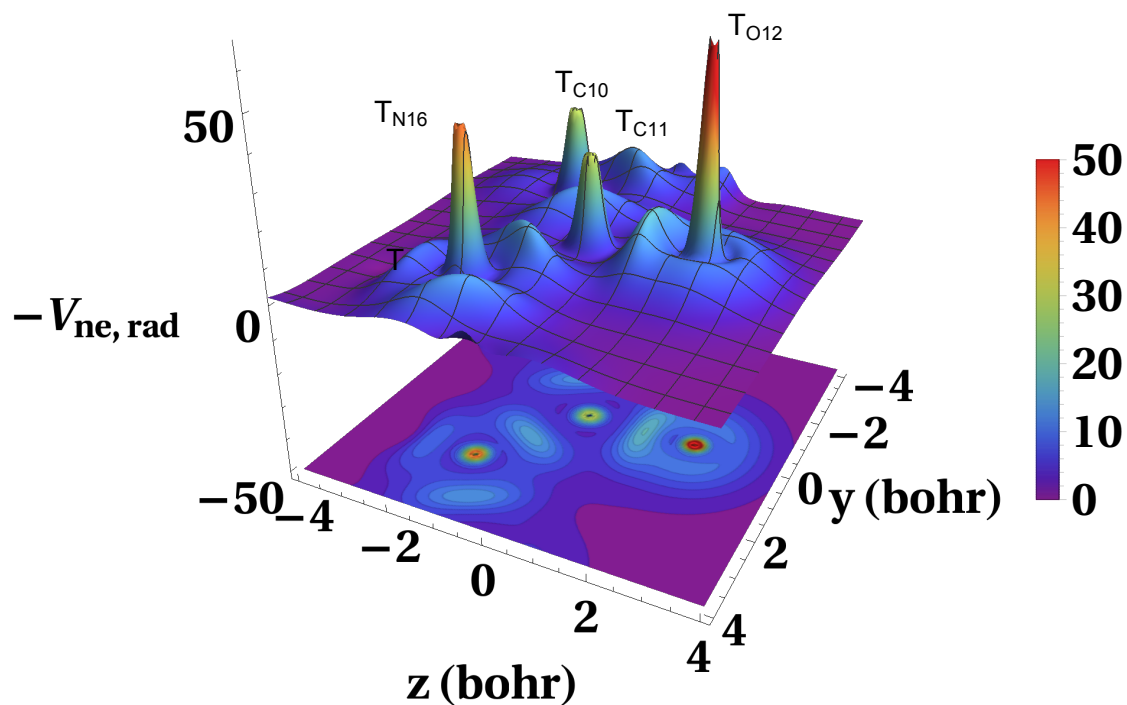


**Figure 7.11:** Effect of size of the fragment on the  $K_{HF}^A$  (Equation 6.34) of target carbon atom.



**Table 7.4:** Comparing the molecular HF exchange energy densities calculated using the sum over atomic HF exchange energy densities that are stored in the database ( $K_{HF,database}^{sum} = \sum_{A=1} K_{HF,database}^A$ ) and by using the direct method ( $K_{HF,direct}^{sum} = \sum_{A=1} K_{HF,direct}^A$ ) with those calculated numerically (Equation 6.31) for the target molecule.  $\mathbf{F}^1$ ,  $\mathbf{F}^2$ , and  $\mathbf{F}^3$  are the size of fragments as first, second, and third neighbor atoms, respectively.

| Target molecule |   | Fragments (Sum over atoms)   |                   |                    |  |                   |                    |
|-----------------|---|--|-------------------|--------------------|--|-------------------|--------------------|
| Molecule        | $\mathbf{K}_{HF}$<br>(Analytical) (Numerical) | $\mathbf{K}_{HF,database}^{sum} = \sum_A \mathbf{K}_{HF,database}^A$ |                   |                    | $\mathbf{K}_{HF,direct}^{sum} = \sum_A \mathbf{K}_{HF,direct}^A$ |                   |                    |
|                 |   | $\mathbf{F}^1$   | $\mathbf{F}^2$    | $\mathbf{F}^3$     | $\mathbf{F}^1$   | $\mathbf{F}^2$    | $\mathbf{F}^3$     |
| Gly             | -35.32928                                     | -35.32928  | -35.334 (-0.012%) | -35.358 (-0.081%)  | -35.334 (-0.013%)  | -35.332 (-0.008%) | -35.355 (-0.074%)  |
| Ala             | -41.24681                                     | -41.24680  | -41.229 (0.044%)  | -41.273 (-0.063%)  | -41.266 (-0.047%)  | -41.226 (0.051%)  | -41.271 (-0.058%)  |
| Thr             | -55.43881                                     | -55.43875  | -55.406 (0.059%)  | -55.464 (-0.046%)  | -55.460 (-0.038%)  | -55.391 (0.086%)  | -55.468 (-0.053%)  |
| GlyGly          | -61.72763                                     | -61.72759  | -61.712 (0.026%)  | -61.777 (-0.080%)  | -61.746 (-0.030%)  | -61.711 (0.027%)  | -61.776 (-0.079%)  |
| AlaAla          | -73.56106                                     | -73.56094  | -73.502 (0.080%)  | -73.602 (-0.056%)  | -73.610 (-0.067%)  | -73.497 (0.086%)  | -73.603 (-0.058%)  |
| ThrThr          | -101.94512                                    | -101.94481   | -101.857 (0.086%) | -101.975 (-0.029%) | -101.979 (-0.033%)   | -101.811 (0.131%) | -101.973 (-0.028%) |
| GlyAla          | -67.64293                                     | -67.64281  | -67.607 (0.053%)  | -67.688 (-0.067%)  | -67.682 (-0.058%)  | -67.605 (0.056%)  | -67.690 (-0.070%)  |
| GlyThr          | -81.83706                                     | -81.83680  | -81.784 (0.064%)  | -81.874 (-0.046%)  | -81.854 (-0.022%)  | -81.761 (0.093%)  | -81.871 (-0.042%)  |
| AlaThr          | -87.75464                                     | -87.75452  | -87.679 (0.086%)  | -87.786 (-0.036%)  | -87.800 (-0.052%)  | -87.655 (0.114%)  | -87.788 (-0.039%)  |
| GlyGlyGly1      | -88.12133                                     | -88.12117  | -88.090 (0.036%)  | -88.183 (-0.070%)  | -88.112 (0.010%)   | -88.088 (0.037%)  | -88.189 (-0.077%)  |
| GlyGlyGly2      | -88.12523                                     | -88.12508  | -88.090 (0.040%)  | -88.183 (-0.066%)  | -88.112 (0.015%)   | -88.079 (0.052%)  | -88.181 (-0.063%)  |
| MAPE =          |   | 0.053%   | 0.058%            | 0.035%             | 0.067%   | 0.058%            | 0.032%             |

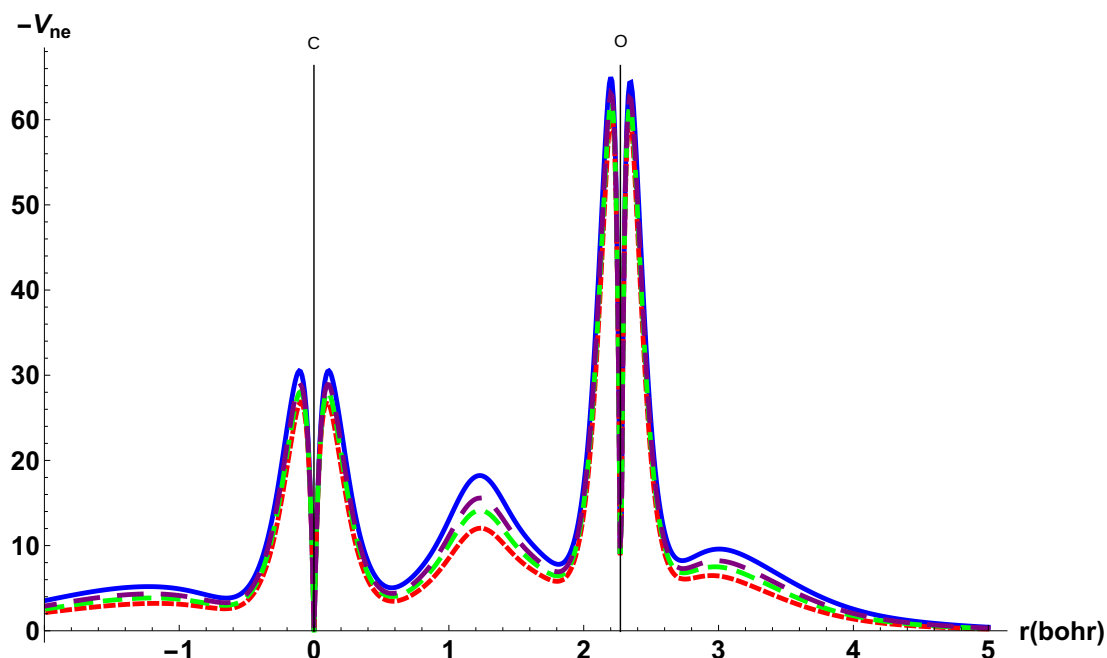


**Figure 7.12:** The molecular radial potential energy density of molecule D (Figure 7.3),  $T_{C11}$  is the target carbon atom.

represents the interaction between an electron at those coordinates with all nuclei in the molecule. This interaction increases as the electron becomes closer to a nucleus in the molecule. Moreover, the potential energy density is proportional to the nuclear charge of the atom, where the potential energy density near the oxygen atom has the largest values followed by nitrogen, then carbon (Figure 7.12).

The radial potential energy density (Equation 6.76) along the bond between the target carbon atom and oxygen atom for  $F_{C11}^1$  (red dash line),  $F_{C11}^2$  (green dash line),  $F_{C11}^3$  (purple dash line), and the D molecule (blue solid line) are compared in Figure 7.13. As shown in Figure 7.13 the potential energy density is proportional to the size of molecules. For example, the molecular radial potential energy density for D molecule is larger than the

fragment  $\mathbf{F}_{\text{C11}}^1$ .



**Figure 7.13:** The radial potential energy density along the bond between the target carbon atom and the oxygen atom for  $\mathbf{F}_{\text{C11}}^1$  (red dash line),  $\mathbf{F}_{\text{C11}}^2$  (green dash line),  $\mathbf{F}_{\text{C11}}^3$  (purple dash line), and the molecule of interest, D molecule, (blue solid line). The carbon is located at 0.0 bohr and oxygen is located at 2.2739 bohr.

The potential energy of the target carbon atoms  $\text{C}^1\mathbf{F}_{\text{C11}}^1$ ,  $\text{C}^1\mathbf{F}_{\text{C11}}^2$ ,  $\text{C}^1\mathbf{F}_{\text{C11}}^3$ , and  $\mathbf{T}_{\text{C11}}$  are displayed in Figure 7.14. The results show that the atomic potential energy of the carbon atom is proportional to the size of the molecule.

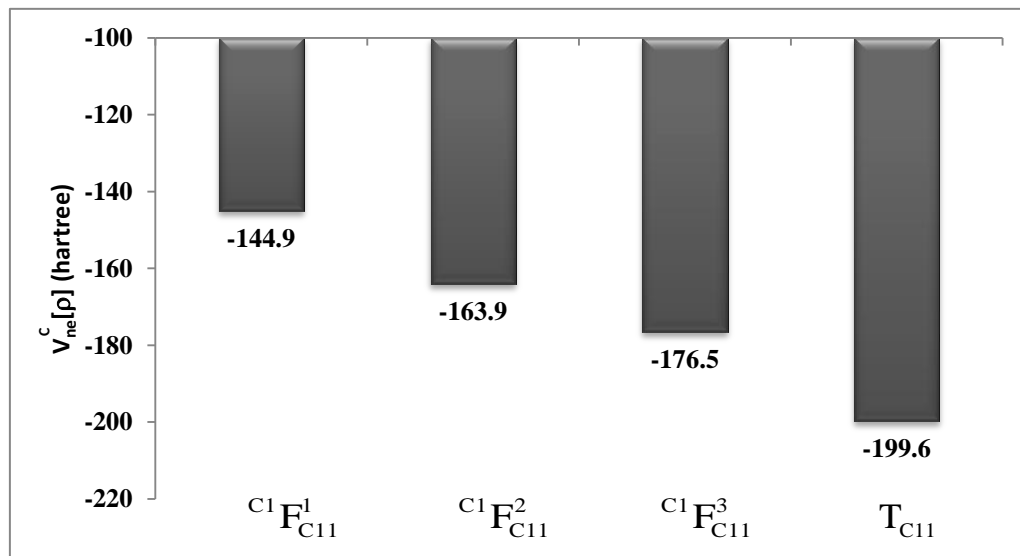
The molecular potential energy is calculated by summing over atomic potential energies,

$$V_{ne}^{sum}[\rho] = \sum_{A=1}^M V_{ne}^A[\rho] \quad (7.6)$$

where  $V_{ne}^A$  is the potential energy density of atom  $A$  computing using Equation 6.74 and  $M$  is the number of atoms in the target molecule. Table 7.5 compares the potential energy den-

**Table 7.5:** Comparing the molecular potential energy densities calculated using the sum over atomic potential energy densities that are stored in the database ( $V_{ne,database}^{sum}[\rho] = \sum_{A=1}^A V_{ne,database}^A[\rho]$ ) and by using the direct method ( $V_{ne,direct}^{sum}[\rho] = \sum_{A=1}^A V_{ne,direct}^A[\rho]$ ) with those calculated numerically (Equation 6.73) for the target molecule.  $\mathbf{F}^1$ ,  $\mathbf{F}^2$ , and  $\mathbf{F}^3$  are the size of fragments as first, second, and third neighbor atoms, respectively.

| Target molecule |                                      | Fragments (Sum over atoms)                                       |                   |                   |   |                   |                   |
|-----------------|--------------------------------------|--|-------------------|-------------------|---|-------------------|-------------------|
| Molecule        | $V_{ne}$<br>(Analytical) (Numerical) | $V_{ne, database}^{sum}[\rho] = \sum_A V_{ne, database}^A[\rho]$ |                   |                   | $V_{ne, direct}^{sum}[\rho] = \sum_A V_{ne, direct}^A[\rho]$ (%error <sub>numerical</sub> ) |                   |                   |
|                 |                                      | F <sup>1</sup>   | F <sup>2</sup>    | F <sup>3</sup>    | F <sup>1</sup>  | F <sup>2</sup>    | F <sup>3</sup>    |
| Gly             | -1028.734 -1028.736                  | -872.49 (15.19%)   | -975.53 (5.17%)   | -1023.52 (0.51%)  | -872.47 (15.19%)  | -975.47 (5.18%)   | -1023.09 (0.55%)  |
| Ala             | -1259.491 -1259.493                  | -1010.16 (19.80%)  | -1156.02 (8.22%)  | -1246.36 (1.04%)  | -1010.01 (19.81%)   | -1155.89 (8.23%)  | -1246.33 (1.04%)  |
| Thr             | -1847.851 -1847.852                  | -1373.81 (25.65%)  | -1590.24 (13.94%) | -1756.55 (4.94%)  | -1372.50 (25.72%)   | -1590.13 (13.95%) | -1755.89 (4.98%)  |
| GlyGly          | -2067.008 -2067.014                  | -1542.80 (25.36%)  | -1761.69 (14.77%) | -1901.82 (7.99%)  | -1542.39 (25.38%)   | -1761.44 (14.78%) | -1901.32 (8.02%)  |
| AlaAla          | -2637.297 -2637.298                  | -1818.15 (31.06%)  | -2122.72 (19.51%) | -2358.44 (10.57%) | -1817.46 (31.09%)   | -2122.72 (19.51%) | -2358.90 (10.56%) |
| ThrThr          | -4080.834 -4080.823                  | -2545.44 (37.62%)  | -2990.35 (26.72%) | -3372.24 (17.36%) | -2541.73 (37.72%)   | -2989.29 (26.75%) | -3374.76 (17.30%) |
| GlyAla          | -2332.480 -2332.480                  | -1680.48 (27.95%)  | -1942.37 (16.73%) | -2123.56 (8.96%)  | -1680.06 (27.97%)   | -1942.61 (16.71%) | -2124.03 (8.94%)  |
| GlyThr          | -3048.499 -3048.490                  | -2044.12 (32.95%)  | -2375.64 (22.07%) | -2642.65 (13.31%) | -2042.09 (33.01%)   | -2375.79 (22.07%) | -2643.80 (13.28%) |
| AlaThr          | -3357.328 -3357.332                  | -2181.79 (35.01%)  | -2556.31 (23.86%) | -2868.63 (14.56%) | -2179.67 (35.08%)   | -2556.24 (23.86%) | -2865.50 (14.65%) |
| GlyGlyGly1      | -3201.414 -3201.417                  | -2213.12 (30.87%)  | -2547.02 (20.44%) | -2776.16 (13.28%) | -2212.37 (30.89%)   | -2548.45 (20.40%) | -2781.38 (13.12%) |
| GlyGlyGly2      | -3330.655 -3330.660                  | -2213.12 (33.55%)  | -2547.02 (23.53%) | -2776.16 (16.65%) | -2211.48 (33.60%)   | -2544.68 (23.60%) | -2777.47 (16.61%) |
| MAPE =          |                                      | 28.64%   | 17.72%            | 9.93%             | 28.68%  | 17.73%            | 9.91%             |



**Figure 7.14:** Effect of size of the fragment on the atomic potential energy density ( $V_{ne}[\rho]$ ) of target carbon atom.

sity of three mono-amino acids (glycine Gly, alanine Ala, and threonine Thr), six di-amino acids (GlyGly, AlaAla, ThrThr, GlyAla, GlyThr, and AlaThr), and two isomers of tri-amino acid (GlyGlyGly1 and GlyGlyGly2) calculated using the sum over atomic potential energy densities that stored in the database ( $V_{ne, database}^{sum}[\rho] = \sum_{A=1} V_{ne, database}^A[\rho]$ ) and by using the direct method ( $V_{ne, direct}^{sum}[\rho] = \sum_{A=1} V_{ne, direct}^A[\rho]$ ) of  $F^1$ ,  $F^2$ , and  $F^3$  fragments with those calculated numerically using Equation 6.73 for the target molecule. Unfortunately, the MAPE for these molecules is very large. The MAPE of  $V_{ne, database}^{sum}[\rho]$  are about 28.64%, 17.72%, and 9.93% for  $F^1$ ,  $F^2$ , and  $F^3$ , respectively, and the MAPE of  $V_{ne, direct}^{sum}[\rho]$  are about 28.68%, 17.73%, and 9.91% for  $F^1$ ,  $F^2$ , and  $F^3$ , respectively. Thus, we cannot consider the potential energy as an atomic property as its value depends very much on the geometry and other atoms of the molecule.

In order to compute the molecular potential energy correctly for the molecule of interest,

one can calculate the molecular potential energy from its atomic electron densities ( $V_{ne}[\rho]$ ) by using Equation 6.73. The atomic electron densities can be built using two different ways: from the database or by using the direct way. In the first way the Cartesian coordinates for the radial grids of the target atom are transformed from the database into the coordinates of the molecule of interest as discussed in section 5.5. In the direct way, the electron densities for each atom are computed in a specific environment (fragment) by keeping the same Cartesian coordinates of the atoms and their radial grids in the molecule of interest. Computing the molecular potential energy for the molecule from its atomic electron densities involves performing numerical integration calculations. Thus, these calculations need computer time and may produce large errors.

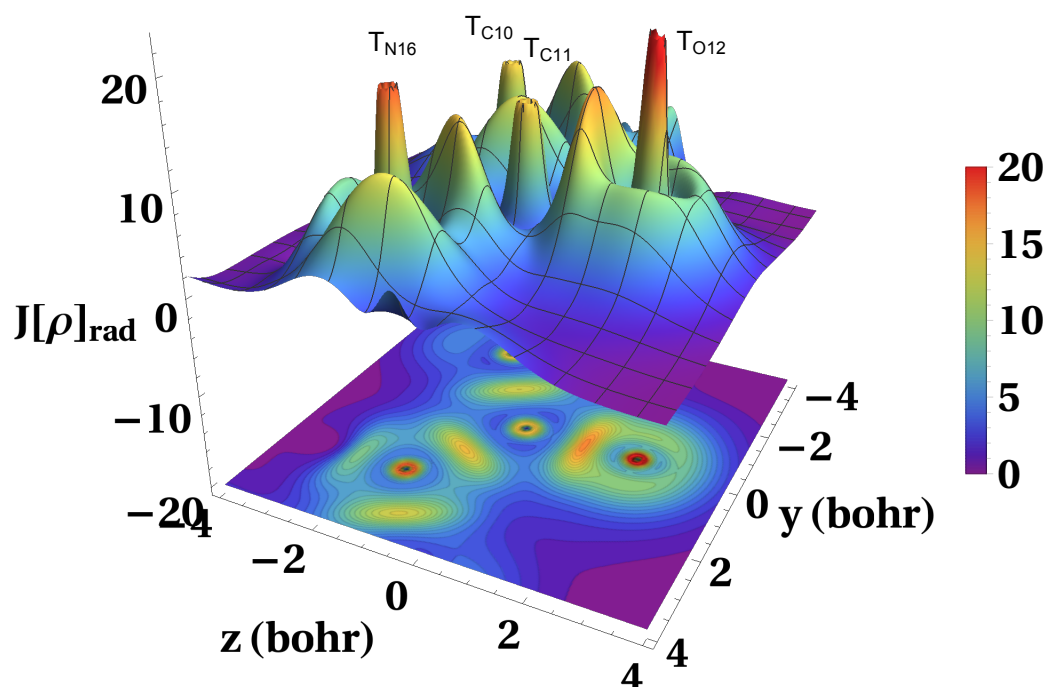
To investigate the validity of our results, we compare the calculated potential energies using the atomic electron densities stored in the database ( $V_{ne}[\rho_{database}]$ ) and using the direct method ( $V_{ne}[\rho_{direct}]$ ) for the  $\mathbf{F}^1$ ,  $\mathbf{F}^2$ , and  $\mathbf{F}^3$  fragments with those computed numerically ( $V_{ne}[\rho_{target}]$ ) for the target molecule (Equation 6.31). The results are tabulated in Table 7.6. The MAPE of  $V_{ne}[\rho_{database}]$  are about 0.046%, 0.087%, and 0.066% for  $\mathbf{F}^1$ ,  $\mathbf{F}^2$ , and  $\mathbf{F}^3$ , respectively, and the MAPE of  $V_{ne}[\rho_{direct}]$  are about 0.034%, 0.085%, and 0.051% for  $\mathbf{F}^1$ ,  $\mathbf{F}^2$ , and  $\mathbf{F}^3$ , respectively. The results are encouraging, in which the %errors are small and the %errors of  $V_{ne}[\rho_{database}]$  and  $V_{ne}[\rho_{direct}]$  are almost same.

## 7.6 Coulomb Energy ( $J$ )

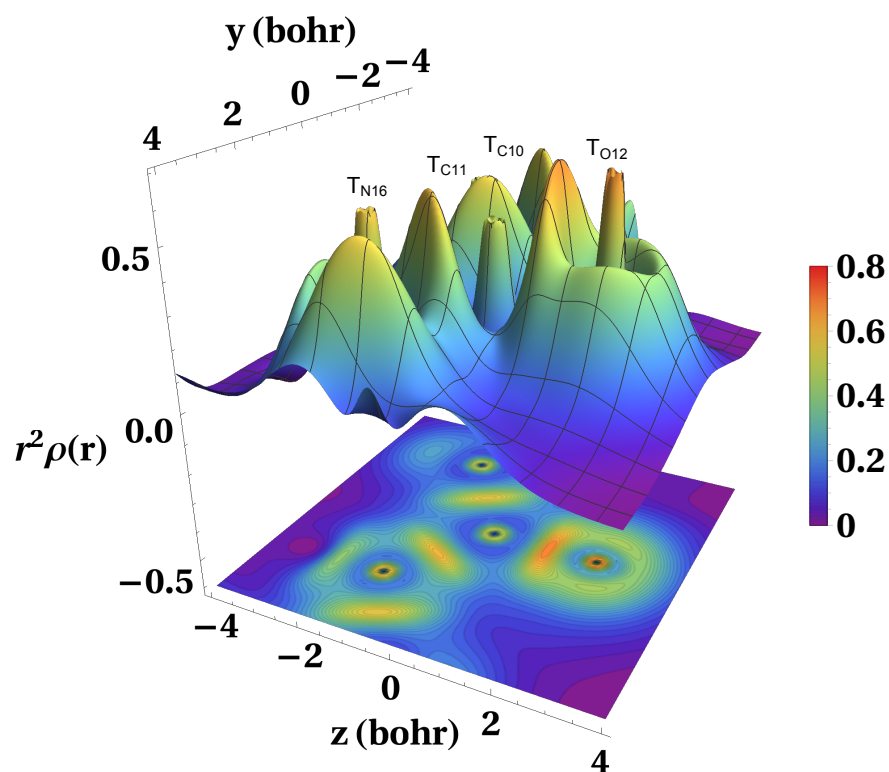
3D-contour plots of the molecular radial Coulomb energy density (Equation 6.68) and molecular radial electron density (RDEN) of the target carbon ( $\mathbf{T}_{C11}$ ) atom in the D molecule are shown in Figures 7.15a and b, respectively. As shown in Figures 7.15a and b, the ra-

**Table 7.6:** Comparing the molecular potential energy densities using the atomic electron densities that are stored in the database ( $V_{ne}[\rho_{database}]$ ) and by using the direct method ( $V_{ne}[\rho_{direct}]$ ) with those calculated numerically (Equation 6.73) for the target molecule.  $\mathbf{F}^1$ ,  $\mathbf{F}^2$ , and  $\mathbf{F}^3$  are the size of fragments as first, second, and third neighbor atoms, respectively.

| Molecule   | Target molecule<br>$V_{ne}$ |             | Fragments (Using the electron densities and the fragments partitioning weights) |                     |                     |                     |
|------------|-----------------------------|-------------|---|---------------------|---------------------|---------------------|
|            | (Analytical)                | (Numerical) | $V_{ne}[\rho_{database}]$   | $\mathbf{F}^1$      | $\mathbf{F}^2$      | $\mathbf{F}^3$      |
| Gly        | -1028.734                   | -1028.736   | -1028.755 (-0.002%)   | -1028.754 (-0.002%) | -1029.436 (-0.068%) | -1028.883 (-0.014%) |
|            | -1259.491                   | -1259.493   | -1258.967 (0.042%)  | -1258.949 (0.043%)  | -1260.214 (-0.057%) | -1259.963 (-0.037%) |
| Ala        | -1847.851                   | -1847.852   | -1846.398 (0.079%)  | -1846.642 (0.065%)  | -1848.838 (-0.053%) | -1848.201 (-0.019%) |
|            | -2067.008                   | -2067.014   | -2066.854 (0.008%)  | -2066.854 (0.008%)  | -2067.938 (-0.045%) | -2067.985 (-0.047%) |
| GlyGly     | -2637.297                   | -2637.298   | -2635.765 (0.058%)  | -2639.641 (-0.089%) | -2639.792 (-0.095%) | -2639.311 (-0.076%) |
|            | -4080.834                   | -4080.823   | -4076.722 (0.100%)  | -4084.093 (-0.080%) | -4084.009 (-0.078%) | -4082.602 (-0.044%) |
| AlaAla     | -2332.480                   | -2332.480   | -2331.687 (0.034%)  | -2334.602 (-0.091%) | -2334.339 (-0.080%) | -2334.037 (-0.067%) |
|            | -3048.499                   | -3048.490   | -3046.485 (0.066%)  | -3051.753 (-0.107%) | -3051.035 (-0.083%) | -3050.179 (-0.055%) |
| AlaThr     | -3357.328                   | -3357.332   | -3354.497 (0.084%)  | -3360.457 (-0.093%) | -3360.834 (-0.104%) | -3359.568 (-0.067%) |
|            | -3201.414                   | -3201.417   | -3201.157 (0.008%)  | -3205.321 (-0.122%) | -3203.989 (-0.080%) | -3203.830 (-0.075%) |
| GlyGlyGly1 | -3330.655                   | -3330.660   | -3329.984 (0.020%)  | -3334.231 (-0.107%) | -3332.867 (-0.066%) | -3332.496 (-0.055%) |
|            |                             |             |   |                     |                     |                     |
| MAPE =     |                             |             | 0.046%  | 0.087%              | 0.066%              | 0.051%              |
|            |                             |             |   | 0.034%              | 0.085%              |                     |



(a) The molecular radial Coulomb energy density using Equation 6.68.



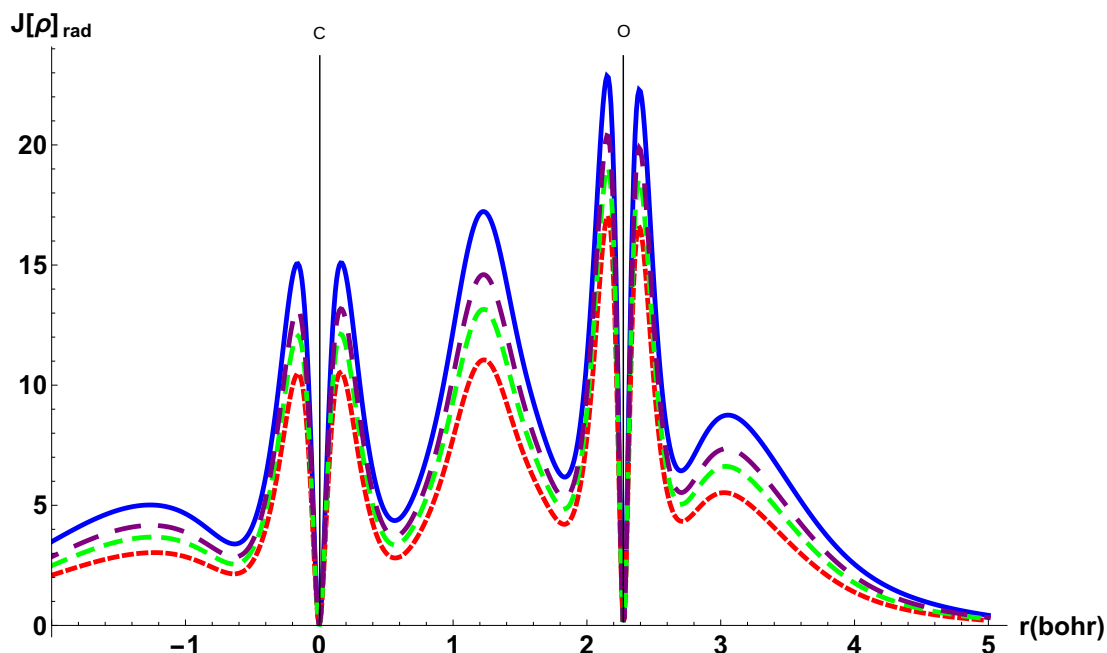
(b) The molecular radial electron density (RDEN).

**Figure 7.15:** Comparing the molecular radial electron density (RDEN) and the molecular radial Coulomb energy density  $J[\rho]_{rad}$  (Equation 6.68) of molecule D (Figure 7.3).  $T_{C11}$  is the target carbon atom.



dial Coulomb energy density  $J[\rho]_{rad}$  and RDEN have a similar shape. The similarity of Coulomb energy with RDEN may be due to the proportionality between the radial Coulomb energy and RDEN.

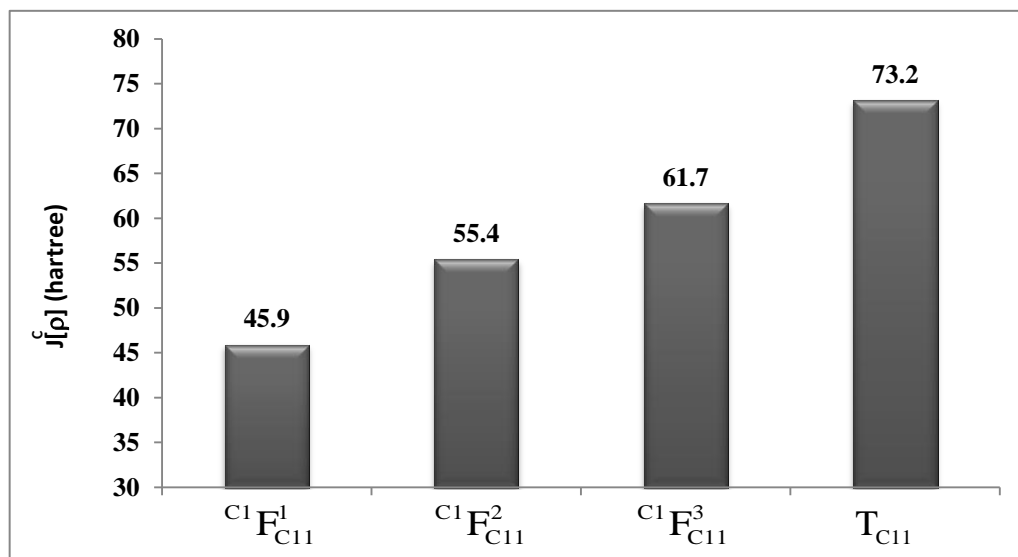
The radial Coulomb energy densities (Equation 6.68) along the bond between the target carbon atom and oxygen atom for:  $F_{C11}^1$  (red dash line),  $F_{C11}^2$  (green dash line),  $F_{C11}^3$  (purple dash line), and the D molecule (blue solid line) are compared in Figure 7.16. As shown in Figure 7.16, the Coulomb energy density is proportional to the size of molecules (e.g., the molecular radial Coulomb energy density for D molecule is larger than the fragment  $F_{C11}^3$ ).



**Figure 7.16:** The radial Coulomb energy density (Equation 6.68) along the bond between the target carbon atom and the oxygen atom for  $F_{C11}^1$  (red dash line),  $F_{C11}^2$  (green dash line),  $F_{C11}^3$  (purple dash line), and the molecule of interest, D molecule, (blue solid line). The carbon is located at 0.0 bohr and oxygen is located at 2.2739 bohr.

Figure 7.17 shows the Coulomb energies of the target carbon atoms:  $C1F_{C11}^1$ ,  $C1F_{C11}^2$ ,

$C^1F_{C11}^3$ , and  $T_{C11}$  obtained using Equation 6.65 ( $V(\mathbf{r})$  is computed analytically). The results show that the atomic Coulomb energies of the target carbon atom are proportional to the size of the molecule.



**Figure 7.17:** Effect of size of the fragment on the atomic Coulomb energy density ( $J[\rho]$ ) of target carbon atom (Equation 6.65).

The molecular Coulomb energy is calculated by summing over atomic Coulomb energies,

$$J^{sum}[\rho] = \sum_{A=1}^M J^A[\rho] \quad (7.7)$$

where  $M$  is the number of atoms in the target molecule and  $J^A[\rho]$  is the Coulomb energy of atom  $A$  computed using Equation 6.65. Table 7.7 compares the Coulomb energy of three mono-amino acids (glycine Gly, alanine Ala, and threonine Thr), six di-amino acids (GlyGly, AlaAla, ThrThr, GlyAla, GlyThr, and AlaThr), and two isomers of tri-amino acid (GlyGlyGly1 and GlyGlyGly2) calculated using the sum over atomic Coulomb ener-

**Table 7.7:** Comparing the molecular Coulomb energies calculated using the sum over atomic Coulomb energies that are stored in the database ( $J_{database}^{sum}[\rho] = \sum_{A=1} J_{database}^A[\rho]$ ) and by using the direct method ( $J_{direct}^{sum}[\rho] = \sum_{A=1} J_{direct}^A[\rho]$ ) with those calculated analytically/numerically (Equation 6.65) for the target molecule.  $\mathbf{F}^1$ ,  $\mathbf{F}^2$ , and  $\mathbf{F}^3$  are the size of fragments as first, second, and third neighbor atoms, respectively.

| Target molecule |   | Fragments (Sum over atoms)  |                 |                  |   |                 |                  |
|-----------------|---|---|-----------------|------------------|---|-----------------|------------------|
| Molecule        | $\mathbf{J}[\rho]$<br>(Analytical) (Analytical/<br>Numerical) | $\mathbf{J}_{database}^{sum}[\rho] = \sum_A \mathbf{J}_{database}^A[\rho]$ (%error) |                 |                  | $\mathbf{J}_{direct}^{sum}[\rho] = \sum_A \mathbf{J}_{direct}^A[\rho]$ (%error) |                 |                  |
|                 |   | $\mathbf{F}^1$  | $\mathbf{F}^2$  | $\mathbf{F}^3$   | $\mathbf{F}^1$  | $\mathbf{F}^2$  | $\mathbf{F}^3$   |
| Gly             | 317.4368 317.4378   | 239.44 (24.57%)   | 290.93 (8.35%)  | 314.85 (0.81%)   | 239.44 (24.57%)   | 290.90 (8.36%)  | 314.66 (0.88%)   |
| Ala             | 405.9641 405.9650   | 281.36 (30.69%)   | 354.08 (12.78%) | 399.47 (1.60%)   | 281.29 (30.71%)   | 354.00 (12.80%) | 399.41 (1.61%)   |
| Thr             | 620.8853 620.8858   | 383.86 (38.17%)   | 491.83 (20.79%) | 575.02 (7.39%)   | 383.33 (38.26%)   | 491.85 (20.78%) | 574.60 (7.45%)   |
| GlyGly          | 692.5468 692.5492   | 430.61 (37.82%)   | 539.98 (22.03%) | 609.96 (11.93%)  | 430.49 (37.84%)   | 539.92 (22.04%) | 609.73 (11.96%)  |
| AlaAla          | 923.9177 923.9179   | 514.45 (44.32%)   | 666.40 (27.87%) | 784.59 (15.08%)  | 514.20 (44.35%)   | 666.40 (27.87%) | 784.70 (15.07%)  |
| ThrThr          | 1487.2822 1487.2767   | 719.45 (51.63%)   | 941.49 (36.70%) | 1135.34 (23.66%) | 717.96 (51.73%)   | 941.12 (36.72%) | 1133.59 (23.78%) |
| GlyAla          | 798.3259 798.3258   | 472.53 (40.81%)   | 603.19 (24.44%) | 694.14 (13.05%)  | 472.40 (40.83%)   | 603.34 (24.42%) | 694.29 (13.03%)  |
| GlyThr          | 1076.9553 1076.9508   | 575.03 (46.61%)   | 740.60 (31.23%) | 873.89 (18.85%)  | 574.27 (46.68%)   | 740.76 (31.22%) | 874.50 (18.80%)  |
| AlaThr          | 1204.5755 1204.5772   | 616.95 (48.78%)   | 803.81 (33.27%) | 960.28 (20.28%)  | 616.14 (48.85%)   | 803.92 (33.26%) | 958.60 (20.42%)  |
| GlyGlyGly1      | 1115.4045 1115.4058   | 621.77 (44.26%)   | 788.57 (29.30%) | 903.13 (19.03%)  | 621.57 (44.27%)   | 789.29 (29.24%) | 905.67 (18.80%)  |
| GlyGlyGly2      | 1180.0865 1180.0886   | 621.77 (47.31%)   | 788.57 (33.18%) | 903.13 (23.47%)  | 621.17 (47.36%)   | 787.56 (33.26%) | 903.72 (23.42%)  |
| MAPE =          |   | 41.36%  | 25.45%          | 14.11%           | 41.40%  | 25.45%          | 14.11%           |

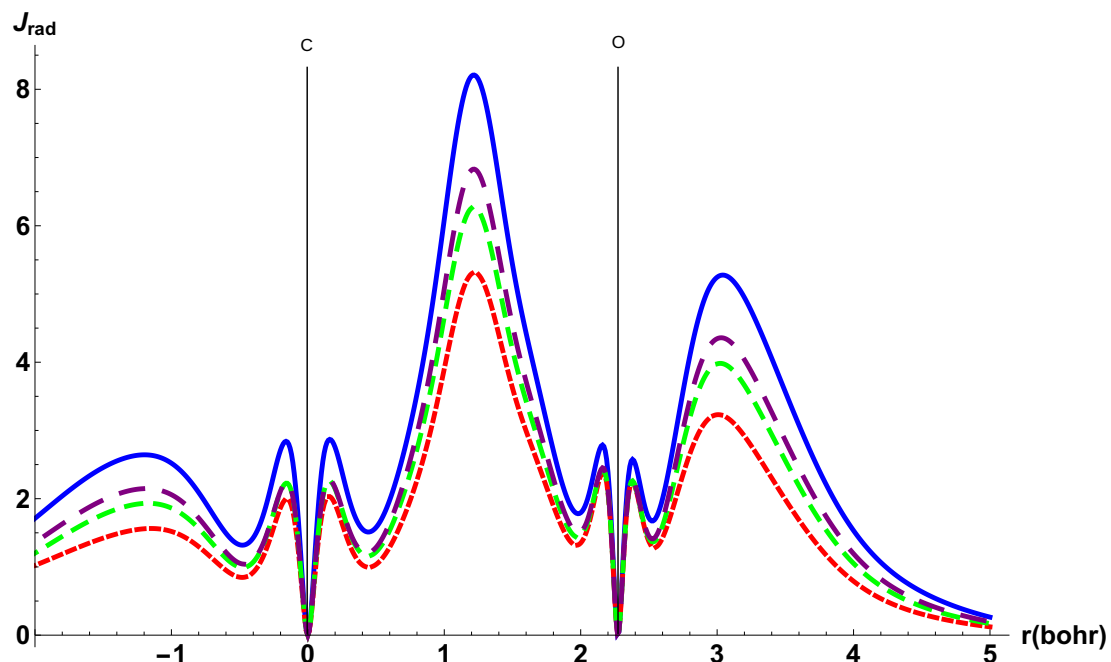
gies stored in the database ( $J_{database}^{sum}[\rho] = \sum_{A=1} J_{database}^A[\rho]$ ) and by using the direct method ( $J_{direct}^{sum}[\rho] = \sum_{A=1} J_{direct}^A[\rho]$ ) of  $\mathbf{F}^1$ ,  $\mathbf{F}^2$ , and  $\mathbf{F}^3$  fragments size with those calculated analytically/numerically using Equation 6.65 for the target molecule. Unfortunately, the MAPEs for these molecules is very large. The MAPE of  $J_{database}^{sum}[\rho]$  are about 41.36%, 25.45%, and 14.11% for  $\mathbf{F}^1$ ,  $\mathbf{F}^2$ , and  $\mathbf{F}^3$ , respectively, and the MAPE of  $J_{direct}^{sum}[\rho]$  are about 41.40%, 25.45%, and 14.11% for  $\mathbf{F}^1$ ,  $\mathbf{F}^2$ , and  $\mathbf{F}^3$ , respectively. Thus, we cannot consider the Coulomb energy,  $J[\rho]$ , as an atomic property as its value depends on the geometry and other atoms of the molecule.

Figure 7.18 compares the molecular radial Coulomb energy density obtained by using the molecular orbital expansion (Equation 6.39) along the bond between the target carbon atom and the oxygen atom for  $\mathbf{F}_{C11}^1$  (red dash line),  $\mathbf{F}_{C11}^2$  (green dash line),  $\mathbf{F}_{C11}^3$  (purple dash line), and the D molecule (blue solid line). As shown in Figure 7.18, the molecular radial Coulomb energy density is proportional to the size of molecules (e.g., the Coulomb energy density for the D molecule is larger than the fragment  $\mathbf{F}_{C11}^3$ ).

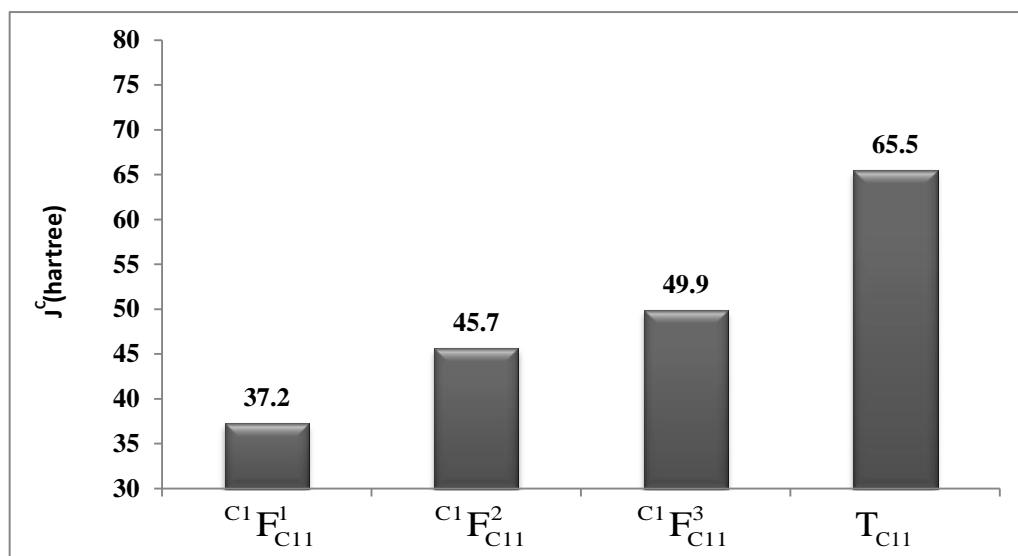
The computed Coulomb energies obtained by using the molecular orbital expansion (Equation 6.52) of the target carbon atom for the four molecules  $\mathbf{F}_{C11}^1$ ,  $\mathbf{F}_{C11}^2$ ,  $\mathbf{F}_{C11}^3$ , and the D molecule are displayed in Figure 7.19. The results as displayed in Figure 7.19 show that the atomic Coulomb energies,  $J$ , (Equation 6.52) of the carbon atom are proportional to the size of the molecule.

The molecular Coulomb energy is calculated by summing over atomic Coulomb energies,

$$J^{sum} = \sum_{A=1}^M J^A \quad (7.8)$$



**Figure 7.18:** The radial Coulomb energy density obtained by using the molecular orbital expansion (Equation 6.39) along the bond between the target carbon atom and the oxygen atom for  $F_{C11}^1$  (red dash line),  $F_{C11}^2$  (green dash line),  $F_{C11}^3$  (purple dash line), and the molecule of interest, D molecule, (blue solid line). The carbon is located at 0.0 bohr and oxygen is located at 2.2739 bohr.



**Figure 7.19:** Effect of size of the fragment on the atomic Coulomb energy ( $J$ ) of target carbon atom.

where  $J^A$  is the Coulomb energy of atom  $A$  computed using Equation 6.52 and  $M$  is the number of atoms in the target molecule. Table 7.8 compares the Coulomb energies,  $J$ , computed using the sum over atomic Coulomb energies stored in the database ( $J_{database}^{sum} = \sum_{A=1} J_{database}^A$ ) and by using the direct method ( $J_{direct}^{sum} = \sum_{A=1} J_{direct}^A$ ) of  $\mathbf{F}^1$ ,  $\mathbf{F}^2$ , and  $\mathbf{F}^3$  fragments size with those calculated using Equation 6.52 for the target molecule. Unfortunately, the MAPE for these molecules is very large. The MAPE of  $J_{database}^{sum}$  are about 46.95%, 29.12%, and 16.48% for  $\mathbf{F}^1$ ,  $\mathbf{F}^2$ , and  $\mathbf{F}^3$ , respectively, and the MAPE of  $J_{direct}^{sum}$  are about 47.08%, 29.14%, and 16.48% for  $\mathbf{F}^1$ ,  $\mathbf{F}^2$ , and  $\mathbf{F}^3$ , respectively. Thus we cannot consider the Coulomb energy obtained by using the molecular orbital expansion as an atomic property as its value depends on the geometry and other atoms of the molecule.

Figure 7.20 compares the  $J_{HF,rad}$  (i.e., the radial Coulomb energy contain  $J_{aa}$  term) along the bond between the target carbon atom and the oxygen atom for  $\mathbf{F}_{C11}^1$  (red dash line),  $\mathbf{F}_{C11}^2$  (green dash line),  $\mathbf{F}_{C11}^3$  (purple dash line), and the D molecule (blue solid line). As shown in Figure 7.20, the  $J_{HF,rad}$  is depending on the size of the molecule (e.g., the  $J_{HF,rad}$  for the D molecule is larger than the fragment  $\mathbf{F}_{C11}^1$ ).

The computed  $J_{HF}^A$  (Equation 6.54) of the target carbon atom for the four molecules  $\mathbf{F}_{C11}^1$ ,  $\mathbf{F}_{C11}^2$ ,  $\mathbf{F}_{C11}^3$ , and the D molecule are displayed in Figure 7.21. The results as displayed in Figure 7.21 show that the  $J_{HF}^A$  (Equation 6.54) of the carbon atom are directly proportional to the size of the molecules.

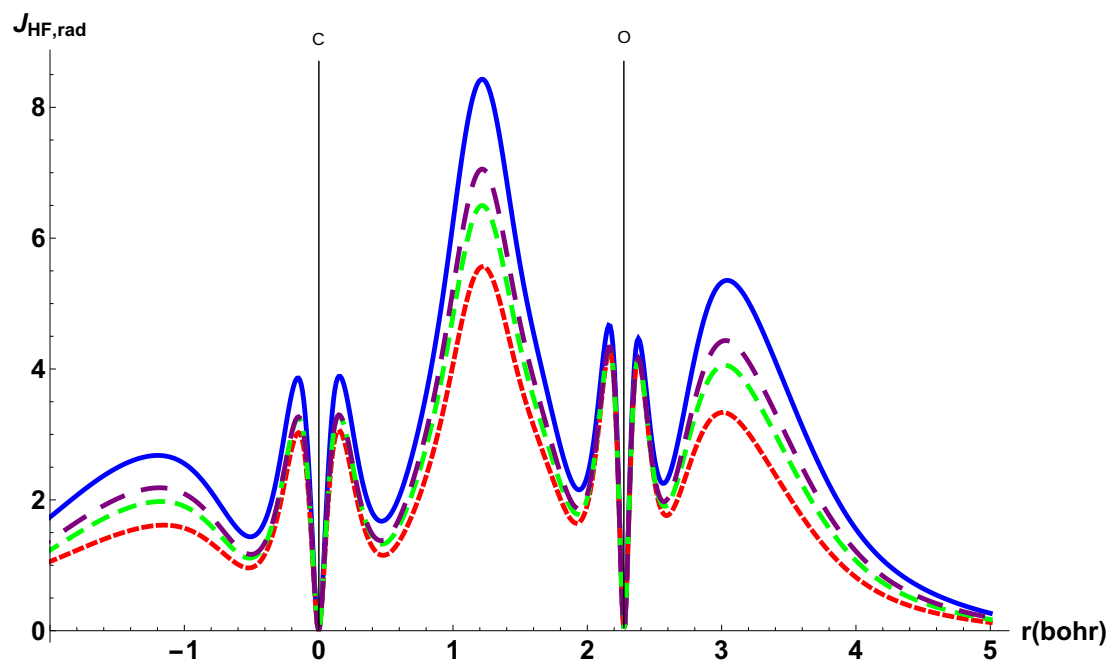
The  $J_{HF}$  is calculated by summing over  $J_{HF}^A$ ,

$$J_{HF}^{sum} = \sum_{A=1}^M J_{HF}^A \quad (7.9)$$

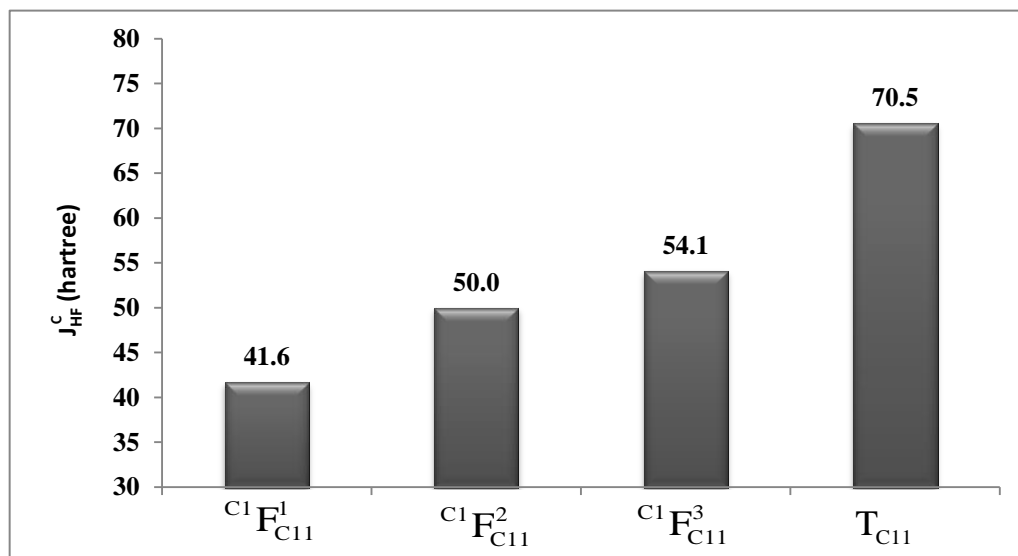
where  $M$  is the number of atoms in the target molecule.

**Table 7.8:** Comparing the molecular Coulomb energy densities calculated using the sum over atomic Coulomb energy densities that are stored in the database ( $J_{database}^{sum} = \sum_{A=1} J_{database}^A$ ) and by using the direct method ( $J_{direct}^{sum} = \sum_{A=1} J_{direct}^A$ ) with those calculated numerically (Equation 6.30) for the target molecule.  $\mathbf{F}^1$ ,  $\mathbf{F}^2$ , and  $\mathbf{F}^3$  are the size of fragments as first, second, and third neighbor atoms, respectively.

| Target molecule |               | $\mathbf{J}_{database}^{sum} = \sum_A \mathbf{J}_{database}^A$ (%error) |                 |                  | Fragments (Sum over atoms) |                 |                  | $\mathbf{J}_{direct}^{sum} = \sum_A \mathbf{J}_{direct}^A$ (%error) |                 |                  |
|-----------------|---------------|---|-----------------|------------------|----------------------------|-----------------|------------------|---|-----------------|------------------|
| Molecule        | J (Numerical) | $\mathbf{F}^1$  | $\mathbf{F}^2$  | $\mathbf{F}^3$   | $\mathbf{F}^1$             | $\mathbf{F}^2$  | $\mathbf{F}^3$   | $\mathbf{F}^1$  | $\mathbf{F}^2$  | $\mathbf{F}^3$   |
| Gly             | 290.199002    | 202.42 (30.25%)   | 259.77 (10.49%) | 286.88 (1.14%)   | 202.42 (30.25%)            | 259.71 (10.51%) | 286.45 (1.29%)   | 202.42 (30.25%)   | 259.71 (10.51%) | 286.45 (1.29%)   |
| Ala             | 374.521914    | 238.90 (36.21%)   | 314.99 (15.89%) | 365.19 (2.49%)   | 238.88 (36.22%)            | 314.58 (16.00%) | 365.05 (2.53%)   | 238.88 (36.22%)   | 314.58 (16.00%) | 365.05 (2.53%)   |
| Thr             | 579.717747    | 325.06 (43.93%)   | 437.69 (24.50%) | 524.28 (9.56%)   | 323.13 (44.26%)            | 437.48 (24.54%) | 524.05 (9.60%)   | 323.13 (44.26%)   | 437.48 (24.54%) | 524.05 (9.60%)   |
| GlyGly          | 646.396794    | 363.78 (43.72%)   | 481.22 (25.55%) | 555.35 (14.09%)  | 363.70 (43.73%)            | 481.18 (25.56%) | 554.99 (14.14%)  | 363.70 (43.73%)   | 481.18 (25.56%) | 554.99 (14.14%)  |
| AlaAla          | 869.305727    | 436.75 (49.76%)   | 591.60 (31.95%) | 715.80 (17.66%)  | 435.55 (49.90%)            | 591.05 (32.01%) | 715.75 (17.66%)  | 435.55 (49.90%)   | 591.05 (32.01%) | 715.75 (17.66%)  |
| ThrThr          | 1413.300372   | 609.06 (56.91%)   | 836.42 (40.82%) | 1028.11 (27.25%) | 605.89 (57.13%)            | 836.22 (40.83%) | 1029.45 (27.16%) | 605.89 (57.13%)   | 836.22 (40.83%) | 1029.45 (27.16%) |
| GlyAla          | 747.778136    | 400.26 (46.47%)   | 536.57 (28.24%) | 633.28 (15.31%)  | 400.30 (46.47%)            | 536.55 (28.25%) | 633.27 (15.31%)  | 400.30 (46.47%)   | 536.55 (28.25%) | 633.27 (15.31%)  |
| GlyThr          | 1016.955671   | 486.42 (52.17%)   | 658.59 (35.24%) | 791.25 (22.19%)  | 484.44 (52.36%)            | 658.77 (35.22%) | 793.13 (22.01%)  | 484.44 (52.36%)   | 658.77 (35.22%) | 793.13 (22.01%)  |
| AlaThr          | 1140.623478   | 522.90 (54.16%)   | 713.94 (37.41%) | 871.94 (23.56%)  | 521.24 (54.30%)            | 713.98 (37.40%) | 871.56 (23.59%)  | 521.24 (54.30%)   | 713.98 (37.40%) | 871.56 (23.59%)  |
| GlyGlyGly1      | 1049.620738   | 525.15 (49.97%)   | 702.05 (33.11%) | 822.20 (21.67%)  | 524.98 (49.98%)            | 702.77 (33.05%) | 823.36 (21.56%)  | 524.98 (49.98%)   | 702.77 (33.05%) | 823.36 (21.56%)  |
| GlyGlyGly2      | 1116.168212   | 525.15 (52.95%)   | 702.05 (37.10%) | 822.20 (26.34%)  | 521.08 (53.32%)            | 701.23 (37.18%) | 821.00 (26.44%)  | 521.08 (53.32%)   | 701.23 (37.18%) | 821.00 (26.44%)  |
| MAPE =          |               | 46.95%  | 29.12%          | 16.48%           | 47.08%                     | 29.14%          | 16.48%           | 47.08%  | 29.14%          | 16.48%           |



**Figure 7.20:** The  $J_{HF,rad}$  using the molecular orbital expansion (Equation 6.38) along the bond between the target carbon atom and the oxygen atom for  $F_{C11}^1$  (red dash line),  $F_{C11}^2$  (green dash line),  $F_{C11}^3$  (purple dash line), and the molecule of interest, D molecule, (blue solid line). The carbon is located at 0.0 bohr and oxygen is located at 2.2739 bohr.



**Figure 7.21:** Effect of size of the fragment on the  $J_{HF}^A$  of target carbon atom.



**Table 7.9:** Comparing  $J_{HF}$  calculated using the sum over  $J_{HF}^A$  that are stored in the database ( $J_{HF, database}^{sum} = \sum_{A=1} J_{HF, database}^A$ ) and by using the direct method ( $J_{HF, direct}^{sum} = \sum_{A=1} J_{HF, direct}^A$ ) with those calculated using Equation 6.54 for the target molecule.  $\mathbf{F}^1$ ,  $\mathbf{F}^2$ , and  $\mathbf{F}^3$  are the size of fragments as first, second, and third neighbor atoms, respectively.

| Target molecule |                               | Fragments (Sum over atoms)   |                 |                  |  |                 |                  |
|-----------------|-------------------------------|--|-----------------|------------------|--|-----------------|------------------|
| Molecule        | $\mathbf{J}_{HF}$ (Numerical) | $\mathbf{F}^1$   | $\mathbf{F}^2$  | $\mathbf{F}^3$   | $\mathbf{F}^1$   | $\mathbf{F}^2$  | $\mathbf{F}^3$   |
|                 |                               | $\mathbf{J}_{HF, database}^{sum} = \sum_A \mathbf{J}_{HF, database}^A$ |                 |                  | $\mathbf{J}_{HF, direct}^{sum} = \sum_A \mathbf{J}_{HF, direct}^A$ |                 |                  |
|                 |                               | (%error)   |                 |                  | (%error)   |                 |                  |
| Gly             | 317.4368                      | 231.44 (27.09%)  | 287.31 (9.49%)  | 314.15 (1.04%)   | 231.43 (27.09%)  | 287.25 (9.51%)  | 313.74 (1.17%)   |
| Ala             | 405.9635                      | 272.00 (33.00%)  | 346.95 (14.54%) | 396.72 (2.28%)   | 271.98 (33.00%)  | 346.56 (14.63%) | 396.60 (2.31%)   |
| Thr             | 620.8816                      | 369.61 (40.47%)  | 480.43 (22.62%) | 565.98 (8.84%)   | 367.66 (40.78%)  | 480.24 (22.65%) | 565.79 (8.87%)   |
| GlyGly          | 692.5425                      | 414.58 (40.14%)  | 529.27 (23.58%) | 602.32 (13.03%)  | 414.50 (40.15%)  | 529.23 (23.58%) | 601.96 (13.08%)  |
| AlaAla          | 923.9067                      | 495.71 (46.35%)  | 648.59 (29.80%) | 771.71 (16.47%)  | 494.62 (46.46%)  | 648.08 (29.85%) | 771.69 (16.48%)  |
| ThrThr          | 1487.2513                     | 690.93 (53.54%)  | 914.98 (38.48%) | 1104.45 (25.74%) | 688.11 (53.73%)  | 914.81 (38.49%) | 1105.88 (25.64%) |
| GlyAla          | 798.3170                      | 455.15 (42.99%)  | 589.08 (26.21%) | 684.87 (14.21%)  | 454.96 (43.01%)  | 589.08 (26.21%) | 684.90 (14.21%)  |
| GlyThr          | 1076.9370                     | 552.75 (48.67%)  | 721.91 (32.97%) | 852.74 (20.82%)  | 551.11 (48.83%)  | 722.19 (32.94%) | 855.05 (20.60%)  |
| AlaThr          | 1204.5590                     | 593.32 (50.74%)  | 781.73 (35.10%) | 938.05 (22.12%)  | 591.87 (50.86%)  | 781.83 (35.09%) | 937.84 (22.14%)  |
| GlyGlyGly1      | 1115.3876                     | 597.73 (46.41%)  | 770.63 (30.91%) | 889.14 (20.28%)  | 597.57 (46.42%)  | 771.41 (30.84%) | 890.65 (20.15%)  |
| GlyGlyGly2      | 1180.0750                     | 597.73 (49.35%)  | 770.63 (34.70%) | 889.14 (24.65%)  | 593.66 (49.69%)  | 769.85 (34.76%) | 887.63 (24.78%)  |
| MAPE =          |                               | 43.52%   | 27.13%          | 15.41%           | 43.64%   | 27.14%          | 15.40%           |

Table 7.9 compares the  $J_{HF}$  computed using the sum over  $J_{HF}^A$  stored in the database ( $J_{HF,database}^{sum} = \sum_{A=1} J_{HF,database}^A$ ) and by using the direct method ( $J_{HF,direct}^{sum} = \sum_{A=1} J_{HF,direct}^A$ ) of  $\mathbf{F}^1$ ,  $\mathbf{F}^2$ , and  $\mathbf{F}^3$  fragments size with those calculated for the target molecule (Equation 6.54). Unfortunately, the MAPE for these molecules is very large. The MAPE of  $J_{HF,database}^{sum}$  are about 43.52%, 27.13%, and 15.41% for  $\mathbf{F}^1$ ,  $\mathbf{F}^2$ , and  $\mathbf{F}^3$ , respectively, and the MAPE of  $J_{HF,direct}^{sum}$  are about 43.64%, 27.14%, and 15.40% for  $\mathbf{F}^1$ ,  $\mathbf{F}^2$ , and  $\mathbf{F}^3$ , respectively. Thus we cannot consider the  $J_{HF}$  as atomic property as its value depends on the geometry and other atoms of the molecule.

In order to compute the molecular Coulomb energy correctly for the molecule of interest, one can calculate the molecular Coulomb energy from its atomic electron densities ( $J[\rho]$ ) by using Equation 6.69. The atomic electron densities can be built using two different ways: from the database or by using the direct way. In the first way the Cartesian coordinates for the radial grids of the target atom are transformed from the database into the coordinates of the molecule of interest as discussed in section 5.5. In the direct way, the electron densities for each atom are computed in a specific environment (fragment) by keeping the same Cartesian coordinates of the atoms and their radial grids in the molecule of interest. Computing the molecular potential energy for the molecule from its atomic electron densities involves performing full numerical integration calculations (i.e., the two integrals in Equation 6.58). Thus, these calculations need computer time and may produce large errors.

To investigate the validity of our results, we compare the calculated Coulomb energies using the atomic electron densities stored in the database ( $J[\rho_{database}]$ ) and using the direct method ( $J[\rho_{direct}]$ ) for the  $\mathbf{F}^1$ ,  $\mathbf{F}^2$ , and  $\mathbf{F}^3$  fragments with those computed numerically (i.e., the two integrals in Equation 6.58) for the target molecule ( $J[\rho_{target}]$ ). The results are tab-

**Table 7.10:** Comparing the molecular Coulomb energy densities using the atomic electron densities that are stored in the database ( $J[\rho_{database}]$ ) and by using the direct method ( $J[\rho_{direct}]$ ) with those calculated numerically (Equation 6.58) for the target molecule.  $\mathbf{F}^1$ ,  $\mathbf{F}^2$ , and  $\mathbf{F}^3$  are the size of fragments as first, second, and third neighbor atoms, respectively.

| Target molecule |              |                           | Fragments (Using the electron densities and the fragments partitioning weights) |                    |                    |  |                    |                    |  |  |
|-----------------|--------------|---------------------------|---|--------------------|--------------------|--|--------------------|--------------------|--|--|
| Molecule        | J[ρ]         |                           | J[ρ <sub>database</sub> ] (%error <sub>numerical/numerical</sub> )              |                    |                    | J[ρ <sub>direct</sub> ] (%error <sub>numerical/numerical</sub> ) |                    |                    |  |  |
|                 | (Analytical) | (Numerical/<br>Numerical) | F <sup>1</sup>  | F <sup>2</sup>     | F <sup>3</sup>     | F <sup>1</sup>   | F <sup>2</sup>     | F <sup>3</sup>     |  |  |
| Gly             | 317.437      | 317.773                   | 317.880 (-0.034%)   | 318.472 (-0.220%)  | 317.864 (-0.029%)  | 317.882 (-0.034%)  | 318.443 (-0.211%)  | 317.912 (-0.044%)  |  |  |
| Ala             | 405.964      | 406.382                   | 406.010 (0.092%)  | 407.122 (-0.182%)  | 406.976 (-0.146%)  | 406.001 (0.094%)   | 407.086 (-0.173%)  | 406.835 (-0.111%)  |  |  |
| Thr             | 620.885      | 621.436                   | 620.181 (0.202%)  | 622.198 (-0.123%)  | 622.038 (-0.097%)  | 620.503 (0.150%)   | 622.408 (-0.157%)  | 621.774 (-0.054%)  |  |  |
| GlyGly          | 692.547      | 693.175                   | 693.155 (0.003%)  | 694.961 (-0.258%)  | 694.074 (-0.130%)  | 693.481 (-0.044%)  | 695.316 (-0.309%)  | 694.112 (-0.135%)  |  |  |
| AlaAla          | 923.918      | 924.712                   | 923.436 (0.138%)  | 927.020 (-0.250%)  | 927.156 (-0.264%)  | 923.789 (0.100%)   | 927.030 (-0.251%)  | 926.679 (-0.213%)  |  |  |
| ThrThr          | 1487.282     | 1488.316                  | 1484.543 (0.254%)   | 1491.428 (-0.209%) | 1491.340 (-0.203%) | 1485.992 (0.156%)  | 1491.237 (-0.196%) | 1490.073 (-0.118%) |  |  |
| GlyAla          | 798.326      | 799.034                   | 798.437 (0.075%)  | 801.100 (-0.258%)  | 800.850 (-0.227%)  | 798.738 (0.037%)   | 801.213 (-0.273%)  | 800.549 (-0.190%)  |  |  |
| GlyThr          | 1076.955     | 1077.785                  | 1076.010 (0.165%)   | 1080.876 (-0.287%) | 1080.126 (-0.217%) | 1076.943 (0.078%)  | 1080.255 (-0.229%) | 1079.446 (-0.154%) |  |  |
| AlaThr          | 1204.576     | 1205.498                  | 1202.960 (0.211%)   | 1208.497 (-0.249%) | 1208.840 (-0.277%) | 1203.940 (0.129%)  | 1208.138 (-0.219%) | 1207.698 (-0.183%) |  |  |
| GlyGlyGly1      | 1115.404     | 1116.319                  | 1116.234 (0.008%)   | 1120.124 (-0.341%) | 1118.910 (-0.232%) | 1116.892 (-0.051%)   | 1120.079 (-0.337%) | 1118.669 (-0.211%) |  |  |
| GlyGlyGly2      | 1180.087     | 1180.997                  | 1180.492 (0.043%)   | 1184.355 (-0.284%) | 1183.128 (-0.180%) | 1181.402 (-0.034%)   | 1184.571 (-0.303%) | 1182.794 (-0.152%) |  |  |
| MAPE =          |              |                           | 0.111%  | 0.242%             | 0.182%             | 0.083%   | 0.242%             | 0.142%             |  |  |

ulated in Table 7.10, the MAPE of  $J[\rho_{database}]$  are about 0.111%, 0.242%, and 0.182% for  $\mathbf{F}^1$ ,  $\mathbf{F}^2$ , and  $\mathbf{F}^3$ , respectively, and the MAPE of  $J[\rho_{direct}]$  are about 0.083%, 0.242%, and 0.142% for  $\mathbf{F}^1$ ,  $\mathbf{F}^2$ , and  $\mathbf{F}^3$ , respectively. The results are encouraging, the %errors are small and computing of  $J[\rho_{database}]$  and  $J[\rho_{direct}]$  have almost the same %errors.

## 7.7 Total Energy

In this section, we obtain the total molecular energy as follows,

$$E = T^{sum} + V_{ne}[\rho] + J[\rho] - K_{HF}^{sum} + V_{nn} \quad (7.10)$$

where,  $T^{sum}$  is the molecular kinetic energy density calculated by summing over the atomic kinetic energy densities (Equation 7.3),  $K_{HF}^{sum}$  is the molecular exchange energy contain  $J_{aa}$  calculated by summing over the  $K_{HF}^A$  (Equation 7.5),  $V_{ne}[\rho]$  is the molecular potential energy calculated numerically using its atomic electron densities (Equation 6.73),  $J[\rho]$  is the molecular Coulomb energy calculated numerically using its atomic electron densities (Equation 6.69), and  $V_{nn}$  is the molecular nuclear-nuclear repulsion energy calculated directly for the entire molecule as,

$$V_{nn} = \sum_A \sum_{B>A} \frac{Z_A Z_B}{R_{AB}} \quad (7.11)$$

where  $Z_A$  and  $Z_B$  are the atomic numbers of atoms A and B respectively and  $R_{AB}$  is the distance between atoms A and B.

Table 7.11 compares the total energy of three mono-amino acids (glycine Gly, alanine Ala, and threonine Thr), six di-amino acids (GlyGly, AlaAla, ThrThr, GlyAla, GlyThr,

and AlaThr), and two isomers of tri-amino acid (GlyGlyGly1 and GlyGlyGly2) calculated using atomic properties and the electron densities that stored in the database  $E_{database}$  and that obtained using the direct methods  $E_{direct}$  of  $\mathbf{F}^1$ ,  $\mathbf{F}^2$ , and  $\mathbf{F}^3$  fragments size with those calculated numerically for the target molecule. The MAPE for these molecules as tabulated in Table 7.11 are relatively small. The MAPE of  $E_{database}$  are about 0.165%, 0.017%, and 0.008% for  $\mathbf{F}^1$ ,  $\mathbf{F}^2$ , and  $\mathbf{F}^3$ , respectively, and the MAPE of  $E_{direct}$  are about 0.164%, 0.022%, and 0.006% for  $\mathbf{F}^1$ ,  $\mathbf{F}^2$ , and  $\mathbf{F}^3$ , respectively. Further work is needed to improve our results.

**Table 7.11:** Comparing the calculated total energy ( $E$ ) using the atomic properties and the electron densities that are stored in the database  $E_{\text{database}}$  and that obtained using the direct methods  $E_{\text{direct}}$  of  $\mathbf{F}^1$ ,  $\mathbf{F}^2$ , and  $\mathbf{F}^3$  fragments size with those calculated numerically for the target molecule.

| Molecule   | Target molecule |                       | Fragments (Using fragments partitioning weights) |                     |                     |                        |                        |                        |
|------------|-----------------|-----------------------|--|---------------------|---------------------|------------------------|------------------------|------------------------|
|            | (Analytical)    | (Numerical)           | $\mathbf{F}^1$                                   | $\mathbf{F}^2$      | $\mathbf{F}^3$      | $\mathbf{F}^1$         | $\mathbf{F}^2$         | $\mathbf{F}^3$         |
|            | $E$             | $E_{\text{database}}$ | $E_{\text{direct}}$                              | $E_{\text{direct}}$ | $E_{\text{direct}}$ | $E_{\text{numerical}}$ | $E_{\text{numerical}}$ | $E_{\text{numerical}}$ |
| Gly        | -282.816        | -282.487              | -282.110 (0.134%)                                | -282.472 (0.005%)   | -282.484 (0.001%)   | -282.116 (0.132%)      | -282.476 (0.004%)      | -282.485 (0.001%)      |
| Ala        | -321.854        | -321.453              | -320.959 (0.154%)                                | -321.371 (0.025%)   | -321.438 (0.004%)   | -320.976 (0.148%)      | -321.386 (0.021%)      | -321.452 (0.000%)      |
| Thr        | -435.739        | -435.221              | -434.550 (0.154%)                                | -435.092 (0.030%)   | -435.161 (0.014%)   | -434.543 (0.156%)      | -435.119 (0.023%)      | -435.179 (0.010%)      |
| GlyGly     | -489.637        | -489.017              | -488.234 (0.160%)                                | -489.032 (-0.003%)  | -488.996 (0.004%)   | -488.222 (0.163%)      | -488.950 (0.014%)      | -488.996 (0.004%)      |
| AlaAla     | -567.708        | -566.955              | -565.934 (0.180%)                                | -566.733 (0.039%)   | -566.888 (0.012%)   | -565.957 (0.176%)      | -566.772 (0.032%)      | -566.925 (0.005%)      |
| ThrThr     | -795.480        | -794.457              | -793.127 (0.167%)                                | -794.365 (0.012%)   | -794.447 (0.001%)   | -793.064 (0.175%)      | -794.241 (0.027%)      | -794.356 (0.013%)      |
| GlyAla     | -528.671        | -528.005              | -527.087 (0.174%)                                | -527.836 (0.032%)   | -527.949 (0.011%)   | -527.109 (0.170%)      | -527.872 (0.025%)      | -527.986 (0.004%)      |
| GlyThr     | -642.556        | -641.747              | -640.679 (0.166%)                                | -641.723 (0.004%)   | -641.838 (-0.014%)  | -640.664 (0.169%)      | -641.590 (0.025%)      | -641.690 (0.009%)      |
| AlaThr     | -681.593        | -680.709              | -679.529 (0.173%)                                | -680.585 (0.018%)   | -680.717 (-0.001%)  | -679.520 (0.175%)      | -680.500 (0.031%)      | -680.647 (0.009%)      |
| GlyGlyGly1 | -696.450        | -695.580              | -694.367 (0.174%)                                | -695.443 (0.020%)   | -695.630 (-0.007%)  | -694.381 (0.172%)      | -695.439 (0.020%)      | -695.556 (0.003%)      |
| GlyGlyGly2 | -696.457        | -695.585              | -694.384 (0.173%)                                | -695.570 (0.002%)   | -695.736 (-0.022%)  | -694.377 (0.174%)      | -695.473 (0.016%)      | -695.550 (0.005%)      |
| MAPE =     |                 |                       | 0.165%   | 0.017%              | 0.008%              | 0.164%                 | 0.022%                 | 0.006%                 |

# Bibliography

- [1] R. A. Poirier, J. W. Hollett, and P. L. Warburton. MUNgauss, Memorial University, Chemistry Department, St. Johns, NL, A1B 3X7. With contributions from I. E. Awad, A. Alrawashdeh, J. P. Becker, J. Besaw, S. D. Bungay, F. Colonna, A.El-Sherbiny, T. Gosse, D. Keefe, A. Kelly, D. Nippard, C. C. Pye, D. Reid, K.Saputantri, M. Shaw, M. Staveley, O. Stueker, Y. Wang, and J. Xidos.
  
- [2] Wolfram Research, Inc. Mathematica 11.2. URL <https://www.wolfram.com>.

## Chapter 8

### Conclusions and Future Work

*“The truth is, everyone is confused by quantum physics.”*

— David Walton



## 8.1 Conclusions and Future Work

### 8.1.1 Conclusions

In the first part of this thesis, we have successfully developed a new partitioning weight (Awad weight). The Awad weight has flexibility of choosing the core boundary of atoms within the molecule, and has the ability to assign the core to the target atom not to the core or the bond of other atoms. The computer time using the Awad weight is very close to the Becke weight. The effect of choosing the core size on Awad weight is studied visually and numerically. The visualization study includes molecular radial electron density (RDEN) and bond electron density (BDEN). The results show that the topology of RDEN and BDEN are affected by changing the core size. These changes are more pronounced in molecules containing metal atoms such as Li and Na than molecules contain only non-metallic atoms.

The numerical study includes comparing the following molecular properties; number of electrons, potential energy, and Coulomb energy with those calculated by the HF wavefunction (the exact values). The mean absolute error (MAE) for calculated total number of electrons using the Awad weight with  $r_{\langle r \rangle}$  and  $r_{max}$  are 0.01524 and 0.003665  $\mu e$  respectively, and it is 0.4916  $\mu e$  using Becke weight. The MAEs for calculating the potential energy numerically using the Awad weight with  $r_{\langle r \rangle}$  and  $r_{max}$  are 102.45 and 102.48  $\mu\text{hartree}$  respectively which are slightly better than MAE of 115.01  $\mu\text{hartree}$  obtained using Becke weight. The Coulomb potential energy results show that the Awad weight with  $r_{\langle r \rangle}$  and  $r_{max}$  gives better results than Becke weight where the MAEs equal 0.09160, 0.08454, and 6.295  $\mu\text{hartree}$  for  $r_{\langle r \rangle}$ ,  $r_{max}$ , and Becke weight, respectively.

In the second part of this thesis, a new approach for calculating the total energy of molecules called atoms in molecules density (AIMD) is proposed. The AIMD includes three general steps for computing the molecular properties of a target molecule: (i) generating small molecules (fragments) from molecules of interest (ii) storing some properties of the fragments in the database, (iii) using these properties to compute the molecular properties of the target molecule. The studied molecular properties include: kinetic energy, exchange energy, Coulomb energy, and potential energy. The atomic properties are obtained using two different ways: using the data stored in the database and by using the direct method. The MAPE of the molecular kinetic energy calculated using the sum over atomic kinetic energies stored in the database  $T_{database}^{sum}$  are about 0.121%, 0.040%, and 0.019% for **F<sup>1</sup>**, **F<sup>2</sup>**, and **F<sup>3</sup>**, respectively. Whereas, The MAPE of direct method  $T_{direct}^{sum}$  are about 0.107%, 0.036%, and 0.015% for **F<sup>1</sup>**, **F<sup>2</sup>**, and **F<sup>3</sup>**, respectively. For the exchange energy contains  $J_{aa}$  term the MAPE of  $K_{HF,database}^{sum}$  are about 0.053%, 0.058%, and 0.035% for **F<sup>1</sup>**, **F<sup>2</sup>**, **F<sup>3</sup>**, respectively, and the MAPE of  $K_{HF,direct}^{sum}$  are about 0.067%, 0.058%, and 0.032% for **F<sup>1</sup>**, **F<sup>2</sup>**, **F<sup>3</sup>**, respectively. The MAPE of the calculated potential energies using the atomic electron densities stored in the database  $V_{ne}[\rho_{database}]$  are about 0.046%, 0.087%, and 0.066% for **F<sup>1</sup>**, **F<sup>2</sup>**, **F<sup>3</sup>**, respectively, and the MAPE of  $V_{ne}[\rho_{direct}]$  are about 0.034%, 0.085%, and 0.051% for **F<sup>1</sup>**, **F<sup>2</sup>**, **F<sup>3</sup>**, respectively. The MAPE of the calculated Coulomb energies using the atomic electron densities stored in the database  $J[\rho_{database}]$  are about 0.111%, 0.242%, and 0.182% for **F<sup>1</sup>**, **F<sup>2</sup>**, **F<sup>3</sup>**, respectively, and the MAPE of  $J[\rho_{direct}]$  are about 0.083%, 0.242%, and 0.142% for **F<sup>1</sup>**, **F<sup>2</sup>**, **F<sup>3</sup>**, respectively. Also, the calculated total molecular energy using AIMD is compared with those calculated numerically for the target molecule. The MAPEs are relatively small. The MAPE of  $E_{database}$  are about 0.165%, 0.017%, and 0.008% for **F<sup>1</sup>**, **F<sup>2</sup>**, **F<sup>3</sup>**, respectively, and the MAPE of  $E_{direct}$  are about 0.164%, 0.022%,

and 0.006% for  $\mathbf{F}^1$ ,  $\mathbf{F}^2$ ,  $\mathbf{F}^3$ , respectively.

### 8.1.2 Future Work

Further work for the development of AIMD is required. Some future goals include:

- Optimizing the core sizes of Awad weight based on some molecular properties such as: bond order, atomic charges, dipole moments, etc.
- Storing the core sizes of Awad weight in the database according to the environment of the target atom in the molecule of interest (i.e., the neighbour atoms, basis set, level of theory, etc).
- Developing a new numerical method for computing the molecular properties in different regions (e.g., the number of electrons in the core region for a specific atom in a molecule).
- Improving the numerical integration techniques for computing the Coulomb and potential energies to increase the accuracy of the results and reduce the computer time.
- Developing a hybrid QM/AIMD method which can describe large molecules.
- Storing the Coulomb and potential energy densities of the target atom in the core region in the database that can be used as pseudopotential to reduce the computer time.
- Fitting the AIM densities.
- Considering cut-off distances for long range interactions to reduce the computer time.

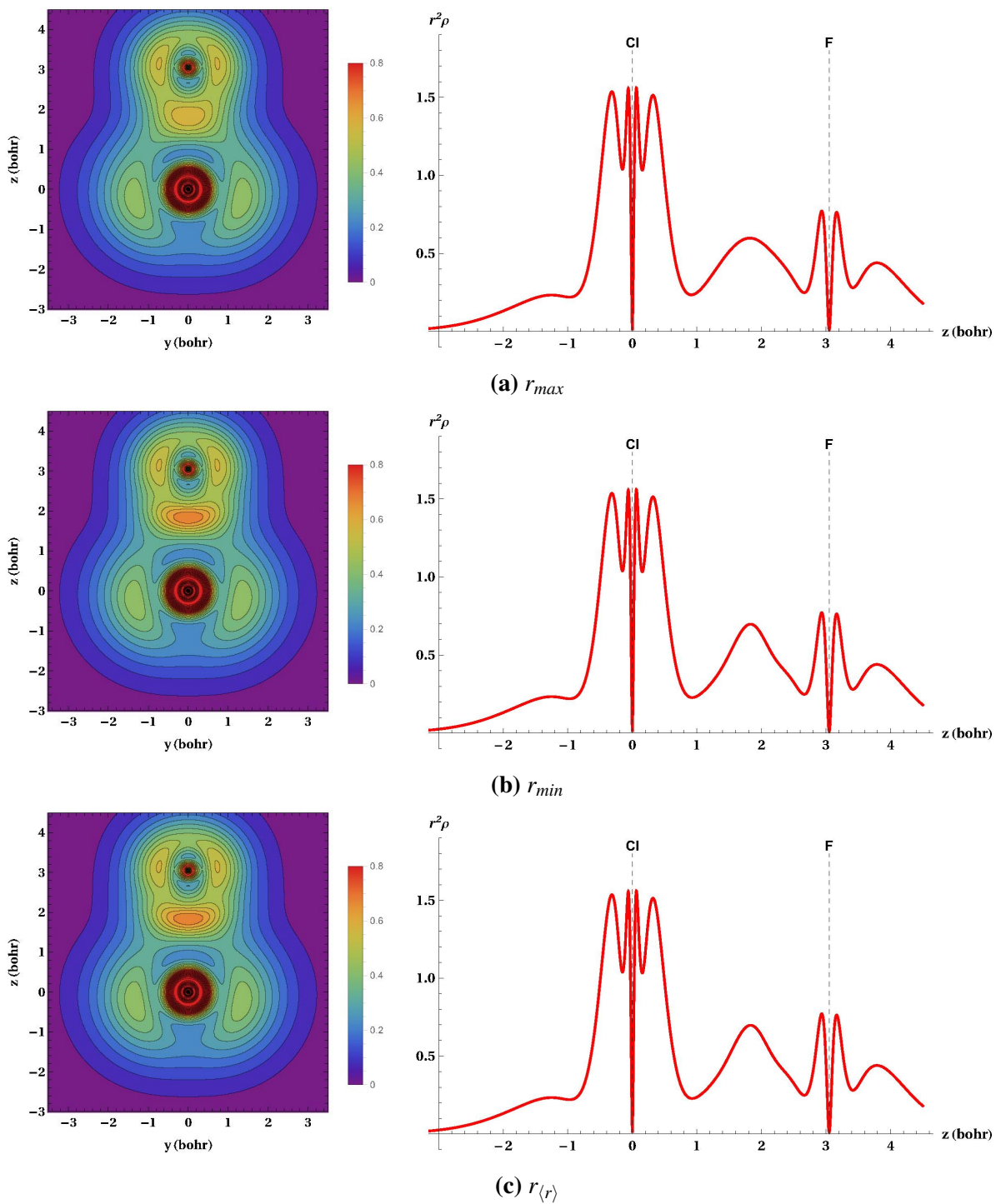
- Improving the descriptions for AIM.
- Improving the visualization tools for molecular properties of AIMD.
- Developing a density functional version of AIMD approach.
- Reducing the computer time by modifying the code and using parallel processing.

# Appendix A

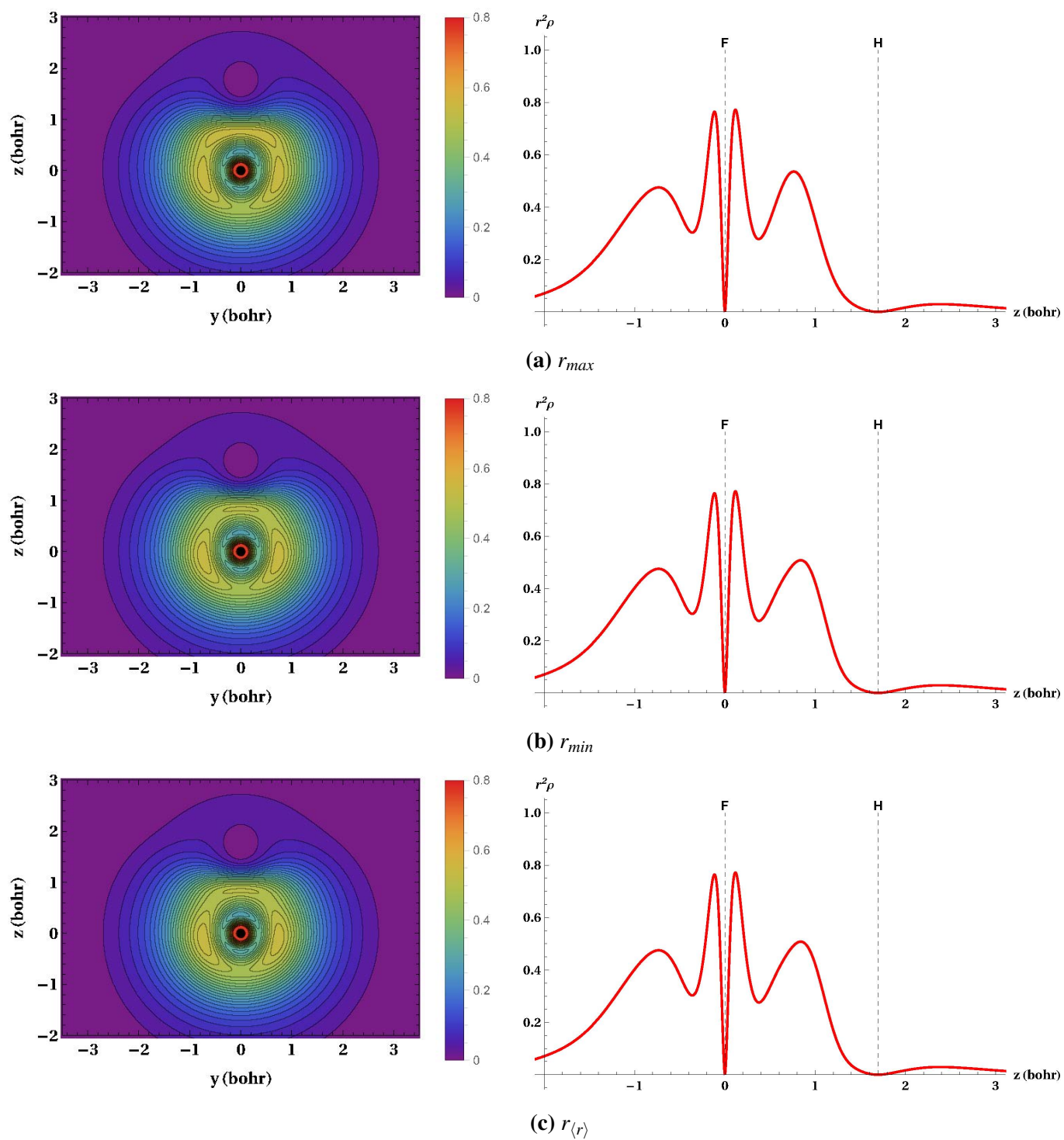
## Figures and Charts

### A.1 Radial Electron Densities

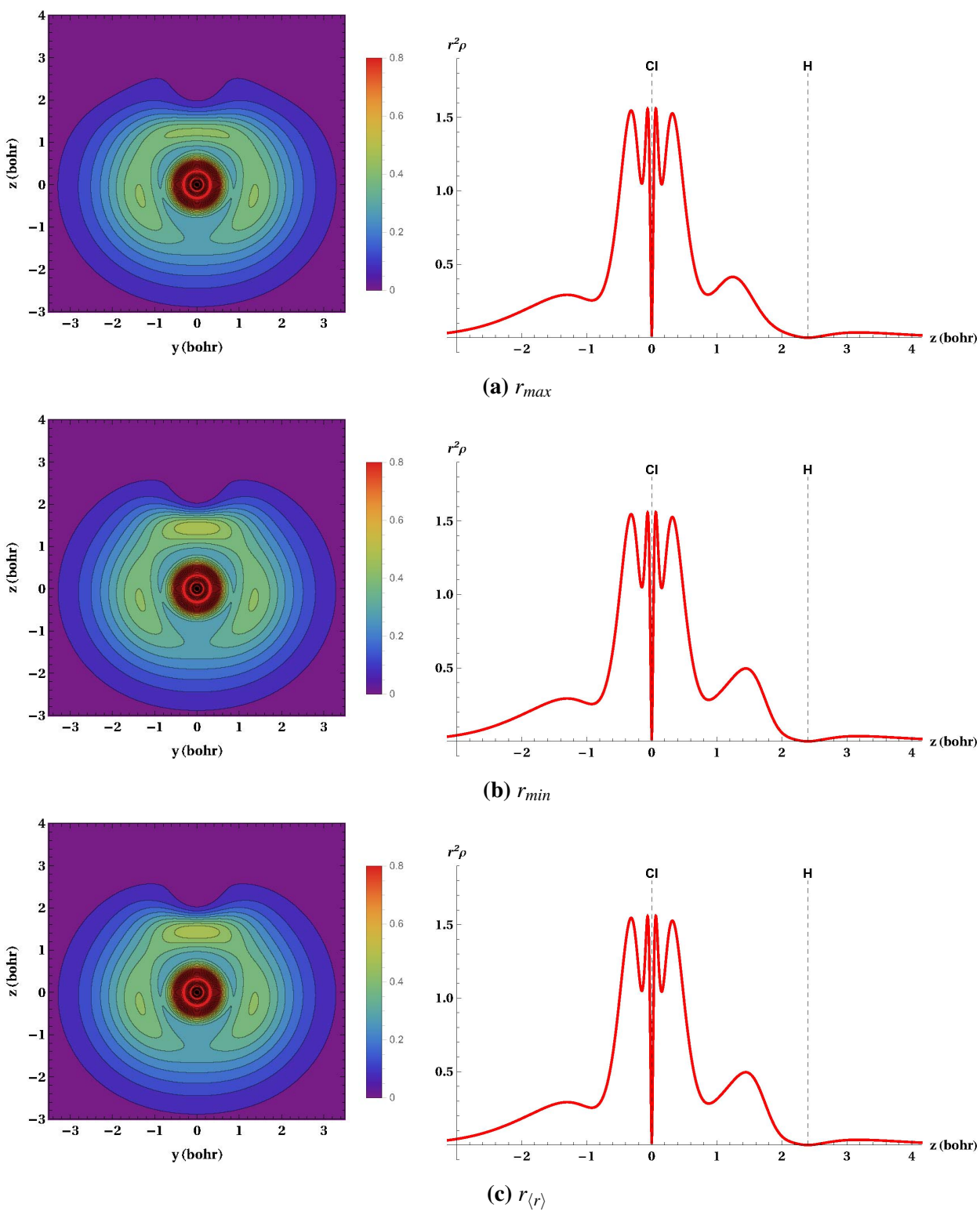
In this section, we give some examples of the molecular radial electron density (RDEN) using Awad weight at:  $r_{max}$ ,  $r_{min}$ , and  $r_{\langle r \rangle}$ .



**Figure A.1:** Molecular radial electron density (RDEN) for FCl is obtained using the Awad weight at: (a)  $r_{max}$ , (b)  $r_{min}$ , and (c)  $r_{\langle r \rangle}$ . The atomic coordinates of Cl at 0.0 bohr and F at 3.0504 bohr along  $z$  axis.

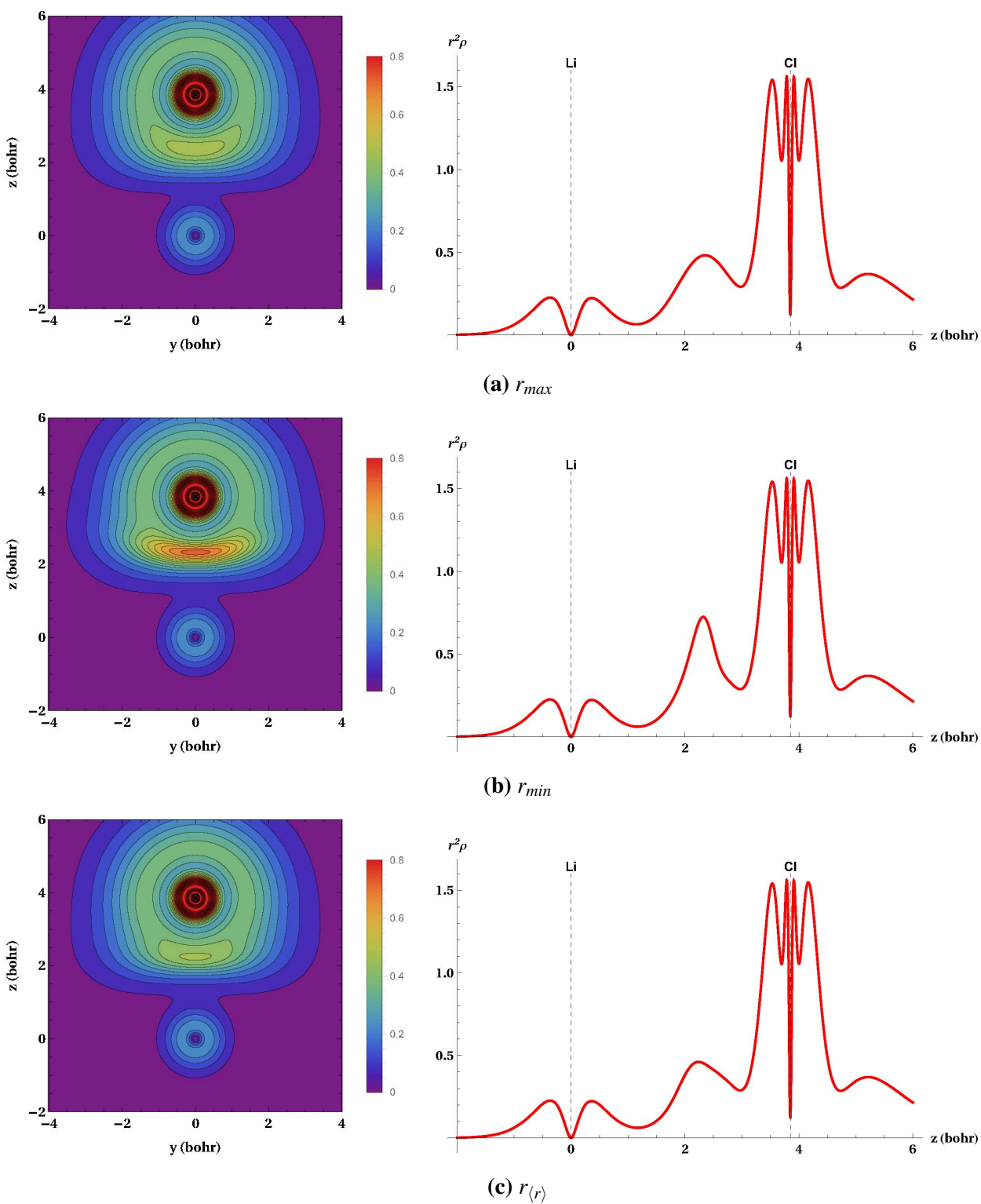


**Figure A.2:** Molecular radial electron density (RDEN) for HF is obtained using the Awad weight at: (a)  $r_{max}$ , (b)  $r_{min}$ , and (c)  $r_{\langle r \rangle}$ . The atomic coordinates of F at 0.0 bohr and H at 1.6960 bohr along  $z$  axis.

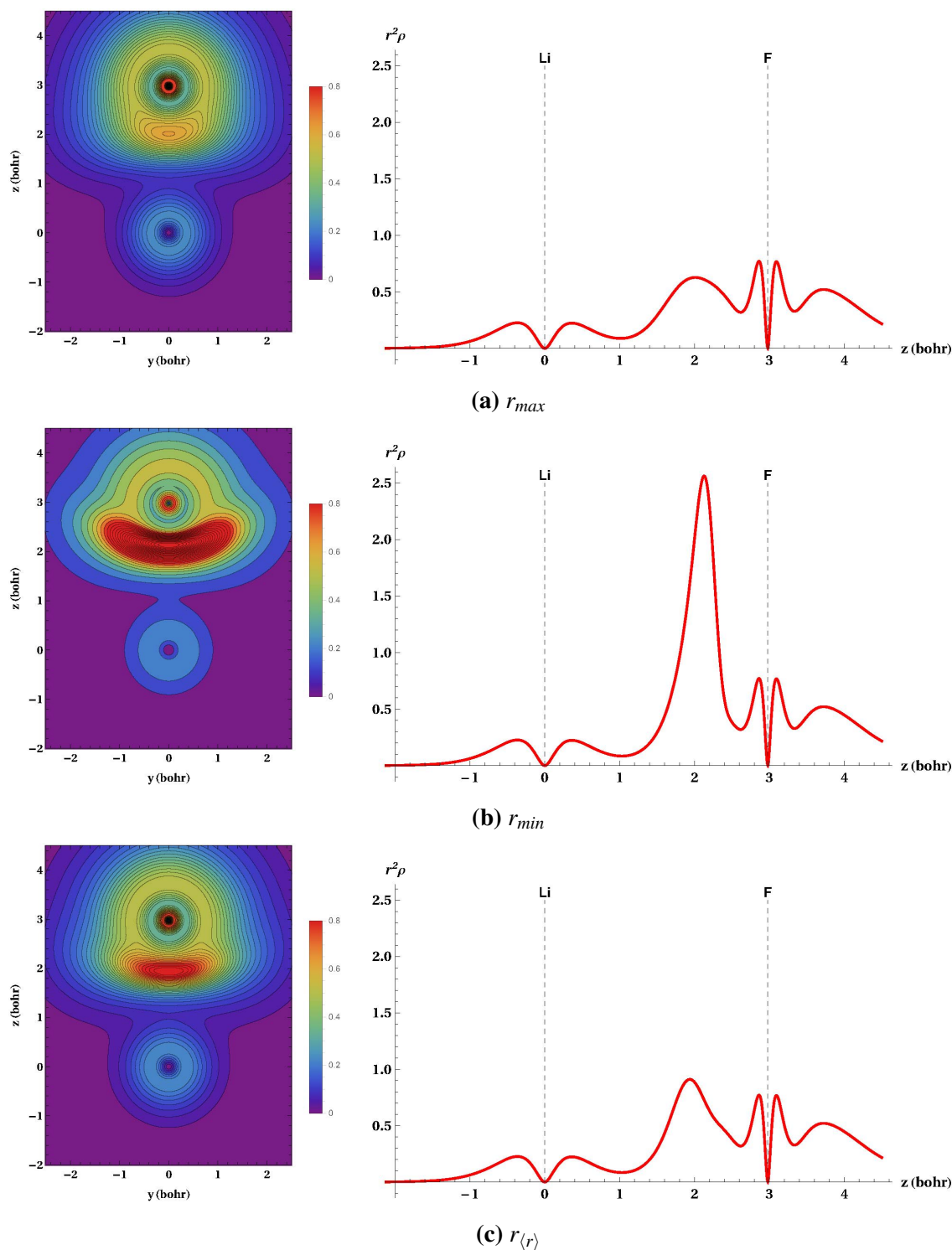


**Figure A.3:** Molecular radial electron density (RDEN) for HCl is obtained using the Awad weight at: (a)  $r_{max}$ , (b)  $r_{min}$ , and (c)  $r_{\langle r \rangle}$ . The atomic coordinates of Cl at 0.0 bohr and H at 2.4000 bohr along  $z$  axis.





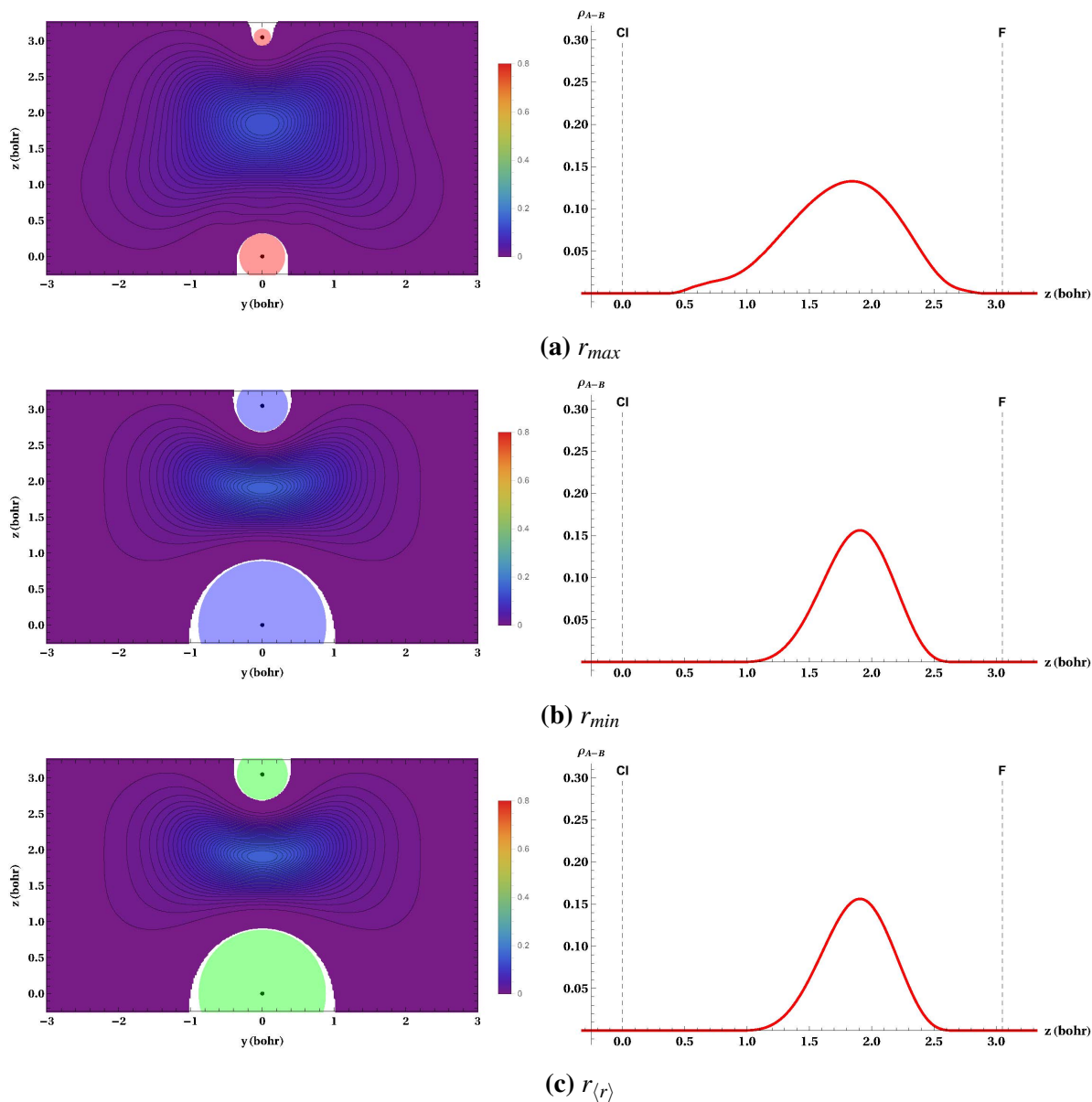
**Figure A.4:** Molecular radial electron density (RDEN) for LiCl is obtained using the Awad weight at: (a)  $r_{max}$ , (b)  $r_{min}$ , and (c)  $r_{\langle r \rangle}$ . The atomic coordinates of Li at 0.0 bohr and Cl at 3.8492 bohr along  $z$  axis.



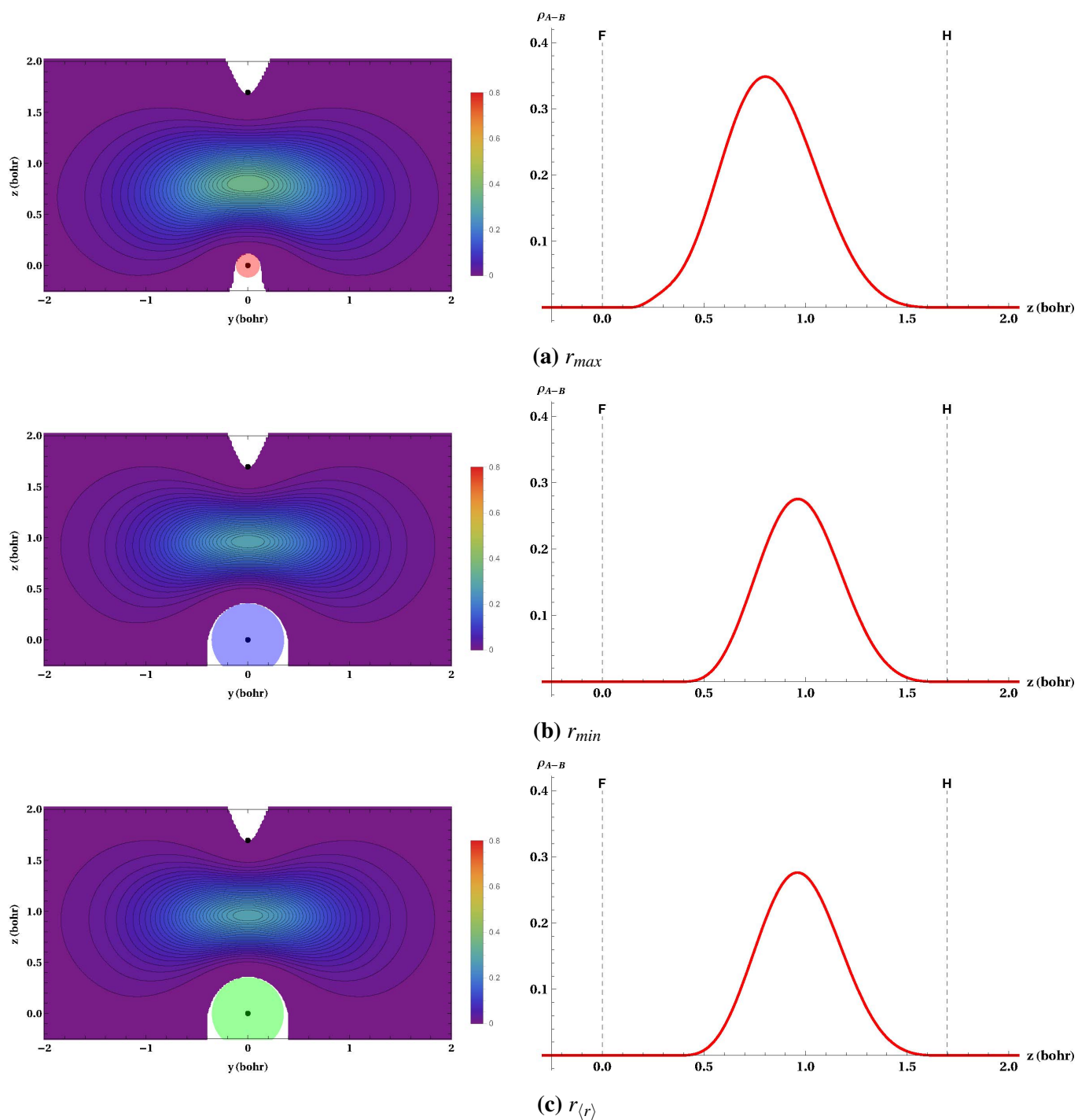
**Figure A.5:** Molecular radial electron density (RDEN) for LiF is obtained using the Awad weight at: (a)  $r_{max}$ , (b)  $r_{min}$ , and (c)  $r_{\langle r \rangle}$ . The atomic coordinates of Li at 0.0 bohr and F at 2.9777 bohr along  $z$  axis.

## A.2 Bond Electron Densities

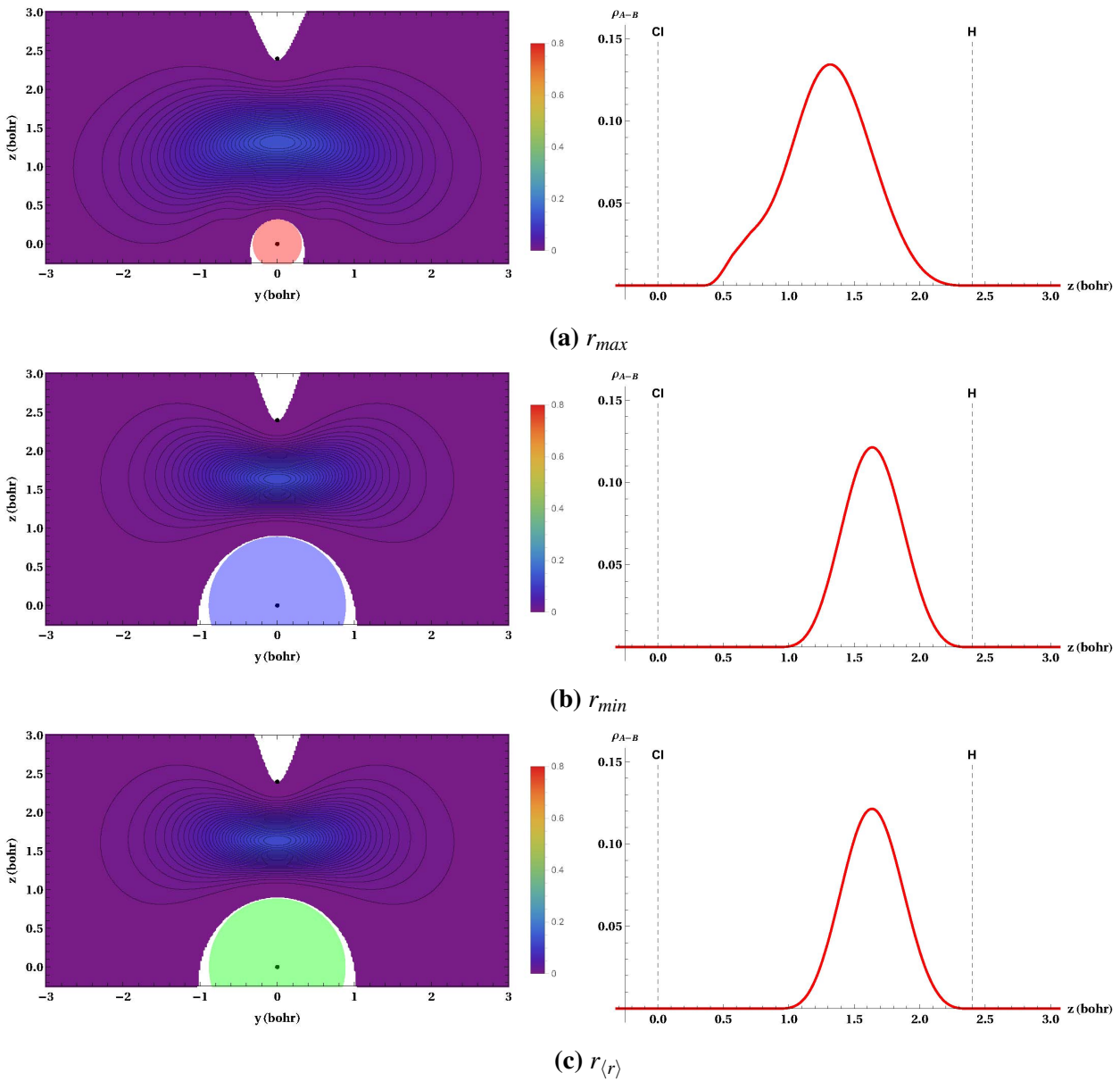
In this section, we give some examples of the bond electron density (BDEN) using Awad weight at:  $r_{max}$ ,  $r_{min}$ , and  $r_{\langle r \rangle}$ .



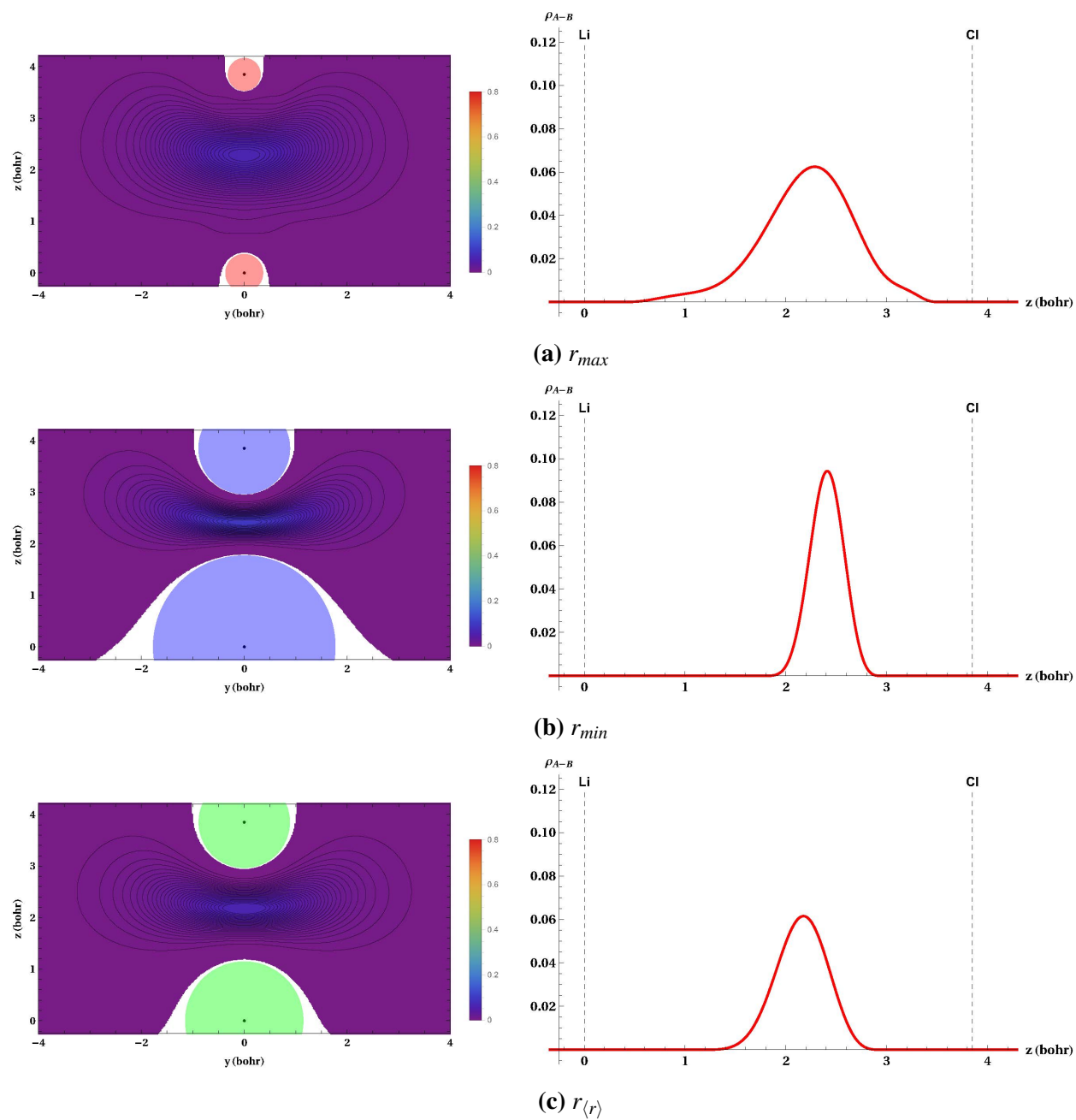
**Figure A.6:** Bond electron density (BDEN) for FCl is obtained using the Awad weight at: (a)  $r_{max}$ , (b)  $r_{min}$ , and (c)  $r_{\langle r \rangle}$ . The atomic coordinates of Cl at 0.0 bohr and F at 3.0504 bohr along  $z$  axis. The red, purple, and green disks are the core regions of  $r_{max}$ ,  $r_{min}$ , and  $r_{\langle r \rangle}$ , respectively.



**Figure A.7:** Bond electron density (BDEN) for HF is obtained using the Awad weight at: (a)  $r_{max}$ , (b)  $r_{min}$ , and (c)  $r_{\langle r \rangle}$ . The atomic coordinates of F at 0.0 bohr and H at 1.6960 bohr along  $z$  axis. The red, purple, and green disks are the core regions of  $r_{max}$ ,  $r_{min}$ , and  $r_{\langle r \rangle}$ , respectively.

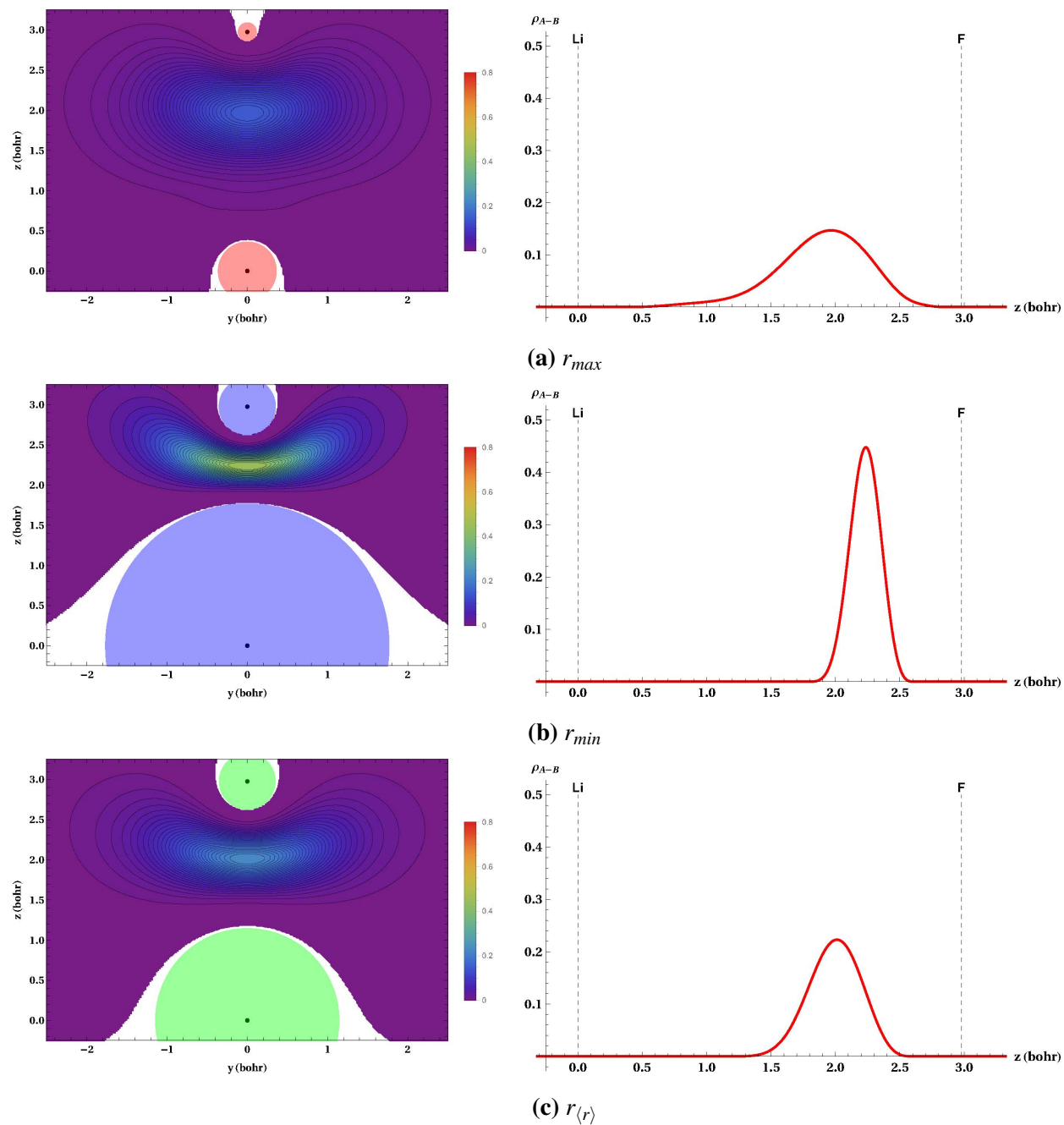


**Figure A.8:** Bond electron density (BDEN) for HCl is obtained using the Awad weight at: (a)  $r_{max}$ , (b)  $r_{min}$ , and (c)  $r_{(r)}$ . The atomic coordinates of Cl at 0.0 bohr and H at 2.4000 bohr along  $z$  axis. The red, purple, and green disks are the core regions of  $r_{max}$ ,  $r_{min}$ , and  $r_{(r)}$ , respectively.



**Figure A.9:** Bond electron density (BDEN) for LiCl is obtained using the Awad weight at: (a)  $r_{max}$ , (b)  $r_{min}$ , and (c)  $r_{(r)}$ . The atomic coordinates of Li at 0.0 bohr and Cl at 3.8492 bohr along  $z$  axis. The red, purple, and green disks are the core regions of  $r_{max}$ ,  $r_{min}$ , and  $r_{(r)}$ , respectively.





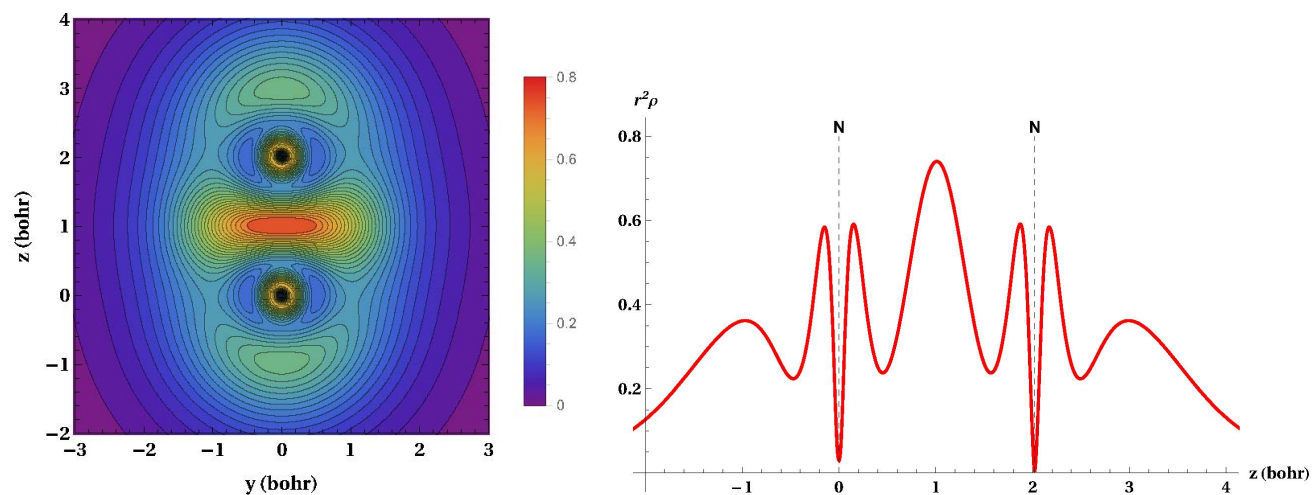
**Figure A.10:** Bond electron density (BDEN) for LiF is obtained using the Awad weight at: (a)  $r_{max}$ , (b)  $r_{min}$ , and (c)  $r_{\langle r \rangle}$ . The atomic coordinates of Li at 0.0 bohr and F at 2.9777 bohr along  $z$  axis. The red, purple, and green disks are the core regions of  $r_{max}$ ,  $r_{min}$ , and  $r_{\langle r \rangle}$ , respectively.

## **A.3 Comparing the Radial and Bond Electron Densities Between Awad and Becke Weights**

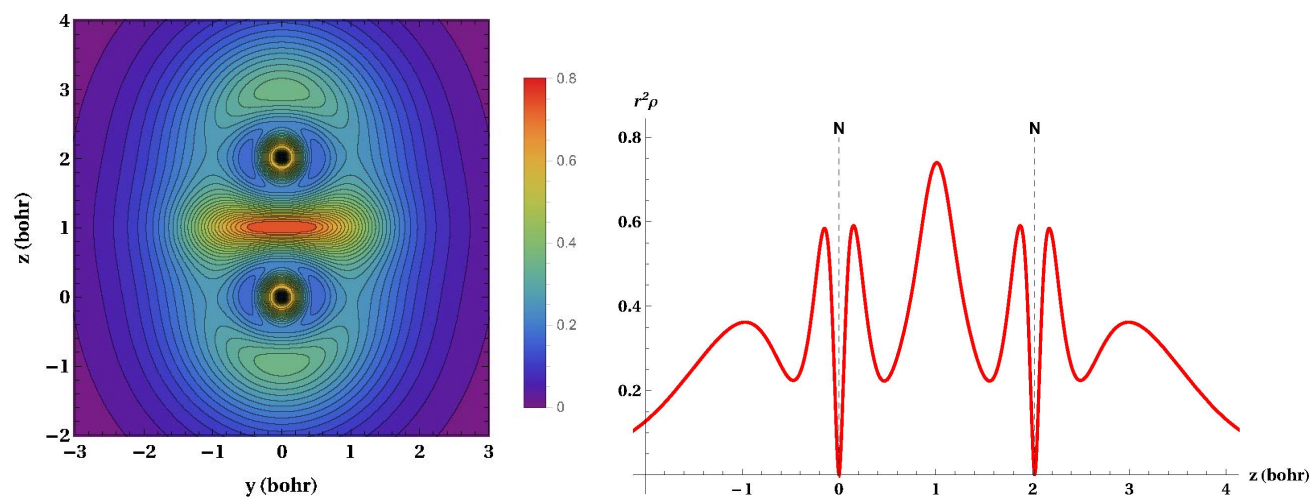
In this section, we compare the molecular radial electron density (RDEN) and bond electron density (BDEN) between using Awad and Becke weights.

### **A.3.1 Comparing the Radial Electron Density Between Awad and Becke Weights**



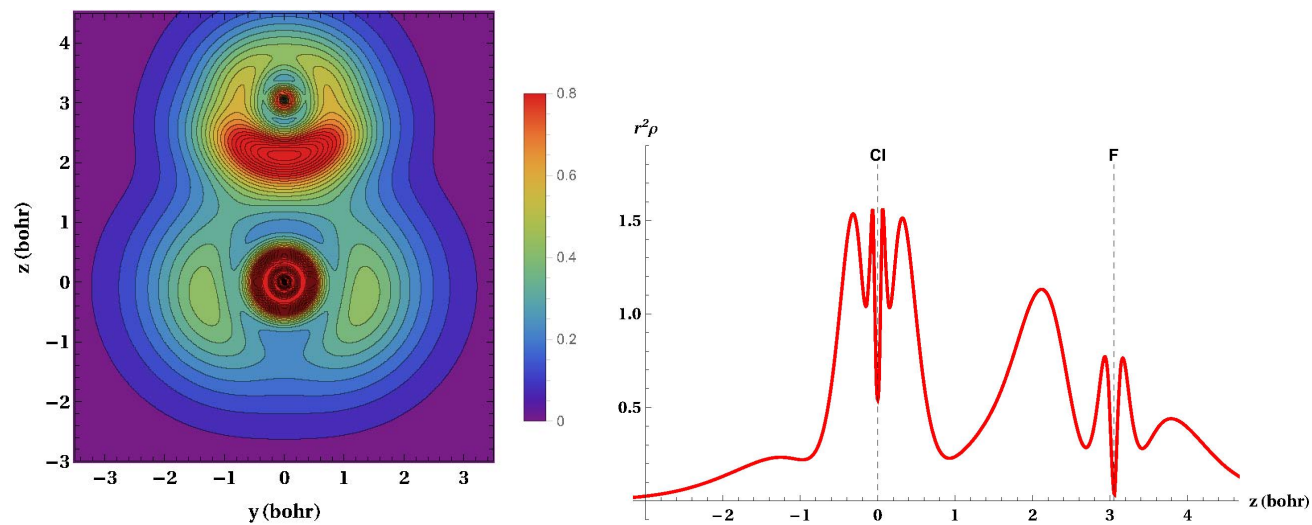


(a) The core size is defined as Becke weight.

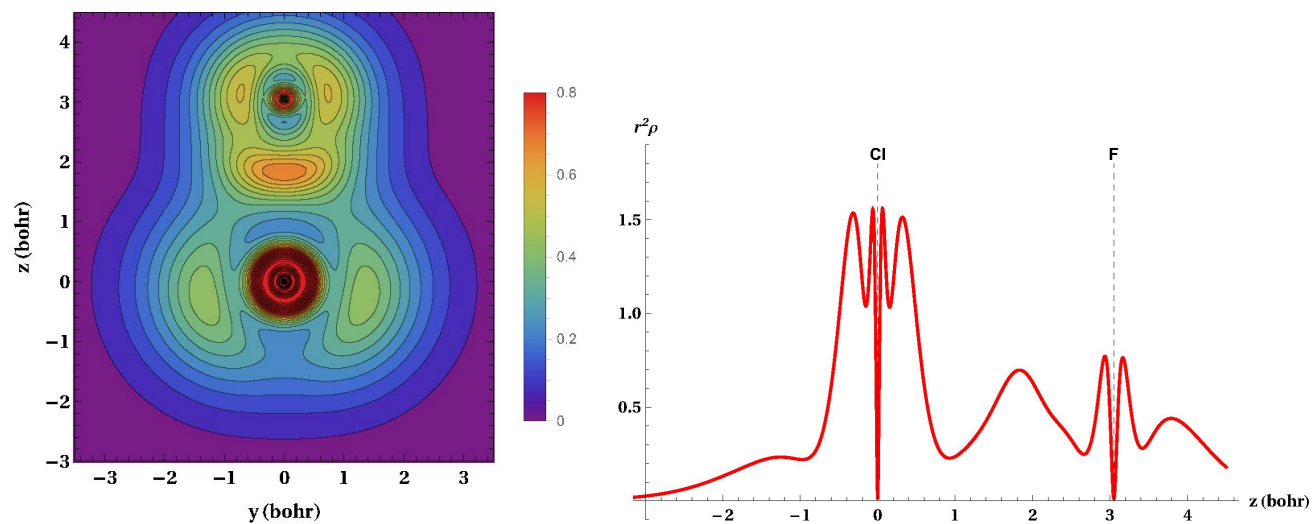


(b) Awad weight at  $r_{\langle r \rangle}$ .

**Figure A.11:** Molecular radial electron density (RDEN) for  $N_2$  is obtained using the Becke and Awad weights. The atomic coordinates of the two N atoms are 0.0 and 2.0229 bohr along  $z$  axis.

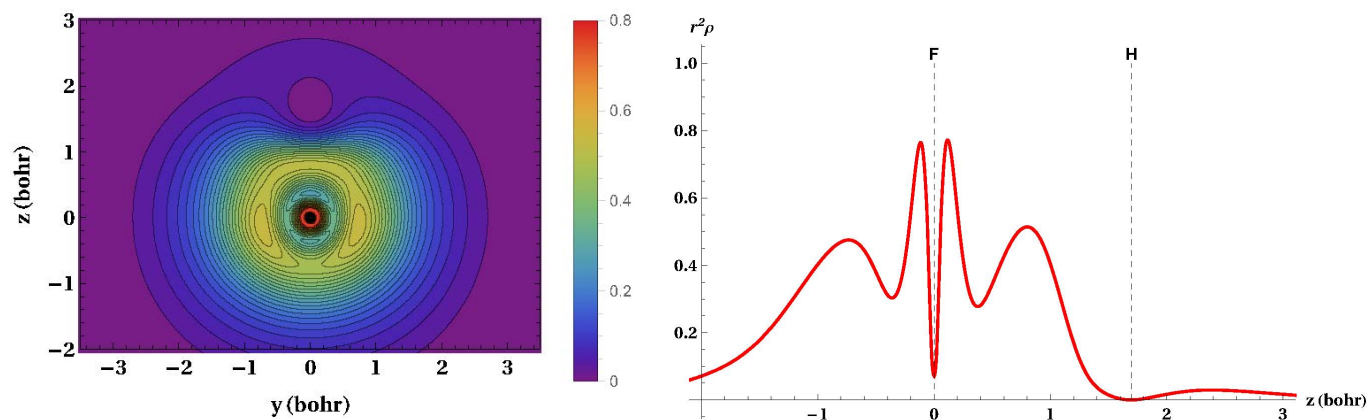


(a) The core size is defined as Becke weight.

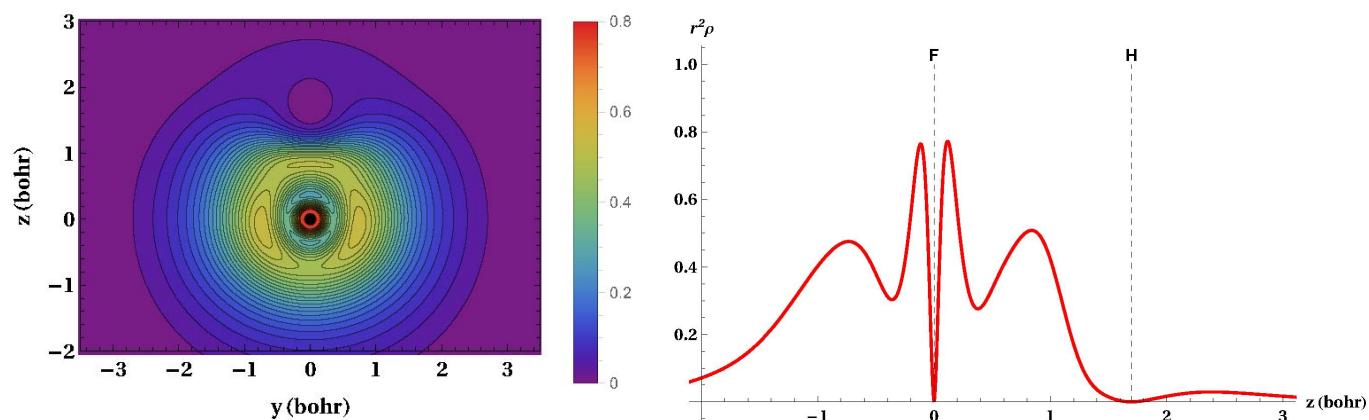


(b) Awad weight at  $r_{(r)}$ .

**Figure A.12:** Molecular radial electron density (RDEN) for FCl is obtained using the Becke and Awad weights. The atomic coordinates of Cl at 0.0 bohr and F at 3.0504 bohr along  $z$  axis.

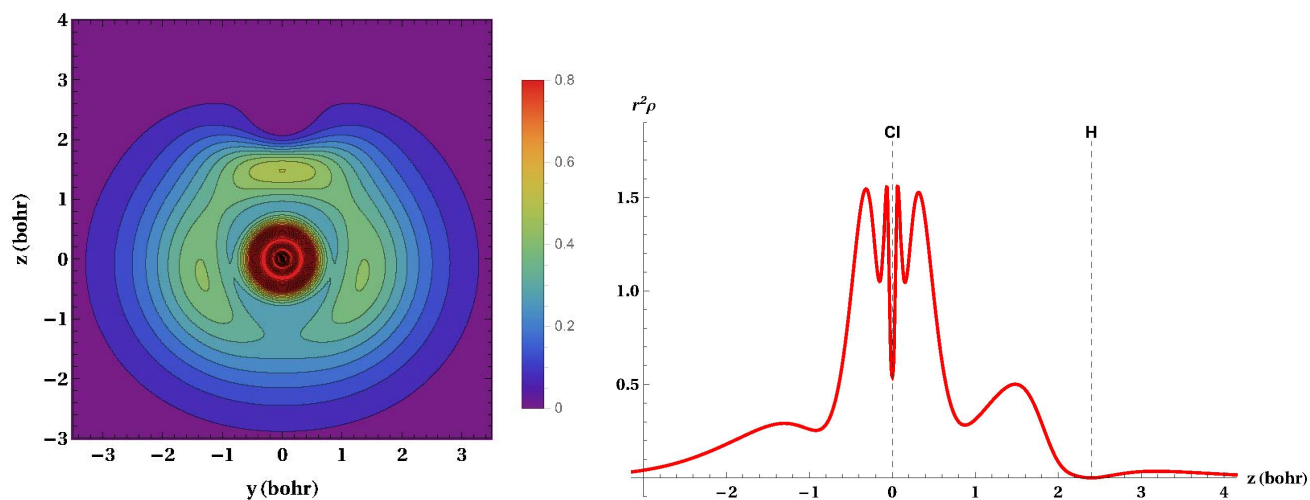


(a) The core size is defined as Becke weight.

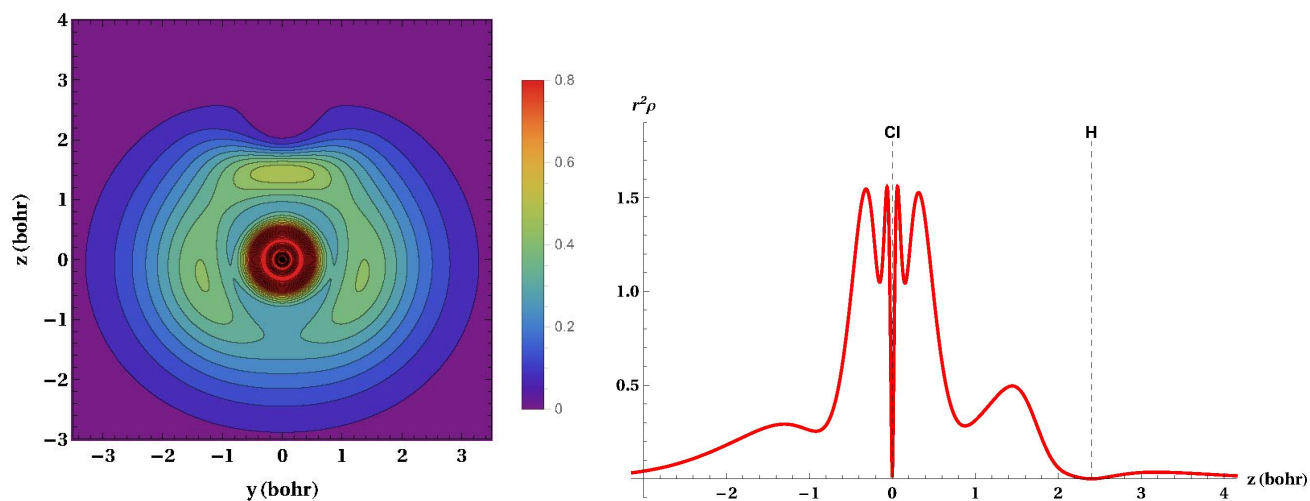


(b) Awad weight at  $r_{(r)}$ .

**Figure A.13:** Molecular radial electron density (RDEN) for HF is obtained using the Becke and Awad weights. The atomic coordinates of F at 0.0 bohr and H at 1.6960 bohr along  $z$  axis.

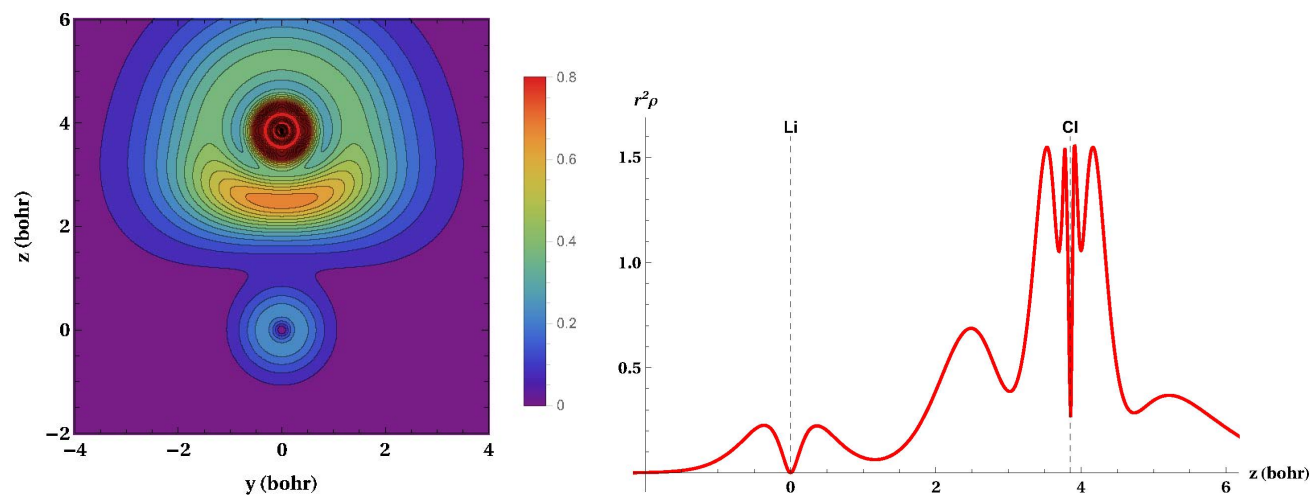


(a) The core size is defined as Becke weight.

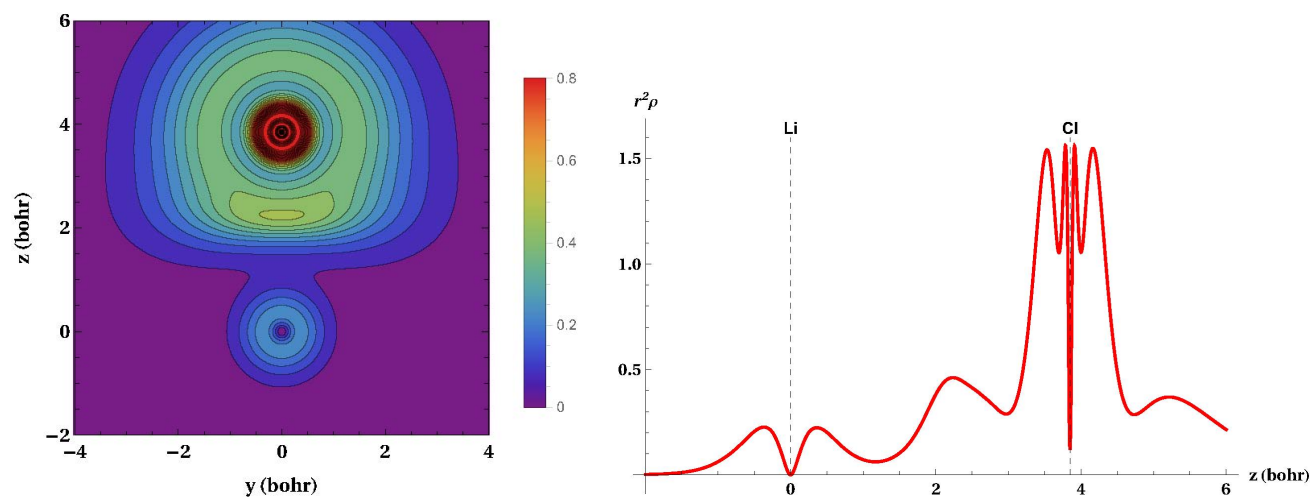


(b) Awad weight at  $r_{(r)}$ .

**Figure A.14:** Molecular radial electron density (RDEN) for HCl is obtained using the Becke and Awad weights. The atomic coordinates of Cl at 0.0 bohr and H at 2.4000 bohr along  $z$  axis.



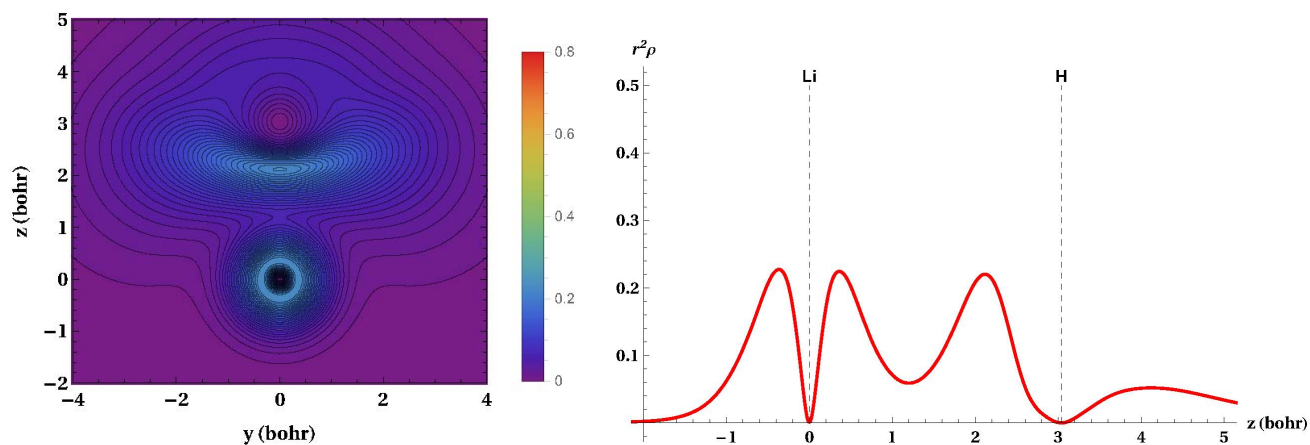
(a) The core size is defined as Becke weight.



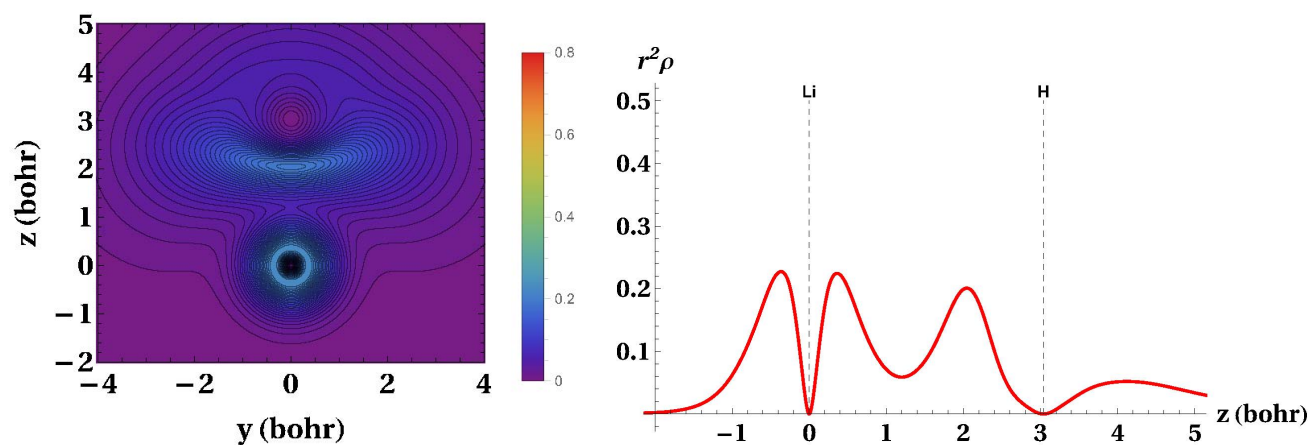
(b) Awad weight at  $r_{(r)}$ .

**Figure A.15:** Molecular radial electron density (RDEN) for LiCl is obtained using the Becke and Awad weights. The atomic coordinates of Li at 0.0 bohr and Cl at 3.8492 bohr along  $z$  axis.



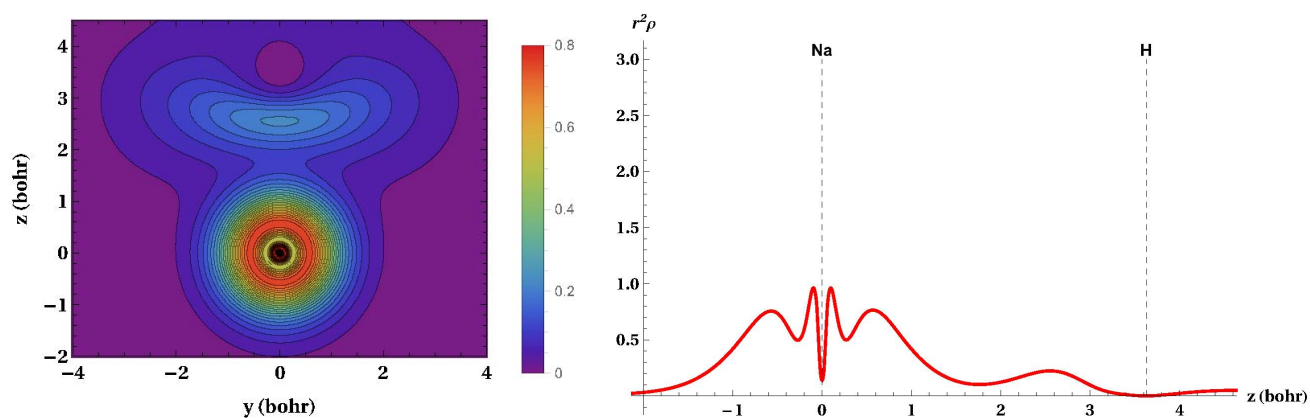


(a) The core size is defined as Becke weight.

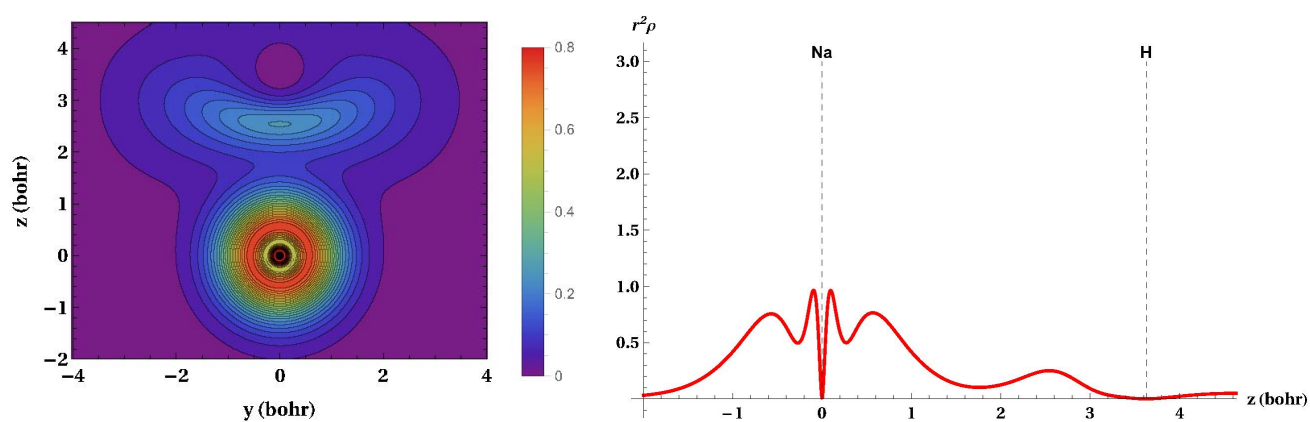


(b) Awad weight at  $r_{(r)}$ .

**Figure A.16:** Molecular radial electron density (RDEN) for LiH is obtained using the Becke and Awad weights. The atomic coordinates of Li at 0.0 bohr and H at 3.0390 bohr along  $z$  axis.



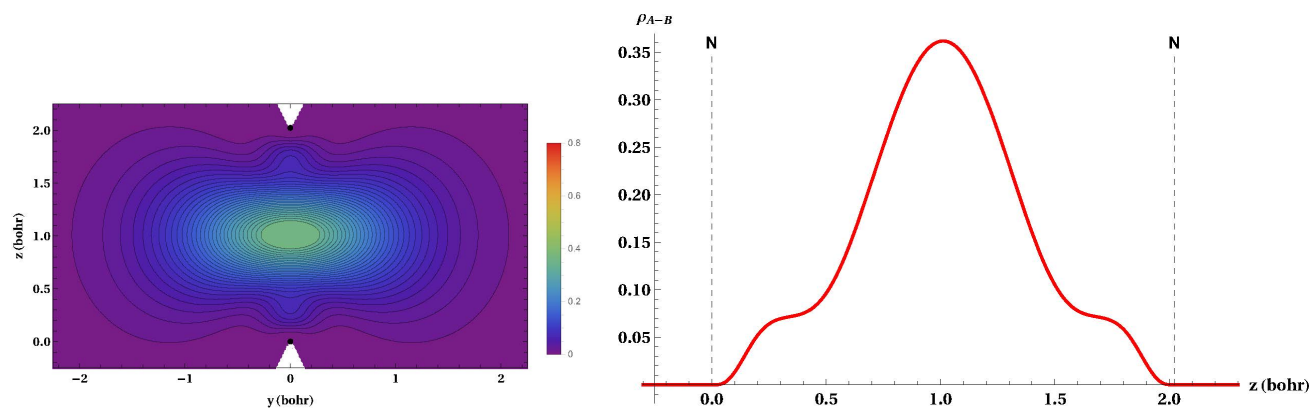
(a) The core size is defined as Becke weight.



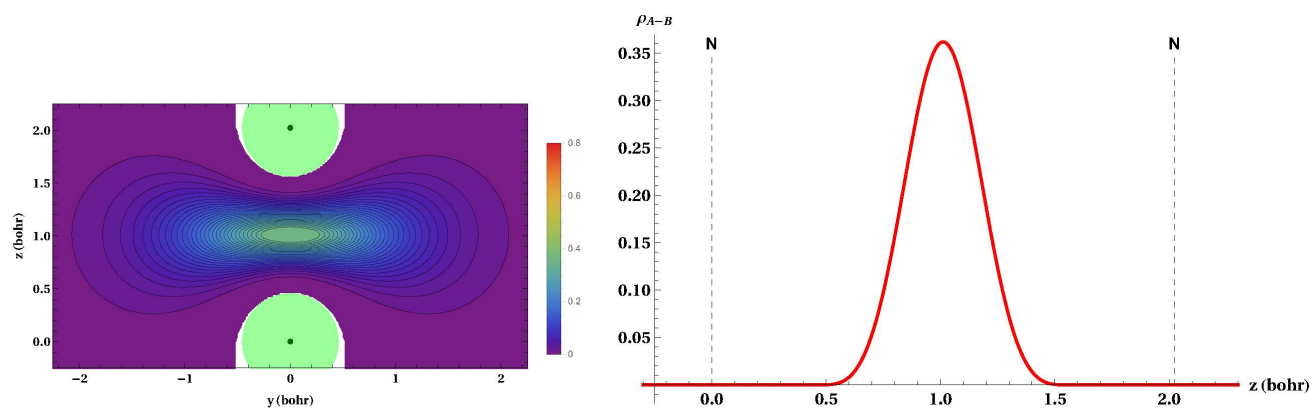
(b) Awad weight at  $r_{(r)}$ .

**Figure A.17:** Molecular radial electron density (RDEN) for NaH is obtained using the Becke and Awad weights. The atomic coordinates of Na at 0.0 bohr and H at 3.6261 bohr along  $z$  axis.

### A.3.2 Comparing the Bond Electron Density Between Awad and Becke Weights



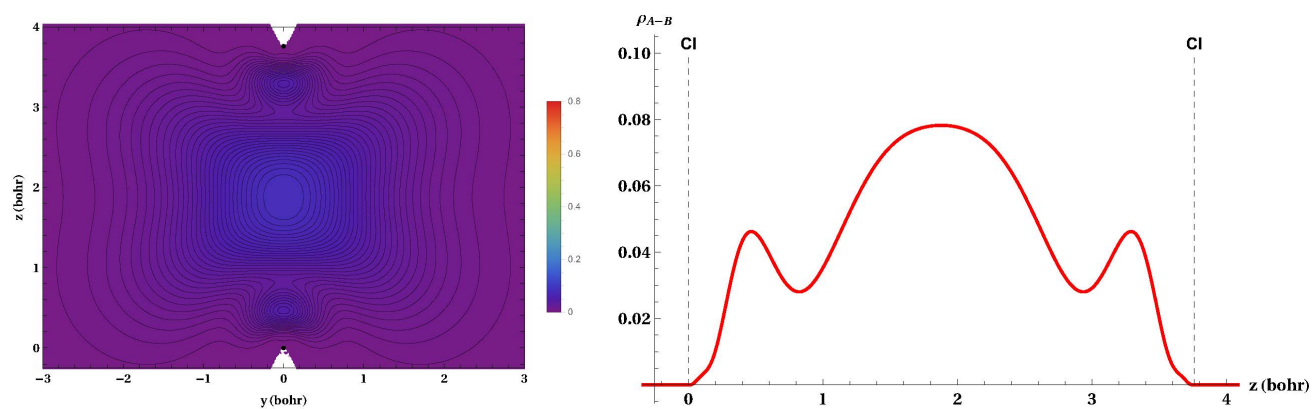
(a) The core size is defined as Becke weight.



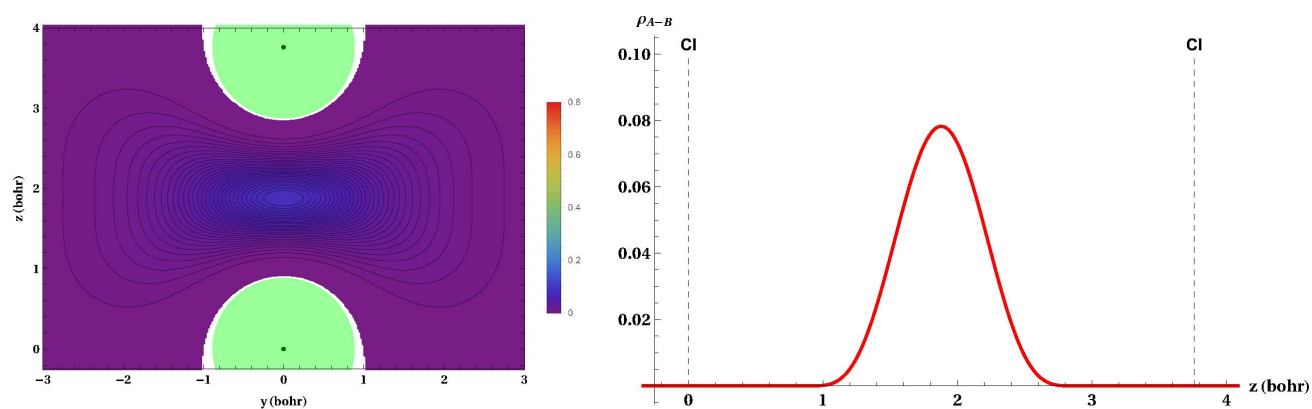
(b) Awad weight at  $r_{\langle r \rangle}$ .

**Figure A.18:** Bond electron density (BDEN) for  $N_2$  is obtained using the Becke and Awad weights. The atomic coordinates of the two N atoms are 0.0 and 2.0229 bohr along  $z$  axis.



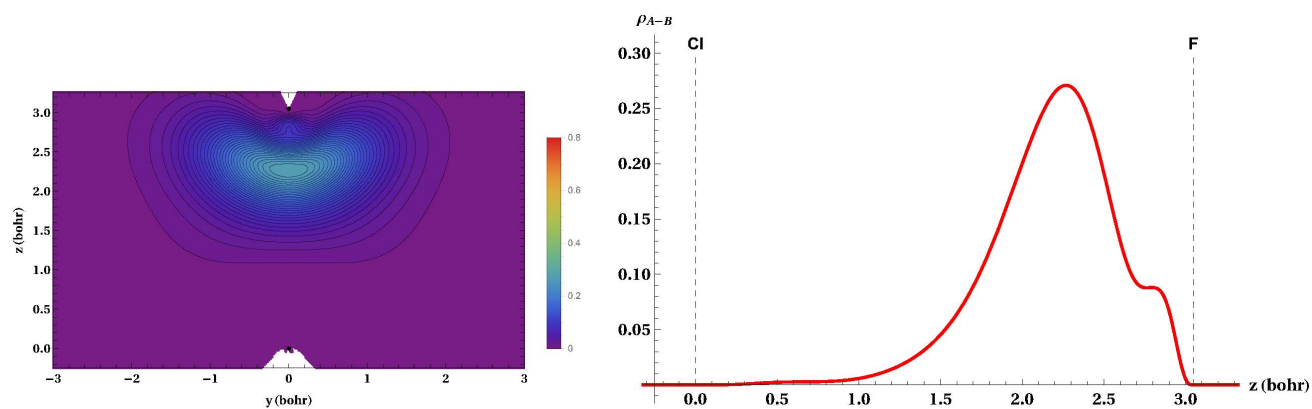


(a) The core size is defined as Becke weight.

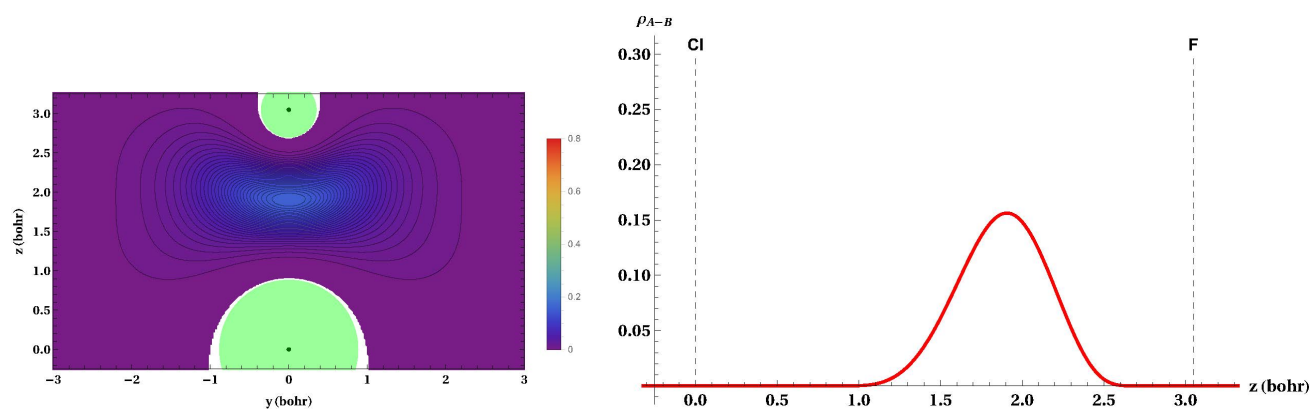


(b) Awad weight at  $r_{\langle r \rangle}$ .

**Figure A.19:** Bond electron density (BDEN) for  $\text{Cl}_2$  is obtained using the Becke and Awad weights. The atomic coordinates of the two Cl atoms are 0.0 and 3.7600 bohr along  $z$  axis.

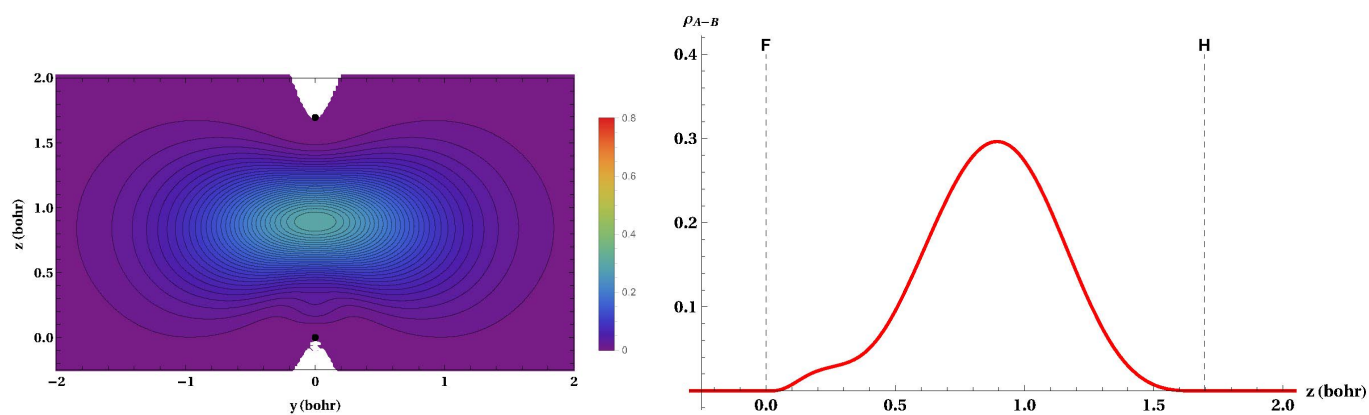


(a) The core size is defined as Becke weight.

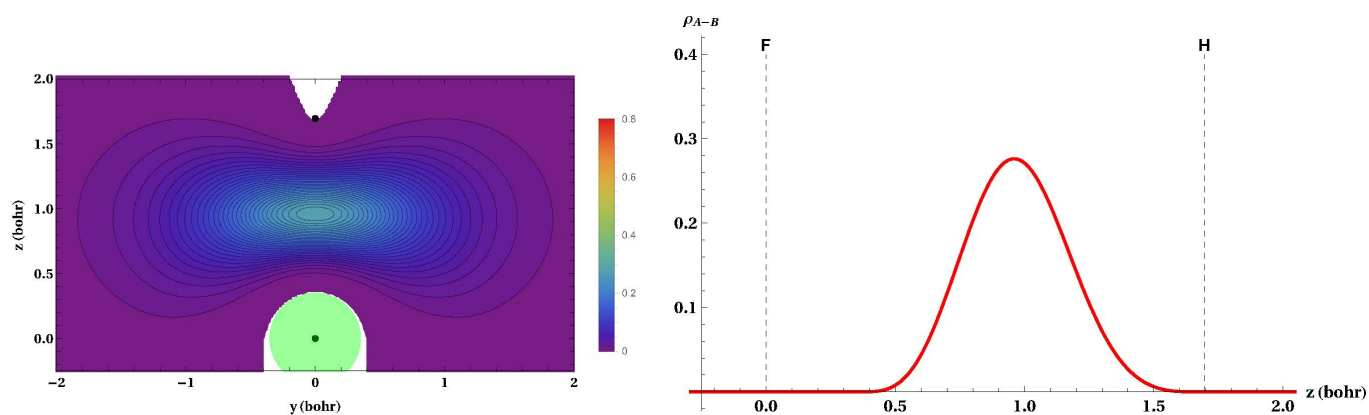


(b) Awad weight at  $r_{\langle r \rangle}$ .

**Figure A.20:** Bond electron density (BDEN) for FCl is obtained using the Becke and Awad weights. The atomic coordinates of Cl at 0.0 bohr and F at 3.0504 bohr along  $z$  axis.

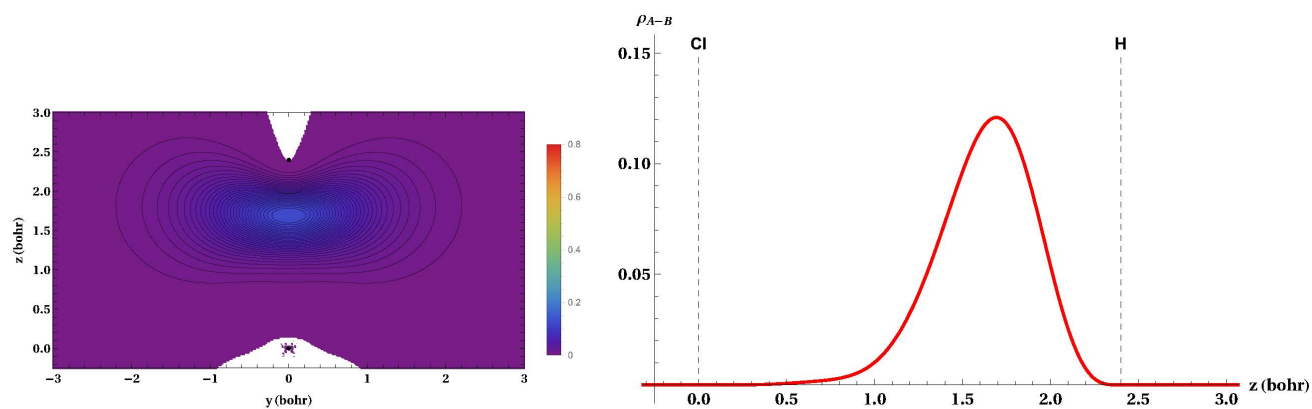


(a) The core size is defined as Becke weight.

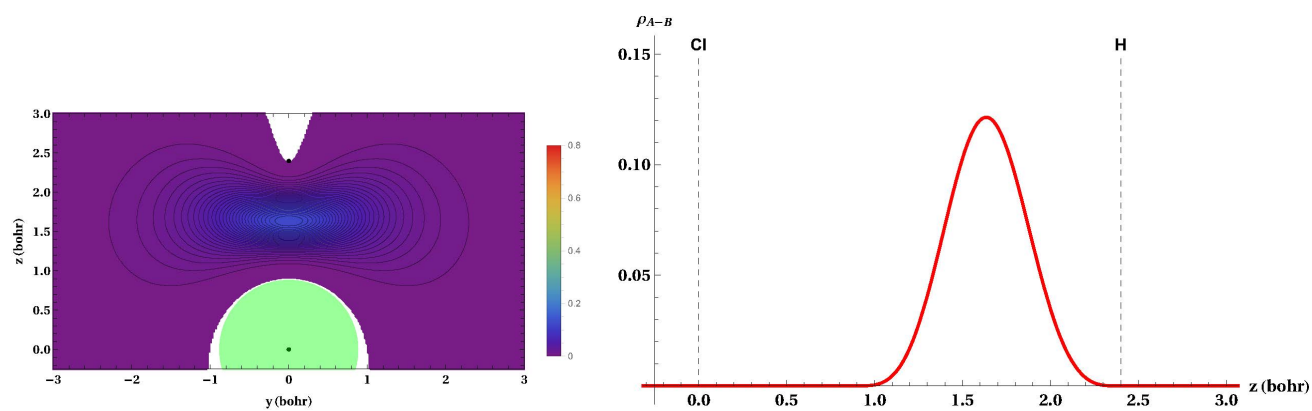


(b) Awad weight at  $r_{(r)}$ .

**Figure A.21:** Bond electron density (BDEN) for HF is obtained using the Becke and Awad weights. The atomic coordinates of F at 0.0 bohr and H at 1.6960 bohr along  $z$  axis.

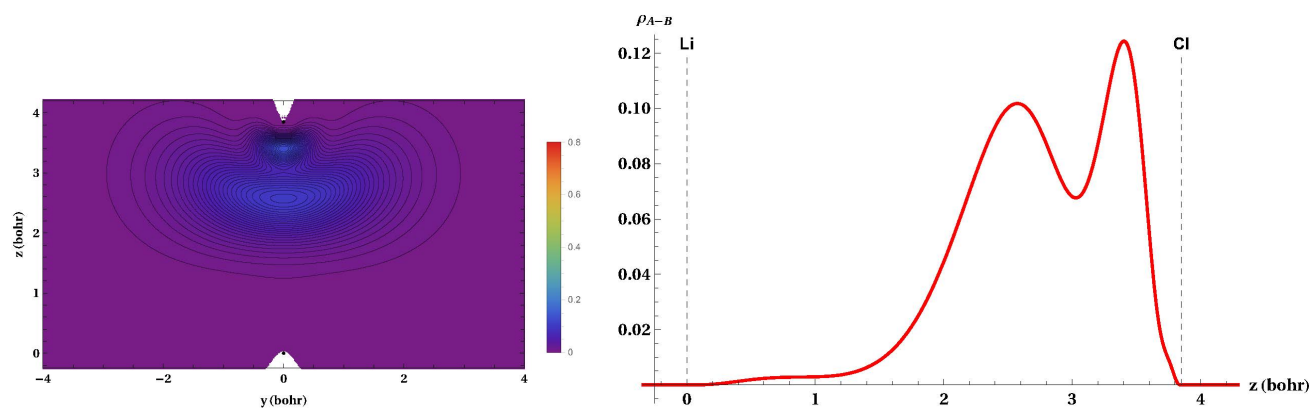


(a) The core size is defined as Becke weight.

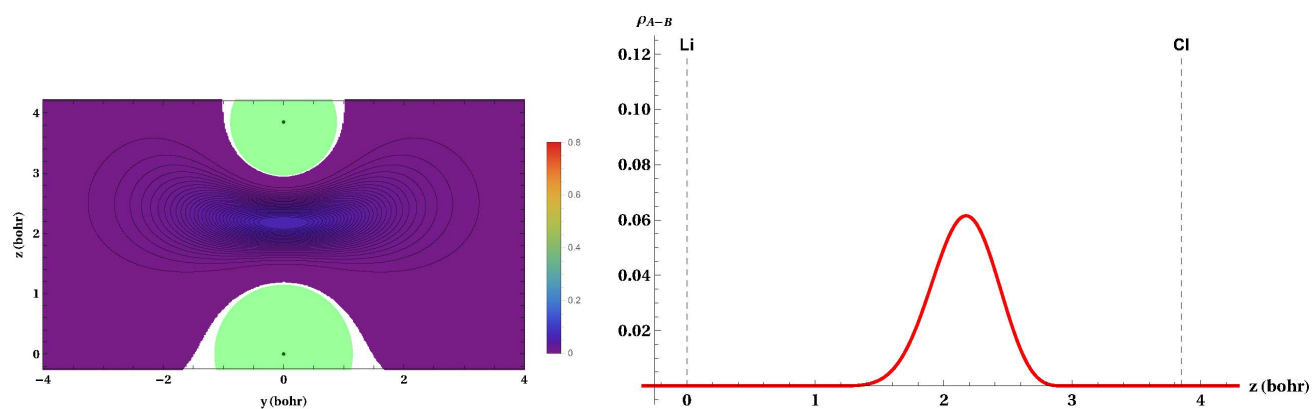


(b) Awad weight at  $r_{\langle r \rangle}$ .

**Figure A.22:** Bond electron density (BDEN) for HCl is obtained using the Becke and Awad weights. The atomic coordinates of Cl at 0.0 bohr and H at 2.4000 bohr along  $z$  axis.

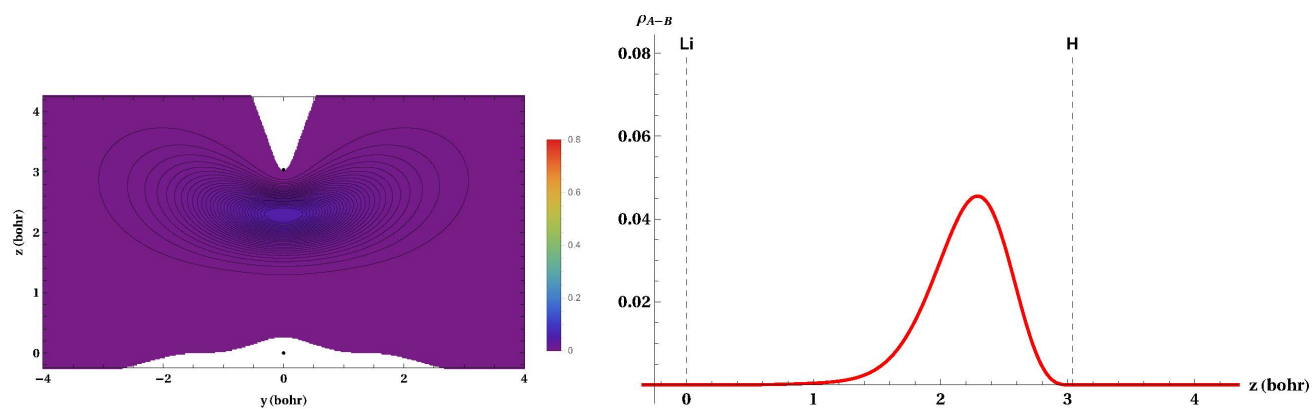


(a) The core size is defined as Becke weight.

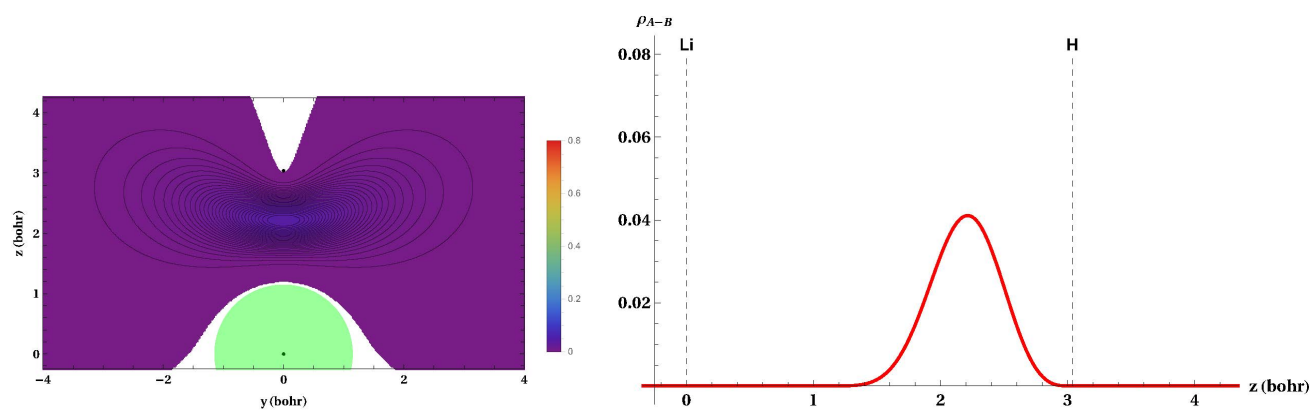


(b) Awad weight at  $r_{\langle r \rangle}$ .

**Figure A.23:** Bond electron density (BDEN) for LiCl is obtained using the Becke and Awad weights. The atomic coordinates of Li at 0.0 bohr and Cl at 3.8492 bohr along  $z$  axis.

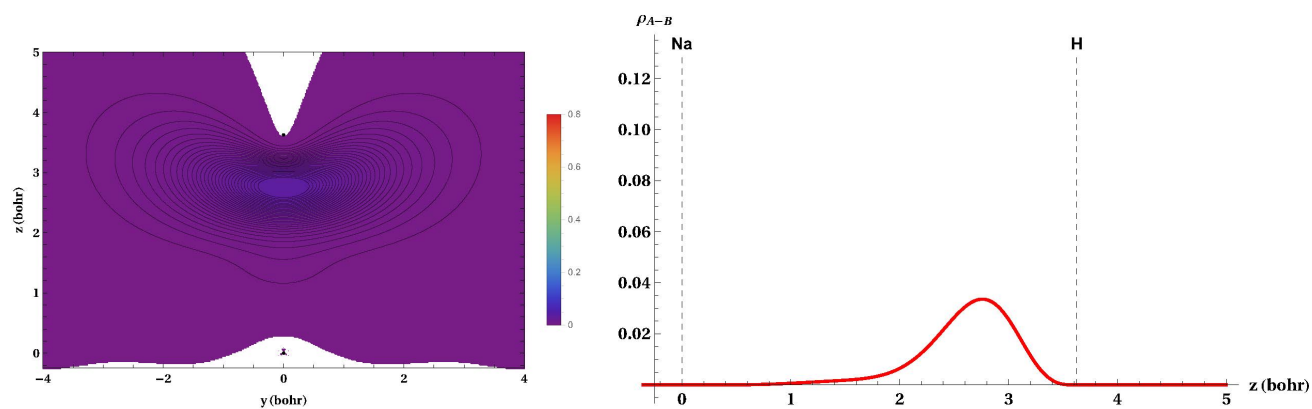


(a) The core size is defined as Becke weight.

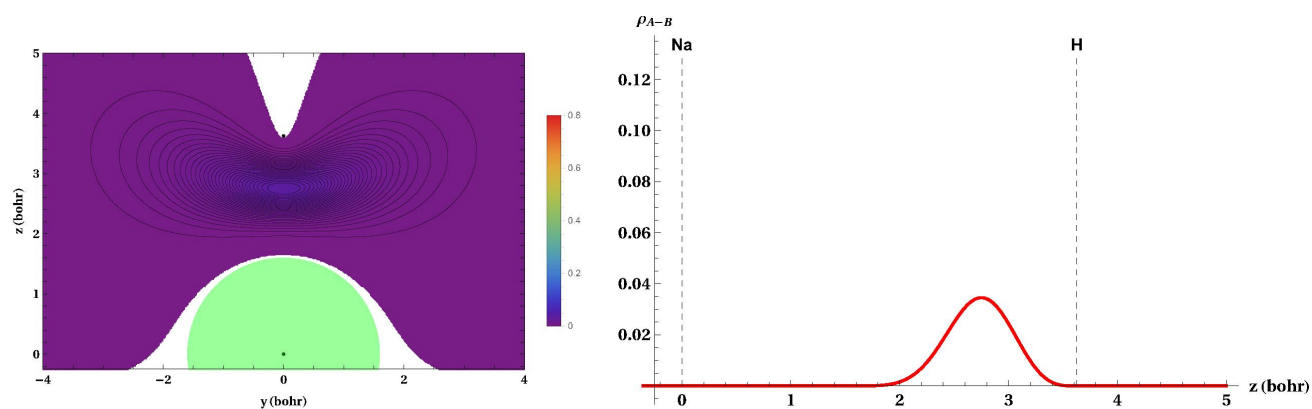


(b) Awad weight at  $r_{\langle r \rangle}$ .

**Figure A.24:** Bond electron density (BDEN) for LiH is obtained using the Becke and Awad weights. The atomic coordinates of Li at 0.0 bohr and H at 3.0390 bohr along  $z$  axis.



(a) The core size is defined as Becke weight.



(b) Awad weight at  $r_{\langle r \rangle}$ .

**Figure A.25:** Bond electron density (BDEN) for NaH is obtained using the Becke and Awad weights. The atomic coordinates of Na at 0.0 bohr and H at 3.6261 bohr along  $z$  axis.

# Appendix B

## AIMD Code

### B.1 Input and Output Examples

In this appendix you could find some example of input and output files that has been used during the research.

#### B.1.1 Displaying the Fragments of the Target Molecule

Here is example of input file to show the cartesian coordinate of the fragments for Glycine molecule,

```
MOLECULE
  UNITS=ANGSTROM !(default)
  CHARGE=0
  MULTIPLICITY=1
FREEZ
N
C   1 B1
C   2 B2 1 A2
O   3 B3 2 A3 1 D3
H   2 B4 1 A4 3 D4
H   2 B5 1 A5 3 D5
H   1 B6 2 A6 3 D6
H   1 B7 2 A7 3 D7
O   3 B8 2 A8 1 D8
H   9 B9 3 A9 2 D9

END
```



```

DEFINE
B1 = 1.44084286      B2 = 1.51781924      B3 = 1.18115749
B4 = 1.08653973      B5 = 1.09386650      B6 = 1.00062635
B7 = 0.99972788      B8 = 1.33306652      B9 = 0.94790958
A2 = 110.31733       A3 = 123.50442       A4 = 109.68864
A5 = 114.69657       A6 = 110.26867       A7 = 111.39727
A8 = 115.54740       A9 = 112.36145       D3 = -21.55248
D4 = 120.68224       D5 = -119.29373      D6 = 38.90011
D7 = 159.88520       D8 = 161.62252      D9 = -5.50811
END
end !molecule

BASIS name=6-31G(d) end

AIMDFT
Level=2
end !AIMDFT

output object=AIMDFT:FRAGMENT%DISPLAY end
stop

```

Here is the output,

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```

N_molecules: 1
Molecule is an asymmetric top.
Point group: C1
Free format Z-Matrix for: C2H5NO2, (C1)
N
C      N      B1
C      C      B2 N      A2
O      C      B3 C      A3 N      D3
H      C      B4 N      A4 C      D4
H      C      B5 N      A5 C      D5
H      N      B6 C      A6 C      D6
H      N      B7 C      A7 C      D7
O      C      B8 C      A8 N      D8
H      O      B9 C      A9 C      D9

```

```

VARIABLES:
B1 = 1.44084286      B2 = 1.51781924      B3 = 1.18115749
B4 = 1.08653973      B5 = 1.09386650      B6 = 1.00062635
B7 = 0.99972788      B8 = 1.33306652      B9 = 0.94790958
A2 = 110.31733       A3 = 123.50442       A4 = 109.68864
A5 = 114.69657       A6 = 110.26867       A7 = 111.39727
A8 = 115.54740       A9 = 112.36145       D3 = -21.55248
D4 = 120.68224       D5 = -119.29373      D6 = 38.90011
D7 = 159.88520       D8 = 161.62252      D9 = -5.50811

```

Z MATRIX FOR: C2H5NO2, (C1)

| ----- |    |    |          |      |    |          |       |    |            |
|-------|----|----|----------|------|----|----------|-------|----|------------|
| I     | AN | Z1 | BL       |      | Z2 | ALPHA    |       | Z3 | BETA       |
|       |    | Z4 |          |      |    |          |       |    |            |
| ----- |    |    |          |      |    |          |       |    |            |
| 1     | 7  |    |          |      |    |          |       |    |            |
| 2     | 6  | 1  | 1.440843 | ( 1) |    |          |       |    |            |
| 3     | 6  | 2  | 1.517819 | ( 2) | 1  | 110.3173 | ( 10) |    |            |
| 4     | 8  | 3  | 1.181157 | ( 3) | 2  | 123.5044 | ( 11) | 1  | -21.5525 ( |
| 18)   |    | 0  |          |      |    |          |       |    |            |

|     |   |   |          |      |   |          |       |   |           |   |
|-----|---|---|----------|------|---|----------|-------|---|-----------|---|
| 5   | 1 | 2 | 1.086540 | ( 4) | 1 | 109.6886 | ( 12) | 3 | 120.6822  | ( |
| 19) |   | 0 |          |      |   |          |       |   |           |   |
| 6   | 1 | 2 | 1.093866 | ( 5) | 1 | 114.6966 | ( 13) | 3 | -119.2937 | ( |
| 20) |   | 0 |          |      |   |          |       |   |           |   |
| 7   | 1 | 1 | 1.000626 | ( 6) | 2 | 110.2687 | ( 14) | 3 | 38.9001   | ( |
| 21) |   | 0 |          |      |   |          |       |   |           |   |
| 8   | 1 | 1 | 0.999728 | ( 7) | 2 | 111.3973 | ( 15) | 3 | 159.8852  | ( |
| 22) |   | 0 |          |      |   |          |       |   |           |   |
| 9   | 8 | 3 | 1.333067 | ( 8) | 2 | 115.5474 | ( 16) | 1 | 161.6225  | ( |
| 23) |   | 0 |          |      |   |          |       |   |           |   |
| 10  | 1 | 9 | 0.947910 | ( 9) | 3 | 112.3615 | ( 17) | 2 | -5.5081   | ( |
| 24) |   | 0 |          |      |   |          |       |   |           |   |

Cartesian coordinates for: C2H5NO2, (C1)

| COORDINATES IN BOHR |    |             | COORDINATES IN ANGSTROMS |             |             |             |
|---------------------|----|-------------|--------------------------|-------------|-------------|-------------|
| I                   | EL | AN          | X                        | Y           | Z           | X           |
|                     |    | Y           | Z                        |             |             |             |
| 1                   | N  | 7           | 0.00000000               | 0.00000000  | 0.00000000  | 0.00000000  |
|                     |    | 0.00000000  | 0.00000000               |             |             |             |
| 2                   | C  | 6           | 0.00000000               | 0.00000000  | 1.44084286  | 0.00000000  |
|                     |    | 0.00000000  | 2.72279820               |             |             |             |
| 3                   | C  | 6           | 1.42338653               | 0.00000000  | 1.96785902  | 2.68981052  |
|                     |    | 0.00000000  | 3.71871433               |             |             |             |
| 4                   | O  | 8           | 2.35288801               | -0.36180632 | 1.33520098  | 4.44631363  |
|                     |    | -0.68371481 | 2.52316398               |             |             |             |
| 5                   | H  | 1           | -0.52202178              | 0.87980599  | 1.80690743  | -0.98647812 |
|                     |    | 1.66259225  | 3.41455992               |             |             |             |
| 6                   | H  | 1           | -0.48626030              | -0.86672789 | 1.89787416  | -0.91889872 |
|                     |    | -1.63787822 | 3.58646212               |             |             |             |
| 7                   | H  | 1           | 0.73050930               | -0.58944902 | -0.34663973 | 1.38046242  |
|                     |    | -1.11389713 | -0.65505411              |             |             |             |
| 8                   | H  | 1           | -0.87404490              | -0.32011104 | -0.36473314 | -1.65170536 |
|                     |    | -0.60492216 | -0.68924570              |             |             |             |
| 9                   | O  | 8           | 1.56620070               | 0.37919230  | 3.23785258  | 2.95969016  |
|                     |    | 0.71656954  | 6.11865417               |             |             |             |
| 10                  | H  | 1           | 0.75176248               | 0.68026803  | 3.61808458  | 1.42062510  |
|                     |    | 1.28552018  | 6.83718846               |             |             |             |

Nuclear repulsion energy: 181.510743561

Distance Matrix for: C2H5NO2, (C1)

|          | 1        | 2        | 3        | 4        | 5        | 6        |
|----------|----------|----------|----------|----------|----------|----------|
| 7        | 8        | 9        | 10       |          |          |          |
| 1        | 0.000000 | 1.440843 | 2.428682 | 2.729423 | 2.076410 | 2.142333 |
| 1.000626 | 0.999728 | 3.616692 | 3.757452 |          |          |          |
| 2        | 1.440843 | 0.000000 | 1.517819 | 2.382886 | 1.086540 | 1.093866 |
| 2.018957 | 2.031386 | 2.413714 | 2.401727 |          |          |          |
| 3        | 2.428682 | 1.517819 | 0.000000 | 1.181157 | 2.141163 | 2.098301 |
| 2.486852 | 3.289627 | 1.333067 | 1.907115 |          |          |          |
| 4        | 2.729423 | 2.382886 | 1.181157 | 0.000000 | 3.166893 | 2.938079 |
| 2.347876 | 3.647549 | 2.188159 | 2.976757 |          |          |          |
| 5        | 2.076410 | 1.086540 | 2.141163 | 3.166893 | 0.000000 | 1.749267 |
| 2.892285 | 2.505942 | 2.580483 | 2.223219 |          |          |          |
| 6        | 2.142333 | 1.093866 | 2.098301 | 2.938079 | 1.749267 | 0.000000 |



```

SCF_CYCLE: 11      -464.326986541      -282.816242980      4.60760E-05
SCF_CYCLE: 12      -464.326987992      -282.816244431
SCF_CYCLE: 13      -464.326987698      -282.816244138      1.87641E-05
At termination total energy is      -282.816244      Hartrees

```

```

*****
Current cartesian coordinates
*****

```

```

Fragment# 1, Atoms# 10
!Atom      X      Y      Z      Ctype      Ftype      Level      Sorted
N      0.00000000      0.00000000      0.00000000      9      0      T
C      0.00000000      0.00000000      1.44084286      29      1      T
H      0.73050930      -0.58944902      -0.34663973      5      1      F
H      -0.87404490      -0.32011104      -0.36473314      5      1      F
C      1.42338653      0.00000000      1.96785902      10      2      T
H      -0.52202178      0.87980599      1.80690743      4      2      F
H      -0.48626030      -0.86672789      1.89787416      4      2      F
H      1.40752011      0.00000000      3.05140244      -CH3      4      3      F
H      1.93964311      0.88478395      1.61422314      -CH3      4      3      F
H      1.93966847      -0.88478388      1.61415464      -CH3      4      3      F

```

```

Fragment# 2, Atoms# 10
!Atom      X      Y      Z      Ctype      Ftype      Level      Sorted
C      0.00000000      0.00000000      1.44084286      29      0      T
N      0.00000000      0.00000000      0.00000000      9      1      T
C      1.42338653      0.00000000      1.96785902      34      1      T
H      -0.52202178      0.87980599      1.80690743      4      1      F
H      -0.48626030      -0.86672789      1.89787416      4      1      F
O      1.56620070      0.37919230      3.23785258      7      2      T
O      2.35288801      -0.36180632      1.33520098      4      2      T
H      0.73050930      -0.58944902      -0.34663973      5      2      F
H      -0.87404490      -0.32011104      -0.36473314      5      2      F
H      2.53932814      0.31941883      3.40629958      -OH      6      3      T

```

```

Fragment# 3, Atoms# 10
!Atom      X      Y      Z      Ctype      Ftype      Level      Sorted
C      1.42338653      0.00000000      1.96785902      34      0      T
O      1.56620070      0.37919230      3.23785258      7      1      T
O      2.35288801      -0.36180632      1.33520098      4      1      T
C      0.00000000      0.00000000      1.44084286      29      1      T
N      0.00000000      0.00000000      0.00000000      9      2      T
H      0.75176248      0.68026803      3.61808458      6      2      T
H      -0.52202178      0.87980599      1.80690743      4      2      F
H      -0.48626030      -0.86672789      1.89787416      4      2      F
H      -0.95777357      0.00000000      -0.29610147      -NH2      5      3      F
H      0.40141812      -0.86960710      -0.29610147      -NH2      5      3      F

```

```

Fragment# 4, Atoms# 8
!Atom      X      Y      Z      Ctype      Ftype      Level      Sorted
O      2.35288801      -0.36180632      1.33520098      4      0      T
C      1.42338653      0.00000000      1.96785902      34      1      T
O      1.56620070      0.37919230      3.23785258      7      2      T
C      0.00000000      0.00000000      1.44084286      10      2      T
H      0.65316574      0.62862176      3.52608235      -OH      6      3      T
H      -0.66879662      0.37530236      2.20646504      -CH3      4      3      F
H      -0.28673497      -1.01059766      1.17457750      -CH3      4      3      F
H      -0.06099919      0.63521777      0.56490118      -CH3      4      3      F

```

```

Fragment# 5, Atoms# 10
!Atom      X      Y      Z      Ctype      Ftype      Level      Sorted
H      -0.52202178      0.87980599      1.80690743      4      0      T
C      0.00000000      0.00000000      1.44084286      29      1      T

```

|                                |             |             |             |       |       |       |        |
|--------------------------------|-------------|-------------|-------------|-------|-------|-------|--------|
| N                              | 0.00000000  | 0.00000000  | 0.00000000  |       | 9     | 2     | T      |
| C                              | 1.42338653  | 0.00000000  | 1.96785902  |       | 10    | 2     | T      |
| H                              | -0.48626030 | -0.86672789 | 1.89787416  |       | 4     | 2     | T      |
| H                              | 0.48872922  | -0.82369531 | -0.29610147 | -NH2  | 5     | 3     | F      |
| H                              | -0.95270540 | -0.09851616 | -0.29610147 | -NH2  | 5     | 3     | F      |
| H                              | 1.94391057  | -0.87751344 | 1.60269742  | -CH3  | 4     | 3     | F      |
| H                              | 1.93528625  | 0.89191159  | 1.62599031  | -CH3  | 4     | 3     | F      |
| H                              | 1.40752416  | -0.01421698 | 3.05139149  | -CH3  | 4     | 3     | F      |
| Fragment# 6, Atoms# 10         |             |             |             |       |       |       |        |
| !Atom                          | X           | Y           | Z           | Ctype | Ftype | Level | Sorted |
| H                              | -0.48626030 | -0.86672789 | 1.89787416  |       | 4     | 0     | T      |
| C                              | 0.00000000  | 0.00000000  | 1.44084286  |       | 29    | 1     | T      |
| N                              | 0.00000000  | 0.00000000  | 0.00000000  |       | 9     | 2     | T      |
| C                              | 1.42338653  | 0.00000000  | 1.96785902  |       | 10    | 2     | T      |
| H                              | -0.52202178 | 0.87980599  | 1.80690743  |       | 4     | 2     | T      |
| H                              | 0.46862617  | 0.83529619  | -0.29610147 | -NH2  | 5     | 3     | F      |
| H                              | -0.95481287 | 0.07540156  | -0.29610147 | -NH2  | 5     | 3     | F      |
| H                              | 1.96357598  | 0.84116568  | 1.54958427  | -CH3  | 4     | 3     | F      |
| H                              | 1.91452219  | -0.92278526 | 1.68207074  | -CH3  | 4     | 3     | F      |
| H                              | 1.40861876  | 0.08144592  | 3.04843517  | -CH3  | 4     | 3     | F      |
| Fragment# 7, Atoms# 7          |             |             |             |       |       |       |        |
| !Atom                          | X           | Y           | Z           | Ctype | Ftype | Level | Sorted |
| H                              | 0.73050930  | -0.58944902 | -0.34663973 |       | 5     | 0     | T      |
| N                              | 0.00000000  | 0.00000000  | 0.00000000  |       | 9     | 1     | T      |
| C                              | 0.00000000  | 0.00000000  | 1.44084286  |       | 14    | 2     | T      |
| H                              | -0.87404490 | -0.32011104 | -0.36473314 |       | 5     | 2     | T      |
| H                              | -0.79508214 | 0.64155293  | 1.80219079  | -CH3  | 4     | 3     | F      |
| H                              | -0.15801825 | -1.00939572 | 1.80219079  | -CH3  | 4     | 3     | F      |
| H                              | 0.95326447  | 0.36771031  | 1.80219079  | -CH3  | 4     | 3     | F      |
| Fragment# 8, Atoms# 7          |             |             |             |       |       |       |        |
| !Atom                          | X           | Y           | Z           | Ctype | Ftype | Level | Sorted |
| H                              | -0.87404490 | -0.32011104 | -0.36473314 |       | 5     | 0     | T      |
| N                              | 0.00000000  | 0.00000000  | 0.00000000  |       | 9     | 1     | T      |
| C                              | 0.00000000  | 0.00000000  | 1.44084286  |       | 14    | 2     | T      |
| H                              | 0.73050930  | -0.58944902 | -0.34663973 |       | 5     | 2     | T      |
| H                              | 0.95932437  | 0.35134388  | 1.80219079  | -CH3  | 4     | 3     | F      |
| H                              | -0.17544770 | -1.00651259 | 1.80219079  | -CH3  | 4     | 3     | F      |
| H                              | -0.78407467 | 0.65509612  | 1.80219079  | -CH3  | 4     | 3     | F      |
| Fragment# 9, Atoms# 8          |             |             |             |       |       |       |        |
| !Atom                          | X           | Y           | Z           | Ctype | Ftype | Level | Sorted |
| O                              | 1.56620070  | 0.37919230  | 3.23785258  |       | 7     | 0     | T      |
| C                              | 1.42338653  | 0.00000000  | 1.96785902  |       | 34    | 1     | T      |
| H                              | 0.75176248  | 0.68026803  | 3.61808458  |       | 6     | 1     | T      |
| O                              | 2.35288801  | -0.36180632 | 1.33520098  |       | 4     | 2     | T      |
| C                              | 0.00000000  | 0.00000000  | 1.44084286  |       | 10    | 2     | T      |
| H                              | -0.00222494 | -0.32209824 | 0.40616133  | -CH3  | 4     | 3     | F      |
| H                              | -0.60406692 | -0.67858912 | 2.03164044  | -CH3  | 4     | 3     | F      |
| H                              | -0.41037651 | 1.00075379  | 1.50851356  | -CH3  | 4     | 3     | F      |
| Fragment# 10, Atoms# 6         |             |             |             |       |       |       |        |
| !Atom                          | X           | Y           | Z           | Ctype | Ftype | Level | Sorted |
| H                              | 0.75176248  | 0.68026803  | 3.61808458  |       | 6     | 0     | T      |
| O                              | 1.56620070  | 0.37919230  | 3.23785258  |       | 7     | 1     | T      |
| C                              | 1.42338653  | 0.00000000  | 1.96785902  |       | 18    | 2     | T      |
| H                              | 2.37886126  | -0.33411421 | 1.58087862  | -CH3  | 4     | 3     | F      |
| H                              | 0.70717887  | -0.81134180 | 1.91135348  | -CH3  | 4     | 3     | F      |
| H                              | 1.06777838  | 0.83714620  | 1.37860065  | -CH3  | 4     | 3     | F      |
| *****                          |             |             |             |       |       |       |        |
| Database cartesian coordinates |             |             |             |       |       |       |        |

|           |              |             |             |       |       |       |        |
|-----------|--------------|-------------|-------------|-------|-------|-------|--------|
| *****     |              |             |             |       |       |       |        |
| Fragment# | 1, Atoms# 10 |             |             |       |       |       |        |
| !Atom     | X            | Y           | Z           | Ctype | Ftype | Level | Sorted |
| N         | 0.00000000   | -0.00000000 | 0.00000000  |       | 9     | 0     | T      |
| C         | -0.00000000  | -0.00000000 | 1.44084286  |       | 29    | 1     | T      |
| H         | 0.00000000   | 0.93866607  | -0.34663973 |       | 5     | 1     | F      |
| H         | 0.79799306   | -0.47920001 | -0.36473314 |       | 5     | 1     | F      |
| C         | -0.89383628  | 1.10773910  | 1.96785902  |       | 10    | 2     | T      |
| H         | -0.35689075  | -0.95874622 | 1.80690743  |       | 4     | 2     | F      |
| H         | 0.97987823   | 0.16584623  | 1.89787416  |       | 4     | 2     | F      |
| H         | -0.88387273  | 1.09539118  | 3.05140244  | -CH3  | 4     | 3     | F      |
| H         | -1.90660309  | 0.95389867  | 1.61422314  | -CH3  | 4     | 3     | F      |
| H         | -0.52946711  | 2.06514427  | 1.61415464  | -CH3  | 4     | 3     | F      |
| Fragment# | 2, Atoms# 10 |             |             |       |       |       |        |
| !Atom     | X            | Y           | Z           | Ctype | Ftype | Level | Sorted |
| C         | -0.00000000  | 0.00000000  | 0.00000000  |       | 29    | 0     | T      |
| N         | 0.00000000   | -0.00000000 | 1.44084286  |       | 9     | 1     | T      |
| C         | 0.00000000   | 1.42338653  | -0.52701616 |       | 34    | 1     | T      |
| H         | 0.87980599   | -0.52202178 | -0.36606457 |       | 4     | 1     | F      |
| H         | -0.86672789  | -0.48626030 | -0.45703130 |       | 4     | 1     | F      |
| O         | 0.37919230   | 1.56620070  | -1.79700972 |       | 7     | 2     | T      |
| O         | -0.36180632  | 2.35288801  | 0.10564188  |       | 4     | 2     | T      |
| H         | -0.58944902  | 0.73050930  | 1.78748259  |       | 5     | 2     | F      |
| H         | -0.32011104  | -0.87404490 | 1.80557600  |       | 5     | 2     | F      |
| H         | 0.31941883   | 2.53932814  | -1.96545672 | -OH   | 6     | 3     | T      |
| Fragment# | 3, Atoms# 10 |             |             |       |       |       |        |
| !Atom     | X            | Y           | Z           | Ctype | Ftype | Level | Sorted |
| C         | -0.00000000  | -0.00000000 | 0.00000000  |       | 34    | 0     | T      |
| O         | -0.00000000  | 0.00000000  | 1.33306652  |       | 7     | 1     | T      |
| O         | 0.00000000   | 1.01381600  | -0.60606116 |       | 4     | 1     | T      |
| C         | 0.07368323   | -1.36743652 | -0.65457115 |       | 29    | 1     | T      |
| N         | -0.35716835  | -1.28888329 | -2.02724171 |       | 9     | 2     | T      |
| H         | -0.03707255  | -0.87584463 | 1.69369704  |       | 6     | 2     | T      |
| H         | -0.55932392  | -2.06380818 | -0.11149008 |       | 4     | 2     | F      |
| H         | 1.10434945   | -1.70738451 | -0.51779972 |       | 4     | 2     | F      |
| H         | -0.29008929  | -2.21219942 | -2.41194164 | -NH2  | 5     | 3     | F      |
| H         | 0.30676556   | -0.71652891 | -2.51368937 | -NH2  | 5     | 3     | F      |
| Fragment# | 4, Atoms# 8  |             |             |       |       |       |        |
| !Atom     | X            | Y           | Z           | Ctype | Ftype | Level | Sorted |
| O         | -0.00000000  | 0.00000000  | -0.00000000 |       | 4     | 0     | T      |
| C         | -0.00000000  | -0.00000000 | 1.18115749  |       | 34    | 1     | T      |
| O         | -0.00000000  | 1.14420319  | 1.86516436  |       | 7     | 2     | T      |
| C         | 0.07368323   | -1.26347662 | 2.01899570  |       | 10    | 2     | T      |
| H         | 0.00000000   | 0.86533235  | 2.81445637  | -OH   | 6     | 3     | T      |
| H         | 0.05839270   | -1.00128368 | 3.07034682  | -CH3  | 4     | 3     | F      |
| H         | 0.99092661   | -1.79429459 | 1.79245948  | -CH3  | 4     | 3     | F      |
| H         | -0.77563740  | -1.89735718 | 1.79239856  | -CH3  | 4     | 3     | F      |
| Fragment# | 5, Atoms# 10 |             |             |       |       |       |        |
| !Atom     | X            | Y           | Z           | Ctype | Ftype | Level | Sorted |
| H         | -0.00000000  | 0.00000000  | -0.00000000 |       | 4     | 0     | T      |
| C         | -0.00000000  | -0.00000000 | 1.08653973  |       | 29    | 1     | T      |
| N         | 0.00000000   | 1.35660739  | 1.57197206  |       | 9     | 2     | T      |
| C         | 1.22412734   | -0.74090902 | 1.59284135  |       | 10    | 2     | T      |
| H         | -0.86045971  | -0.59784588 | 1.40075830  |       | 4     | 2     | T      |
| H         | 0.00000000   | 1.31271584  | 2.57351079  | -NH2  | 5     | 3     | F      |
| H         | -0.86960710  | 1.77063919  | 1.29378103  | -NH2  | 5     | 3     | F      |
| H         | 1.22400910   | -0.74083745 | 2.67650093  | -CH3  | 4     | 3     | F      |
| H         | 2.11948787   | -0.24860401 | 1.23174979  | -CH3  | 4     | 3     | F      |
| H         | 1.20323095   | -1.76248761 | 1.23168092  | -CH3  | 4     | 3     | F      |

|                        |             |             |            |       |       |       |        |
|------------------------|-------------|-------------|------------|-------|-------|-------|--------|
| Fragment# 6, Atoms# 10 |             |             |            |       |       |       |        |
| !Atom                  | X           | Y           | Z          | Ctype | Ftype | Level | Sorted |
| H                      | -0.00000000 | 0.00000000  | 0.00000000 |       | 4     | 0     | T      |
| C                      | 0.00000000  | 0.00000000  | 1.09386650 |       | 29    | 1     | T      |
| N                      | 0.00000000  | 1.30905356  | 1.69586892 |       | 9     | 2     | T      |
| C                      | 1.24136788  | -0.76979504 | 1.50641545 |       | 10    | 2     | T      |
| H                      | -0.88574475 | -0.54645193 | 1.40598042 |       | 4     | 2     | T      |
| H                      | 0.00000000  | 1.17790163  | 2.68975295 | -NH2  | 5     | 3     | F      |
| H                      | -0.86960710 | 1.74578916  | 1.45488219 | -NH2  | 5     | 3     | F      |
| H                      | 1.30090798  | -0.80671695 | 2.58780800 | -CH3  | 4     | 3     | F      |
| H                      | 2.12120534  | -0.27430176 | 1.11297643 | -CH3  | 4     | 3     | F      |
| H                      | 1.18863813  | -1.77819294 | 1.11290613 | -CH3  | 4     | 3     | F      |

|                       |             |             |            |       |       |       |        |
|-----------------------|-------------|-------------|------------|-------|-------|-------|--------|
| Fragment# 7, Atoms# 7 |             |             |            |       |       |       |        |
| !Atom                 | X           | Y           | Z          | Ctype | Ftype | Level | Sorted |
| H                     | 0.00000000  | 0.00000000  | 0.00000000 |       | 5     | 0     | T      |
| N                     | -0.00000000 | 0.00000000  | 1.00062635 |       | 9     | 1     | T      |
| C                     | 0.00000000  | 1.35162371  | 1.49976710 |       | 14    | 2     | T      |
| H                     | 0.79799306  | -0.50815411 | 1.32380172 |       | 5     | 2     | T      |
| H                     | 0.00000000  | 1.33667752  | 2.58332361 | -CH3  | 4     | 3     | F      |
| H                     | 0.88478395  | 1.86757977  | 1.14569291 | -CH3  | 4     | 3     | F      |
| H                     | -0.88478388 | 1.86760507  | 1.14562438 | -CH3  | 4     | 3     | F      |

|                       |             |             |             |       |       |       |        |
|-----------------------|-------------|-------------|-------------|-------|-------|-------|--------|
| Fragment# 8, Atoms# 7 |             |             |             |       |       |       |        |
| !Atom                 | X           | Y           | Z           | Ctype | Ftype | Level | Sorted |
| H                     | 0.00000000  | -0.00000000 | -0.00000000 |       | 5     | 0     | T      |
| N                     | -0.00000000 | -0.00000000 | 0.99972788  |       | 9     | 1     | T      |
| C                     | 0.00000000  | 1.34153017  | 1.52539407  |       | 14    | 2     | T      |
| H                     | 0.80471964  | -0.49904836 | 1.32319369  |       | 5     | 2     | T      |
| H                     | 0.00000000  | 1.30524460  | 2.60844599  | -CH3  | 4     | 3     | F      |
| H                     | 0.88478395  | 1.86436016  | 1.18155110  | -CH3  | 4     | 3     | F      |
| H                     | -0.88478388 | 1.86438681  | 1.18148309  | -CH3  | 4     | 3     | F      |

|                       |             |             |             |       |       |       |        |
|-----------------------|-------------|-------------|-------------|-------|-------|-------|--------|
| Fragment# 9, Atoms# 8 |             |             |             |       |       |       |        |
| !Atom                 | X           | Y           | Z           | Ctype | Ftype | Level | Sorted |
| O                     | -0.00000000 | -0.00000000 | 0.00000000  |       | 7     | 0     | T      |
| C                     | -0.00000000 | -0.00000000 | 1.33306652  |       | 34    | 1     | T      |
| H                     | -0.00000000 | 0.87662888  | -0.36063052 |       | 6     | 1     | T      |
| O                     | 0.04287417  | -1.01290903 | 1.93912768  |       | 4     | 2     | T      |
| C                     | -0.13144606 | 1.36309713  | 1.98763767  |       | 10    | 2     | T      |
| H                     | -0.12044877 | 1.24905513  | 3.06522367  | -CH3  | 4     | 3     | F      |
| H                     | -1.06458625 | 1.82199002  | 1.68253358  | -CH3  | 4     | 3     | F      |
| H                     | 0.69680779  | 1.99187626  | 1.68246767  | -CH3  | 4     | 3     | F      |

|                        |             |             |             |       |       |       |        |
|------------------------|-------------|-------------|-------------|-------|-------|-------|--------|
| Fragment# 10, Atoms# 6 |             |             |             |       |       |       |        |
| !Atom                  | X           | Y           | Z           | Ctype | Ftype | Level | Sorted |
| H                      | 0.00000000  | -0.00000000 | -0.00000000 |       | 6     | 0     | T      |
| O                      | 0.00000000  | -0.00000000 | 0.94790958  |       | 7     | 1     | T      |
| C                      | 0.00000000  | 1.23282287  | 1.45507238  |       | 18    | 2     | T      |
| H                      | -0.00000000 | 1.17831758  | 2.53736036  | -CH3  | 4     | 3     | F      |
| H                      | 0.88478395  | 1.76136479  | 1.12007593  | -CH3  | 4     | 3     | F      |
| H                      | -0.88478388 | 1.76139258  | 1.12000837  | -CH3  | 4     | 3     | F      |

```

*****
Molecule atoms symbols
*****
1 7(6(6,1,1)1()1())
2 6(7(1,1)6(8,8)1()1())
3 6(8(1)8()6(7,1,1))
4 8(6(8,6))
5 1(6(7,6,1))
6 1(6(7,6,1))
7 1(7(6,1))

```

```

      8      1(7(6,1))
      9      8(6(8,6)1())
     10      1(8(6))

PROGRAM> end of inputs

Program terminated normally

Job: C2H5N02_C1_RHF_631Gd ended on :24-Aug-18 at 12:50:13
User: ibrahim
Cpu      time: 00h00m04s46c on ibrahim-Lenovo-IdeaPad-P500
Elapsed time: 00h00m05s00c

```

## B.1.2 Saving the Molecular Properties in the Database

Here is example of input file to save the molecular properties of Glycine into the database,

```

MOLECULE
  UNITS=ANGSTROM !(default)
  CHARGE=0
  MULTIPLICITY=1
FREEZ
N
C   1 B1
C   2 B2 1 A2
O   3 B3 2 A3 1 D3
H   2 B4 1 A4 3 D4
H   2 B5 1 A5 3 D5
H   1 B6 2 A6 3 D6
H   1 B7 2 A7 3 D7
O   3 B8 2 A8 1 D8
H   9 B9 3 A9 2 D9

END
DEFINE
B1 = 1.44084286      B2 = 1.51781924      B3 = 1.18115749
B4 = 1.08653973      B5 = 1.09386650      B6 = 1.00062635
B7 = 0.99972788      B8 = 1.33306652      B9 = 0.94790958
A2 = 110.31733       A3 = 123.50442       A4 = 109.68864
A5 = 114.69657       A6 = 110.26867       A7 = 111.39727
A8 = 115.54740       A9 = 112.36145       D3 = -21.55248
D4 = 120.68224       D5 = -119.29373      D6 = 38.90011
D7 = 159.88520       D8 = 161.62252      D9 = -5.50811
END
end !molecule

BASIS name=6-31G(d) end

SET RUN NAME = "RUN_Gly_" end end
!RUN NAME = "RUN_Gly_" end

Partitioning
Scheme=BECKE ! other examples: ABSw, IAWAD, Fermi, iAwad (default is Becke)
end

AIMDFT
Level=2

```



```

! OPT ! Fragments optimization before save in the database (default is NOOPT)
end !AIMDFT

output object=AIMDFT:FRAGMENT%BUILD end
stop

```

Here is the output,

Welcome to MUNgauss - July 9, 2018 Version 7

N\_molecules: 1  
Molecule is an asymmetric top.  
Point group: C1  
Free format Z-Matrix for: C2H5NO2, (C1)

```

N
C      N      B1
C      C      B2 N      A2
O      C      B3 C      A3 N      D3
H      C      B4 N      A4 C      D4
H      C      B5 N      A5 C      D5
H      N      B6 C      A6 C      D6
H      N      B7 C      A7 C      D7
O      C      B8 C      A8 N      D8
H      O      B9 C      A9 C      D9

```

VARIABLES:

```

B1 = 1.44084286      B2 = 1.51781924      B3 = 1.18115749
B4 = 1.08653973      B5 = 1.09386650      B6 = 1.00062635
B7 = 0.99972788      B8 = 1.33306652      B9 = 0.94790958
A2 = 110.31733       A3 = 123.50442       A4 = 109.68864
A5 = 114.69657       A6 = 110.26867       A7 = 111.39727
A8 = 115.54740       A9 = 112.36145       D3 = -21.55248
D4 = 120.68224       D5 = -119.29373     D6 = 38.90011
D7 = 159.88520       D8 = 161.62252     D9 = -5.50811

```

Z MATRIX FOR: C2H5NO2, (C1)

| I   | AN | Z1<br>Z4 | BL       |      | Z2 | ALPHA    |       | Z3 | BETA      |   |
|-----|----|----------|----------|------|----|----------|-------|----|-----------|---|
| 1   | 7  |          |          |      |    |          |       |    |           |   |
| 2   | 6  | 1        | 1.440843 | ( 1) |    |          |       |    |           |   |
| 3   | 6  | 2        | 1.517819 | ( 2) | 1  | 110.3173 | ( 10) |    |           |   |
| 4   | 8  | 3        | 1.181157 | ( 3) | 2  | 123.5044 | ( 11) | 1  | -21.5525  | ( |
| 18) |    | 0        |          |      |    |          |       |    |           |   |
| 5   | 1  | 2        | 1.086540 | ( 4) | 1  | 109.6886 | ( 12) | 3  | 120.6822  | ( |
| 19) |    | 0        |          |      |    |          |       |    |           |   |
| 6   | 1  | 2        | 1.093866 | ( 5) | 1  | 114.6966 | ( 13) | 3  | -119.2937 | ( |
| 20) |    | 0        |          |      |    |          |       |    |           |   |
| 7   | 1  | 1        | 1.000626 | ( 6) | 2  | 110.2687 | ( 14) | 3  | 38.9001   | ( |
| 21) |    | 0        |          |      |    |          |       |    |           |   |
| 8   | 1  | 1        | 0.999728 | ( 7) | 2  | 111.3973 | ( 15) | 3  | 159.8852  | ( |
| 22) |    | 0        |          |      |    |          |       |    |           |   |
| 9   | 8  | 3        | 1.333067 | ( 8) | 2  | 115.5474 | ( 16) | 1  | 161.6225  | ( |
| 23) |    | 0        |          |      |    |          |       |    |           |   |
| 10  | 1  | 9        | 0.947910 | ( 9) | 3  | 112.3615 | ( 17) | 2  | -5.5081   | ( |
| 24) |    | 0        |          |      |    |          |       |    |           |   |

Cartesian coordinates for: C2H5NO2, (C1)

| COORDINATES IN BOHR |    |             |             | COORDINATES IN ANGSTROMS |             |             |  |
|---------------------|----|-------------|-------------|--------------------------|-------------|-------------|--|
| I                   | EL | AN          | X           | Y                        | Z           | X           |  |
|                     |    | Y           | Z           |                          |             |             |  |
| 1                   | N  | 7           | 0.00000000  | 0.00000000               | 0.00000000  | 0.00000000  |  |
|                     |    | 0.00000000  | 0.00000000  |                          |             |             |  |
| 2                   | C  | 6           | 0.00000000  | 0.00000000               | 1.44084286  | 0.00000000  |  |
|                     |    | 0.00000000  | 2.72279820  |                          |             |             |  |
| 3                   | C  | 6           | 1.42338653  | 0.00000000               | 1.96785902  | 2.68981052  |  |
|                     |    | 0.00000000  | 3.71871433  |                          |             |             |  |
| 4                   | O  | 8           | 2.35288801  | -0.36180632              | 1.33520098  | 4.44631363  |  |
|                     |    | -0.68371481 | 2.52316398  |                          |             |             |  |
| 5                   | H  | 1           | -0.52202178 | 0.87980599               | 1.80690743  | -0.98647812 |  |
|                     |    | 1.66259225  | 3.41455992  |                          |             |             |  |
| 6                   | H  | 1           | -0.48626030 | -0.86672789              | 1.89787416  | -0.91889872 |  |
|                     |    | -1.63787822 | 3.58646212  |                          |             |             |  |
| 7                   | H  | 1           | 0.73050930  | -0.58944902              | -0.34663973 | 1.38046242  |  |
|                     |    | -1.11389713 | -0.65505411 |                          |             |             |  |
| 8                   | H  | 1           | -0.87404490 | -0.32011104              | -0.36473314 | -1.65170536 |  |
|                     |    | -0.60492216 | -0.68924570 |                          |             |             |  |
| 9                   | O  | 8           | 1.56620070  | 0.37919230               | 3.23785258  | 2.95969016  |  |
|                     |    | 0.71656954  | 6.11865417  |                          |             |             |  |
| 10                  | H  | 1           | 0.75176248  | 0.68026803               | 3.61808458  | 1.42062510  |  |
|                     |    | 1.28552018  | 6.83718846  |                          |             |             |  |

Nuclear repulsion energy: 181.510743561

Distance Matrix for: C2H5NO2, (C1)

|          | 1        | 2        | 3        | 4        | 5        | 6        |
|----------|----------|----------|----------|----------|----------|----------|
| 7        | 8        | 9        | 10       |          |          |          |
| 1        | 0.000000 | 1.440843 | 2.428682 | 2.729423 | 2.076410 | 2.142333 |
| 1.000626 | 0.999728 | 3.616692 | 3.757452 |          |          |          |
| 2        | 1.440843 | 0.000000 | 1.517819 | 2.382886 | 1.086540 | 1.093866 |
| 2.018957 | 2.031386 | 2.413714 | 2.401727 |          |          |          |
| 3        | 2.428682 | 1.517819 | 0.000000 | 1.181157 | 2.141163 | 2.098301 |
| 2.486852 | 3.289627 | 1.333067 | 1.907115 |          |          |          |
| 4        | 2.729423 | 2.382886 | 1.181157 | 0.000000 | 3.166893 | 2.938079 |
| 2.347876 | 3.647549 | 2.188159 | 2.976757 |          |          |          |
| 5        | 2.076410 | 1.086540 | 2.141163 | 3.166893 | 0.000000 | 1.749267 |
| 2.892285 | 2.505942 | 2.580483 | 2.223219 |          |          |          |
| 6        | 2.142333 | 1.093866 | 2.098301 | 2.938079 | 1.749267 | 0.000000 |
| 2.568123 | 2.359779 | 2.749628 | 2.623932 |          |          |          |
| 7        | 1.000626 | 2.018957 | 2.486852 | 2.347876 | 2.892285 | 2.568123 |
| 0.000000 | 1.627103 | 3.805947 | 4.163132 |          |          |          |
| 8        | 0.999728 | 2.031386 | 3.289627 | 3.647549 | 2.505942 | 2.359779 |
| 1.627103 | 0.000000 | 4.407091 | 4.416655 |          |          |          |
| 9        | 3.616692 | 2.413714 | 1.333067 | 2.188159 | 2.580483 | 2.749628 |
| 3.805947 | 4.407091 | 0.000000 | 0.947910 |          |          |          |
| 10       | 3.757452 | 2.401727 | 1.907115 | 2.976757 | 2.223219 | 2.623932 |
| 4.163132 | 4.416655 | 0.947910 | 0.000000 |          |          |          |

Charge= 0, Number of electrons= 40

The basis set has now been re-ordered FDPS

The basis set has now been re-ordered FDPS

6-31G(d) Basis Set - Total number of basis functions: 85

```

The basis set has now been re-ordered FDPS
Projecting extended Huckel matrix (ST0-3G) to 6-31G(d)
NOTE: INTEGRALS .LE. 1.00E-07 (I2EACC) WERE NOT KEPT
Exponent cutoff used: 2.00E+01 PQCUT2 cutoff used: 1.00E-16

4124847 TWO-ELECTRON INTEGRALS CALCULATED IN IDFCLC (COMBINATIONS)
TOTAL NUMBER OF EACH OF THE 8 TYPES OF INTEGRALS SAVED:
IIKL: 113538 IJKJ: 113807 IJJL: 114903 IIKK: 3799
IJJJ: 1462 IIIL: 1470 IIII: 30 IJKL: 3775838
Number of integrals in INCORE buffers:
IIKK: 2085 IJJL: 64223 IJKJ: 63543
IIKL: 64237 IJKL: 1535810

937565 TWO-ELECTRON INTEGRALS CALCULATED IN ISPCLC (COMBINATIONS)
TOTAL NUMBER OF EACH OF THE 8 TYPES OF INTEGRALS SAVED:
IIKL: 43244 IJKJ: 44015 IJJL: 43286 IIKK: 2740
IJJJ: 1152 IIIL: 1158 IIII: 55 IJKL: 801915
Number of integrals in INCORE buffers:
IIKK: 3570 IJJL: 88212 IJKJ: 87691
IIKL: 88349 IJKL: 271361
Number of buffers:
IJJJ: 0 IJKJ: 0
IIKL: 0 IJKL: 1
TOTAL OF 5062412 TWO-ELECTRON INTEGRALS CALCULATED (COMBINATIONS)

CLOSED SHELL SCF Nuclear Repulsion Energy is 181.510743561 Hartrees
Convergence on Density Matrix Required to Exit is 5.0000E-06

CYCLE ELECTRONIC ENERGY TOTAL ENERGY CONVERGENCE EXTRAPOLATION
SCF_CYCLE: 1 -463.303052879 -281.792309318
SCF_CYCLE: 2 -464.156082549 -282.645338988 2.78967E-02
SCF_CYCLE: 3 -464.252967556 -282.742223996 1.11306E-02
SCF_CYCLE: 4 -464.282711412 -282.771967851 8.78492E-03
SCF_CYCLE: 5 -464.301085806 -282.790342245 6.67293E-03
SCF_CYCLE: 6 -464.310646467 -282.799902906 5.26956E-03
SCF_CYCLE: 7 -464.325561119 -282.814817558 4-POINT
SCF_CYCLE: 8 -464.326970023 -282.816226462 2.39444E-03
SCF_CYCLE: 9 -464.326980754 -282.816237193 1.12426E-04
SCF_CYCLE: 10 -464.326984887 -282.816241326 7.27041E-05
SCF_CYCLE: 11 -464.326986541 -282.816242980 4.60760E-05
SCF_CYCLE: 12 -464.326987992 -282.816244431 4-POINT
SCF_CYCLE: 13 -464.326987698 -282.816244138 1.87641E-05
At termination total energy is -282.816244 Hartrees

Number of indexes available in the database = 0
Number of indexes NOT available in the database = 8
MaxIndex = 114
*****
The unique Symbols in the molecule
*****
SYM# 1 is 7(6(6,1,1)1()1())
SYM# 2 is 6(7(1,1)6(8,8)1()1())
SYM# 3 is 6(8(1)8()6(7,1,1))
SYM# 4 is 8(6(8,6))
SYM# 5 is 1(6(7,6,1))
SYM# 6 is 1(7(6,1))
SYM# 7 is 8(6(8,6)1())
SYM# 8 is 1(8(6))
AIMDFT-AIMDFT-AIMDFT-AIMDFT-AIMDFT-AIMDFT-AIMDFT-AIMDFT

```

\*\*\* RUNing the inputfile :: INPUT\_0115.dat

\*\*\*\*\*

Working on symbol 7(6(6,1,1)1()1())

IndexNUM 115

\*\*\*\*\*

Welcome to MUNgauss - July 9, 2018 Version 7

N\_molecules: 1

Molecule is an asymmetric top.

Point group: C1

Cartesian coordinates for: 7(6(6,1,1)1()1())

| COORDINATES IN BOHR |             |             | COORDINATES IN ANGSTROMS |             |             |  |
|---------------------|-------------|-------------|--------------------------|-------------|-------------|--|
| I EL                | AN          | X           | Y                        | Z           | X           |  |
|                     | Y           | Z           |                          |             |             |  |
| 1 N                 | 7           | 0.00000000  | -0.00000000              | 0.00000000  | 0.00000000  |  |
|                     |             | -0.00000000 | 0.00000000               |             |             |  |
| 2 C                 | 6           | -0.00000000 | -0.00000000              | 1.44084286  | -0.00000000 |  |
|                     |             | -0.00000000 | 2.72279820               |             |             |  |
| 3 H                 | 1           | 0.00000000  | 0.93866607               | -0.34663973 | 0.00000000  |  |
|                     | 1.77382166  | -0.65505411 |                          |             |             |  |
| 4 H                 | 1           | 0.79799306  | -0.47920001              | -0.36473314 | 1.50798822  |  |
|                     | -0.90555672 | -0.68924570 |                          |             |             |  |
| 5 C                 | 6           | -0.89383628 | 1.10773910               | 1.96785902  | -1.68910566 |  |
|                     | 2.09332336  | 3.71871433  |                          |             |             |  |
| 6 H                 | 1           | -0.35689075 | -0.95874622              | 1.80690743  | -0.67442573 |  |
|                     | -1.81176764 | 3.41455992  |                          |             |             |  |
| 7 H                 | 1           | 0.97987823  | 0.16584623               | 1.89787416  | 1.85170135  |  |
|                     | 0.31340393  | 3.58646212  |                          |             |             |  |
| 8 H                 | 1           | -0.88387273 | 1.09539118               | 3.05140244  | -1.67027728 |  |
|                     | 2.06998919  | 5.76631450  |                          |             |             |  |
| 9 H                 | 1           | -1.90660309 | 0.95389867               | 1.61422314  | -3.60295741 |  |
|                     | 1.80260710  | 3.05043943  |                          |             |             |  |
| 10 H                | 1           | -0.52946711 | 2.06514427               | 1.61415464  | -1.00054777 |  |
|                     | 3.90255681  | 3.05030997  |                          |             |             |  |

Nuclear repulsion energy: 83.927218609

Distance Matrix for: 7(6(6,1,1)1()1())

|   | 1        | 2        | 3        | 4        | 5        | 6        |
|---|----------|----------|----------|----------|----------|----------|
| 7 | 8        | 9        | 10       |          |          |          |
| 1 | 0.000000 | 1.440843 | 1.000626 | 0.999728 | 2.428682 | 2.076410 |
|   | 2.142333 | 3.360382 | 2.674093 | 2.674070 |          |          |
| 2 | 1.440843 | 0.000000 | 2.018957 | 2.031386 | 1.517819 | 1.086540 |
|   | 1.093867 | 2.138928 | 2.138953 | 2.138970 |          |          |
| 3 | 1.000626 | 2.018957 | 0.000000 | 1.627103 | 2.486852 | 2.892285 |
|   | 2.568123 | 3.514610 | 2.735023 | 2.322499 |          |          |
| 4 | 0.999728 | 2.031386 | 1.627103 | 0.000000 | 3.289627 | 2.505942 |
|   | 2.359779 | 4.120436 | 3.644843 | 3.485948 |          |          |
| 5 | 2.428682 | 1.517819 | 2.486852 | 3.289627 | 0.000000 | 2.141163 |
|   | 2.098301 | 1.083660 | 1.083707 | 1.083742 |          |          |

```

6      2.076410      1.086540      2.892285      2.505942      2.141163      0.000000
1.749267      2.458853      2.469199      3.034938
7      2.142333      1.093867      2.568123      2.359779      2.098301      1.749267
0.000000      2.380808      3.005538      2.442530
8      3.360382      2.138928      3.514610      4.120436      1.083660      2.458853
2.380808      0.000000      1.769599      1.769663
9      2.674093      2.138953      2.735023      3.644843      1.083707      2.469199
3.005538      1.769599      0.000000      1.769568
10     2.674070      2.138970      2.322499      3.485948      1.083742      3.034938
2.442530      1.769663      1.769568      0.000000

Charge=      0, Number of electrons=      26

The basis set has now been re-ordered FDPs
The basis set has now been re-ordered FDPs
6-31G(d) Basis Set - Total number of basis functions:      59

Partitioning scheme set to: BECKE
Free format Z-Matrix for: 7(6(6,1,1)1()1())
N1
C2      N1      C2N1
H3      N1      H3N1      C2      H3N1C2
H4      N1      H4N1      C2      H4N1C2      H3      H4N1C2H3
C5      C2      C5C2      N1      C5C2N1      H3      C5C2N1H3
H6      C2      H6C2      N1      H6C2N1      H3      H6C2N1H3
H7      C2      H7C2      N1      H7C2N1      H3      H7C2N1H3
H8      C5      H8C5      C2      H8C5C2      N1      H8C5C2N1
H9      C5      H9C5      C2      H9C5C2      N1      H9C5C2N1
H10     C5      H10C5      C2      H10C5C2      N1      H10C5C2N1

VARIABLES:
C2N1      = 1.44084286      H3N1      = 1.00062635      H4N1      = 0.99972788
C5C2      = 1.51781924      H6C2      = 1.08653973      H7C2      = 1.09386650
H8C5      = 1.08365959      H9C5      = 1.08370744      H10C5     = 1.08374183
H3N1C2    = 110.26867      H4N1C2    = 111.39727      C5C2N1    = 110.31733
H6C2N1    = 109.68864      H7C2N1    = 114.69657      H8C5C2    = 109.47840
H9C5C2    = 109.47751      H10C5C2   = 109.47686      H4N1C2H3  = 120.98509
C5C2N1H3  = 38.90011      H6C2N1H3  = 159.58235      H7C2N1H3  = -80.39362
H8C5C2N1  = 180.00000      H9C5C2N1  = 59.99718      H10C5C2N1 = -59.99363

Cartesian coordinates for: 7(6(6,1,1)1()1())
-----
COORDINATES IN BOHR      COORDINATES IN ANGSTROMS
I EL      AN      X      Y      Z      X
Y      Z
-----
1 N1      7      0.00000000      0.00000000      0.00000000      0.00000000
0.00000000      0.00000000
2 C2      6      0.00000000      0.00000000      1.44084286      0.00000000
0.00000000      2.72279820
3 H3      1      0.93866607      0.00000000      -0.34663973      1.77382166
0.00000000      -0.65505411
4 H4      1      -0.47920001      -0.79799306      -0.36473314      -0.90555672
-1.50798822      -0.68924570
5 C5      6      1.10773910      0.89383628      1.96785902      2.09332336
1.68910566      3.71871433
6 H6      1      -0.95874622      0.35689075      1.80690743      -1.81176764
0.67442573      3.41455992
7 H7      1      0.16584623      -0.97987823      1.89787416      0.31340393
-1.85170135      3.58646212

```

|        |            |            |            |            |            |
|--------|------------|------------|------------|------------|------------|
| 8 H8   | 1          | 1.09539118 | 0.88387273 | 3.05140244 | 2.06998919 |
|        | 1.67027728 | 5.76631450 |            |            |            |
| 9 H9   | 1          | 0.95389867 | 1.90660309 | 1.61422314 | 1.80260710 |
|        | 3.60295741 | 3.05043943 |            |            |            |
| 10 H10 | 1          | 2.06514427 | 0.52946711 | 1.61415464 | 3.90255681 |
|        | 1.00054777 | 3.05030997 |            |            |            |

Nuclear repulsion energy: 83.927218609

The basis set has now been re-ordered FDPS  
 Projecting extended Huckel matrix (STO-3G) to 6-31G(d)  
 All integrals will be kept INCORE  
 NOTE: INTEGRALS .LE. 1.00E-07 (I2EACC) WERE NOT KEPT  
 Exponent cutoff used: 2.00E+01 PQCUT2 cutoff used: 1.00E-16

1059473 TWO-ELECTRON INTEGRALS CALCULATED IN IDFCCL (RAW)  
 TOTAL NUMBER OF EACH OF THE 8 TYPES OF INTEGRALS SAVED:  
 IIKL: 36457 IJKJ: 37167 IJJL: 37781 IIKK: 1748  
 IJJJ: 618 IIIL: 618 IIIL: 18 IJKL: 945066

Number of integrals in INCORE buffers:  
 IIKK: 891 IJJL: 19339 IJKJ: 19030  
 IIKL: 18869 IJKL: 330019

337774 TWO-ELECTRON INTEGRALS CALCULATED IN ISPCCL (RAW)  
 TOTAL NUMBER OF EACH OF THE 8 TYPES OF INTEGRALS SAVED:  
 IIKL: 19034 IJKJ: 19531 IJJL: 19062 IIKK: 1589  
 IJJJ: 666 IIIL: 669 IIIL: 41 IJKL: 277182

Number of integrals in INCORE buffers:  
 IIKK: 1711 IJJL: 29218 IJKJ: 29080  
 IIKL: 28741 IJKL: 427733

TOTAL OF 1397247 TWO-ELECTRON INTEGRALS CALCULATED (RAW)

CLOSED SHELL SCF Nuclear Repulsion Energy is 83.927218609 Hartrees  
 Convergence on Density Matrix Required to Exit is 5.0000E-06

|            | CYCLE | ELECTRONIC ENERGY | TOTAL ENERGY   | CONVERGENCE | EXTRAPOLATION |
|------------|-------|-------------------|----------------|-------------|---------------|
| SCF_CYCLE: | 1     | -217.903421853    | -133.976203244 |             |               |
| SCF_CYCLE: | 2     | -218.150803679    | -134.223585070 | 1.47540E-02 |               |
| SCF_CYCLE: | 3     | -218.169358850    | -134.242140241 | 4.55808E-03 |               |
| SCF_CYCLE: | 4     | -218.171840447    | -134.244621838 | 2.08155E-03 |               |
| SCF_CYCLE: | 5     | -218.172279553    | -134.245060944 | 9.12817E-04 |               |
| SCF_CYCLE: | 6     | -218.172372771    | -134.245154162 | 4.87492E-04 |               |
| SCF_CYCLE: | 7     | -218.172394640    | -134.245176031 | 2.28184E-04 |               |
| SCF_CYCLE: | 8     | -218.172400105    | -134.245181496 | 1.25070E-04 |               |
| SCF_CYCLE: | 9     | -218.172404659    | -134.245186051 |             | 4-POINT       |
| SCF_CYCLE: | 10    | -218.172402051    | -134.245183442 | 9.80013E-05 |               |

At termination total energy is -134.245183 Hartrees

Energy components:  
 Kinetic = 134.181219713  
 Potential = -481.008943387  
 Kinetic + Potential = -346.827723674  
 Coulomb repulsion = 148.186452299  
 Exchange = -19.531130678  
 Coulomb+Exchange = 128.655321622  
 Nuclear = 83.927218609  
 Total electronic = -218.172402052  
 Total energy = -134.245183443

Virial = 2.000476697

```

Atom      J      K      Vee      JHF      KHF
VeeHF
  1      38.966053      -1.235830      37.730223      44.377644      -6.647421
37.730223
Atom      Coulomb
  1      46.801541
J_total      46.801541

Atom      Kinetic(x)      Kinetic(y)      Kinetic(z)      Total
  1      17.933375      18.175964      17.900265      54.009604

Ttotal:      17.933375      18.175964      17.900265      54.009604

Atom      Vne
  1      -169.306443
Vne_total:      -169.306443

*****
The fragment with symbol 7(6(6,1,1)1()1())
has been added to the database with index #      115
*****

Atomic properties for atom #      1
+++++
Number of Electrons, N      =      7.1649528615
Pure Exchange, K ( 2K_ab)      =      -1.2358296867
HF Exchange, KHF ( 2K_ab+ Kaa)      =      -6.6474207082
Kinetic energy Numerical, T      =      54.0096039530
Potential Energy Analytical, Vne = -167.9539733452
Potential Energy Numerical, Vne = -169.3064434906
Coulomb Energy Anal/Num, Vee      =      46.8015413684
Pure Coulomb, J ( 4J_ab+ Jaa)      =      38.9660529960
HF Coulomb, JHF ( 4J_ab+ 2Jaa)      =      44.3776440175
Jaa = Kaa      =      5.4115910215
Coulomb Numerically Over A      =      27.8333661426

PROGRAM> end of inputs

Program terminated normally

Job: RUN_FRAG_0115 ended on :24-Aug-18 at 15:37:05
User: ibrahim
Cpu      time: 00h00m12s91c on ibrahim-Lenovo-IdeaPad-P500
Elapsed time: 00h00m13s00c

*** RUNing the inputfile :: INPUT_0116.dat

*****
Working on symbol 6(7(1,1)6(8,8)1()1())
IndexNUM      116
*****

Welcome to MUNgauss - July 9, 2018 Version 7

N_molecules: 1
Molecule is an asymmetric top.

```

Point group: C1  
 Cartesian coordinates for: 6(7(1,1)6(8,8)1()1())

| COORDINATES IN BOHR |    |             |             | COORDINATES IN ANGSTROMS |             |             |  |
|---------------------|----|-------------|-------------|--------------------------|-------------|-------------|--|
| I                   | EL | AN          | X           | Y                        | Z           | X           |  |
|                     |    | Y           | Z           |                          |             |             |  |
| 1                   | C  | 6           | -0.00000000 | 0.00000000               | 0.00000000  | -0.00000000 |  |
|                     |    | 0.00000000  | 0.00000000  |                          |             |             |  |
| 2                   | N  | 7           | 0.00000000  | -0.00000000              | 1.44084286  | 0.00000000  |  |
|                     |    | -0.00000000 | 2.72279820  |                          |             |             |  |
| 3                   | C  | 6           | 0.00000000  | 1.42338653               | -0.52701616 | 0.00000000  |  |
|                     |    | 2.68981052  | -0.99591613 |                          |             |             |  |
| 4                   | H  | 1           | 0.87980599  | -0.52202178              | -0.36606457 | 1.66259225  |  |
|                     |    | -0.98647812 | -0.69176172 |                          |             |             |  |
| 5                   | H  | 1           | -0.86672789 | -0.48626030              | -0.45703130 | -1.63787822 |  |
|                     |    | -0.91889872 | -0.86366392 |                          |             |             |  |
| 6                   | O  | 8           | 0.37919230  | 1.56620070               | -1.79700972 | 0.71656954  |  |
|                     |    | 2.95969016  | -3.39585598 |                          |             |             |  |
| 7                   | O  | 8           | -0.36180632 | 2.35288801               | 0.10564188  | -0.68371481 |  |
|                     |    | 4.44631363  | 0.19963421  |                          |             |             |  |
| 8                   | H  | 1           | -0.58944902 | 0.73050930               | 1.78748259  | -1.11389713 |  |
|                     |    | 1.38046242  | 3.37785231  |                          |             |             |  |
| 9                   | H  | 1           | -0.32011104 | -0.87404490              | 1.80557600  | -0.60492216 |  |
|                     |    | -1.65170536 | 3.41204390  |                          |             |             |  |
| 10                  | H  | 1           | 0.31941883  | 2.53932814               | -1.96545672 | 0.60361407  |  |
|                     |    | 4.79863437  | -3.71417465 |                          |             |             |  |

Nuclear repulsion energy: 181.273968331

Distance Matrix for: 6(7(1,1)6(8,8)1()1())

|    | 1        | 2        | 3        | 4        | 5        | 6        |
|----|----------|----------|----------|----------|----------|----------|
| 7  | 8        | 9        | 10       |          |          |          |
| 1  | 0.000000 | 1.440843 | 1.517819 | 1.086540 | 1.093866 | 2.413714 |
|    | 2.382886 | 2.018957 | 2.031386 | 3.226955 |          |          |
| 2  | 1.440843 | 0.000000 | 2.428682 | 2.076410 | 2.142333 | 3.616692 |
|    | 2.729423 | 1.000626 | 0.999728 | 4.260645 |          |          |
| 3  | 1.517819 | 2.428682 | 0.000000 | 2.141163 | 2.098301 | 1.333067 |
|    | 1.181157 | 2.486852 | 3.289627 | 1.848368 |          |          |
| 4  | 1.086540 | 2.076410 | 2.141163 | 0.000000 | 1.749267 | 2.580483 |
|    | 3.166893 | 2.892285 | 2.505942 | 3.499136 |          |          |
| 5  | 1.093866 | 2.142333 | 2.098301 | 1.749267 | 0.000000 | 2.749628 |
|    | 2.938079 | 2.568123 | 2.359779 | 3.582803 |          |          |
| 6  | 2.413714 | 3.616692 | 1.333067 | 2.580483 | 2.749628 | 0.000000 |
|    | 2.188159 | 3.805947 | 4.407091 | 0.989406 |          |          |
| 7  | 2.382886 | 2.729423 | 1.181157 | 3.166893 | 2.938079 | 2.188159 |
|    | 0.000000 | 2.347876 | 3.647549 | 2.188213 |          |          |
| 8  | 2.018957 | 1.000626 | 2.486852 | 2.892285 | 2.568123 | 3.805947 |
|    | 2.347876 | 0.000000 | 1.627103 | 4.264085 |          |          |
| 9  | 2.031386 | 0.999728 | 3.289627 | 2.505942 | 2.359779 | 4.407091 |
|    | 3.647549 | 1.627103 | 0.000000 | 5.126480 |          |          |
| 10 | 3.226955 | 4.260645 | 1.848368 | 3.499136 | 3.582803 | 0.989406 |
|    | 2.188213 | 4.264085 | 5.126480 | 0.000000 |          |          |

Charge= 0, Number of electrons= 40

The basis set has now been re-ordered FDPS  
 The basis set has now been re-ordered FDPS



```

6-31G(d) Basis Set - Total number of basis functions:      85

Partitioning scheme set to: BECKE
Free format Z-Matrix for: 6(7(1,1)6(8,8)1()1())
C1
N2      C1      N2C1
C3      C1      C3C1      N2      C3C1N2
H4      C1      H4C1      N2      H4C1N2      C3      H4C1N2C3
H5      C1      H5C1      N2      H5C1N2      C3      H5C1N2C3
O6      C3      O6C3      C1      O6C3C1      N2      O6C3C1N2
O7      C3      O7C3      C1      O7C3C1      N2      O7C3C1N2
H8      N2      H8N2      C1      H8N2C1      C3      H8N2C1C3
H9      N2      H9N2      C1      H9N2C1      C3      H9N2C1C3
H10     O6      H10O6     C3      H10O6C3     C1      H10O6C3C1

VARIABLES:
N2C1      = 1.44084286      C3C1      = 1.51781924      H4C1      = 1.08653973
H5C1      = 1.09386650      O6C3      = 1.33306652      O7C3      = 1.18115749
H8N2      = 1.00062635      H9N2      = 0.99972788      H10O6     = 0.98940602
C3C1N2     = 110.31733      H4C1N2     = 109.68864      H5C1N2     = 114.69657
O6C3C1     = 115.54740      O7C3C1     = 123.50442      H8N2C1     = 110.26867
H9N2C1     = 111.39727      H10O6C3    = 104.50001      H4C1N2C3   = 120.68224
H5C1N2C3   = -119.29373      O6C3C1N2   = 161.62252      O7C3C1N2   = -21.55248
H8N2C1C3   = 38.90011      H9N2C1C3   = 159.88520      H10O6C3C1  = -180.00000

Cartesian coordinates for: 6(7(1,1)6(8,8)1()1())
-----

```

| COORDINATES IN BOHR |     |    |             | COORDINATES IN ANGSTROMS |             |             |   |
|---------------------|-----|----|-------------|--------------------------|-------------|-------------|---|
| I                   | EL  | AN | X           | Y                        | Z           | X           | Y |
| 1                   | C1  | 6  | 0.00000000  | 0.00000000               | 0.00000000  | 0.00000000  |   |
| 2                   | N2  | 7  | 0.00000000  | 0.00000000               | 1.44084286  | 0.00000000  |   |
| 3                   | C3  | 6  | 1.42338653  | 0.00000000               | -0.52701616 | 2.68981052  |   |
| 4                   | H4  | 1  | -0.52202178 | -0.87980599              | -0.36606457 | -0.98647812 |   |
| 5                   | H5  | 1  | -0.48626030 | 0.86672789               | -0.45703130 | -0.91889872 |   |
| 6                   | O6  | 8  | 1.56620070  | -0.37919230              | -1.79700972 | 2.95969016  |   |
| 7                   | O7  | 8  | 2.35288801  | 0.36180632               | 0.10564188  | 4.44631363  |   |
| 8                   | H8  | 1  | 0.73050930  | 0.58944902               | 1.78748259  | 1.38046242  |   |
| 9                   | H9  | 1  | -0.87404490 | 0.32011104               | 1.80557600  | -1.65170536 |   |
| 10                  | H10 | 1  | 2.53932814  | -0.31941883              | -1.96545672 | 4.79863437  |   |

```

-----
Nuclear repulsion energy:      181.273968331

The basis set has now been re-ordered FDPS
Projecting extended Huckel matrix (STO-3G) to 6-31G(d)
NOTE: INTEGRALS .LE. 1.00E-07 (I2EACC) WERE NOT KEPT
Exponent cutoff used: 2.00E+01 PQCUT2 cutoff used: 1.00E-16

```

```

4104066 TWO-ELECTRON INTEGRALS CALCULATED IN IDFCLC (RAW)
TOTAL NUMBER OF EACH OF THE 8 TYPES OF INTEGRALS SAVED:
IIKL:      113140 IJKJ:      113572 IJJL:      114756 IIKK:      3793
IJJJ:      1462 IIIL:      1470 IIII:      30 IJKL:      3755843
Number of integrals in INCORE buffers:
IIKK:      2085 IJJL:      64260 IJKJ:      63526
IIKL:      64095 IJKL:      1534076

930964 TWO-ELECTRON INTEGRALS CALCULATED IN ISPCLC (RAW)
TOTAL NUMBER OF EACH OF THE 8 TYPES OF INTEGRALS SAVED:
IIKL:      43052 IJKJ:      43862 IJJL:      43150 IIKK:      2734
IJJJ:      1151 IIIL:      1157 IIII:      55 IJKL:      795803
Number of integrals in INCORE buffers:
IIKK:      3570 IJJL:      88256 IJKJ:      87663
IIKL:      88149 IJKL:      269361
Number of buffers:
IJJJ:      0 IJKJ:      0
IIKL:      0 IJKL:      1
TOTAL OF      5035030 TWO-ELECTRON INTEGRALS CALCULATED (RAW)

CLOSED SHELL SCF                      Nuclear Repulsion Energy is      181.273968331 Hartrees
Convergence on Density Matrix Required to Exit is  5.0000E-06

      CYCLE      ELECTRONIC ENERGY      TOTAL ENERGY      CONVERGENCE      EXTRAPOLATION
SCF_CYCLE:  1      -463.075382663      -281.801414332
SCF_CYCLE:  2      -463.938846750      -282.664878419      2.63503E-02
SCF_CYCLE:  3      -464.033491357      -282.759523025      1.04825E-02
SCF_CYCLE:  4      -464.061188697      -282.787220365      8.09175E-03
SCF_CYCLE:  5      -464.077105997      -282.803137666      6.06597E-03
SCF_CYCLE:  6      -464.085279580      -282.811311249      4.81216E-03
SCF_CYCLE:  7      -464.098916367      -282.824948035
SCF_CYCLE:  8      -464.098969593      -282.825001262      2.15105E-03
SCF_CYCLE:  9      -464.098999780      -282.825031449      1.85118E-04
SCF_CYCLE: 10      -464.099011460      -282.825043129      1.17512E-04
SCF_CYCLE: 11      -464.099021079      -282.825052747
SCF_CYCLE: 12      -464.099019232      -282.825050900      4.77928E-05
At termination total energy is      -282.825051 Hartrees

Energy components:
Kinetic =      282.246594234
Potential =      -1028.372855515
Kinetic + Potential =      -746.126261281
Coulomb repulsion =      317.344749503
Exchange =      -35.317507461
Coulomb+Exchange =      282.027242042
Nuclear =      181.273968331
Total electronic =      -464.099019239
Total energy =      -282.825050908

Virial =      2.002049473

Atom      J      K      Vee      JHF      KHF
VeeHF
  1      43.478567      -0.933213      42.545354      47.819244      -5.273891
42.545354
Atom      Coulomb
  1      48.232671
J_total      48.232671

Atom      Kinetic(x)      Kinetic(y)      Kinetic(z)      Total
  1      12.644472      12.655040      12.669234      37.968746

```

Ttotal: 12.644472 12.655040 12.669234 37.968746

Atom Vne  
1 -149.312739  
Vne\_total: -149.312739

\*\*\*\*\*  
The fragment with symbol 6(7(1,1)6(8,8)1()1())  
has been added to the database with index # 116  
\*\*\*\*\*

Atomic properties for atom # 1  
\*\*\*\*\*  
Number of Electrons, N = 6.1012052215  
Pure Exchange, K ( 2K\_ab) = -0.9332131891  
HF Exchange, KHF ( 2K\_ab+ Kaa) = -5.2738907810  
Kinetic energy Numerical, T = 37.9687460325  
Potential Energy Analytical, Vne = -147.1754484683  
Potential Energy Numerical, Vne = -149.3127391196  
Coulomb Energy Anal/Num, Vee = 48.2326712819  
Pure Coulomb, J ( 4J\_ab+ Jaa) = 43.4785667800  
HF Coulomb, JHF ( 4J\_ab+ 2Jaa) = 47.8192443719  
Jaa = Kaa = 4.3406775919  
Coulomb Numerically Over A = 19.3994839860

PROGRAM> end of inputs

Program terminated normally

Job: RUN\_FRAG\_0116 ended on :24-Aug-18 at 15:37:44  
User: ibrahim  
Cpu time: 00h00m38s54c on ibrahim-Lenovo-IdeaPad-P500  
Elapsed time: 00h00m39s00c

\*\*\* RUNing the inputfile :: INPUT\_0117.dat

\*\*\*\*\*  
Working on symbol 6(8(1)8()6(7,1,1))  
IndexNUM 117  
\*\*\*\*\*

Welcome to MUNgauss - July 9, 2018 Version 7

N\_molecules: 1  
Molecule is an asymmetric top.  
Point group: C1  
Cartesian coordinates for: 6(8(1)8()6(7,1,1))

-----  
COORDINATES IN BOHR COORDINATES IN ANGSTROMS  
I EL AN X Y Z X  
Y Z  
-----

|    |   |   |             |             |             |             |
|----|---|---|-------------|-------------|-------------|-------------|
| 1  | C | 6 | -0.00000000 | -0.00000000 | 0.00000000  | -0.00000000 |
|    |   |   | -0.00000000 | 0.00000000  |             |             |
| 2  | O | 8 | -0.00000000 | 0.00000000  | 1.33306652  | -0.00000000 |
|    |   |   | 0.00000000  | 2.51913045  |             |             |
| 3  | O | 8 | 0.00000000  | 1.01381600  | -0.60606116 | 0.00000000  |
|    |   |   | 1.91583445  | -1.14528952 |             |             |
| 4  | C | 6 | 0.07368323  | -1.36743652 | -0.65457115 | 0.13924111  |
|    |   |   | -2.58408033 | -1.23696012 |             |             |
| 5  | N | 7 | -0.35716835 | -1.28888329 | -2.02724171 | -0.67495031 |
|    |   |   | -2.43563626 | -3.83093134 |             |             |
| 6  | H | 1 | -0.03707255 | -0.87584463 | 1.69369704  | -0.07005695 |
|    |   |   | -1.65510637 | 3.20062332  |             |             |
| 7  | H | 1 | -0.55932392 | -2.06380818 | -0.11149008 | -1.05696894 |
|    |   |   | -3.90003195 | -0.21068570 |             |             |
| 8  | H | 1 | 1.10434945  | -1.70738451 | -0.51779972 | 2.08691785  |
|    |   |   | -3.22648888 | -0.97849960 |             |             |
| 9  | H | 1 | -0.29008929 | -2.21219942 | -2.41194164 | -0.54818927 |
|    |   |   | -4.18045073 | -4.55790880 |             |             |
| 10 | H | 1 | 0.30676556  | -0.71652891 | -2.51368937 | 0.57970285  |
|    |   |   | -1.35404331 | -4.75018412 |             |             |

Nuclear repulsion energy: 181.310954311

Distance Matrix for: 6(8(1)8(1)6(7,1,1))

|    | 1        | 2        | 3        | 4        | 5        | 6        |
|----|----------|----------|----------|----------|----------|----------|
| 7  | 8        | 9        | 10       |          |          |          |
| 1  | 0.000000 | 1.333067 | 1.181157 | 1.517819 | 2.428682 | 1.907115 |
| 2  | 1.141163 | 2.098301 | 3.285642 | 2.631759 |          |          |
| 3  | 1.333067 | 0.000000 | 2.188159 | 2.413714 | 3.616692 | 0.947910 |
| 4  | 2.580483 | 2.749628 | 4.359250 | 3.924927 |          |          |
| 5  | 1.181157 | 2.188159 | 0.000000 | 2.382886 | 2.729423 | 2.976757 |
| 6  | 3.166893 | 2.938079 | 3.708441 | 2.593693 |          |          |
| 7  | 1.517819 | 2.413714 | 2.382886 | 0.000000 | 1.440843 | 2.401727 |
| 8  | 1.086540 | 1.093866 | 1.983508 | 1.983514 |          |          |
| 9  | 2.428682 | 3.616692 | 2.729423 | 1.440843 | 0.000000 | 3.757452 |
| 10 | 2.076410 | 2.142333 | 1.002500 | 1.002511 |          |          |
| 1  | 1.907115 | 0.947910 | 2.976757 | 2.401727 | 3.757452 | 0.000000 |
| 2  | 2.223219 | 2.623932 | 4.325058 | 4.224418 |          |          |
| 3  | 2.141163 | 2.580483 | 3.166893 | 1.086540 | 2.076410 | 2.223219 |
| 4  | 0.000000 | 1.749267 | 2.320902 | 2.887184 |          |          |
| 5  | 2.098301 | 2.749628 | 2.938079 | 1.093866 | 2.142333 | 2.623932 |
| 6  | 1.749267 | 0.000000 | 2.405633 | 2.366751 |          |          |
| 7  | 3.285642 | 4.359250 | 3.708441 | 1.983508 | 1.002500 | 4.325058 |
| 8  | 2.320902 | 2.405633 | 0.000000 | 1.613573 |          |          |
| 9  | 2.631759 | 3.924927 | 2.593693 | 1.983514 | 1.002511 | 4.224418 |
| 10 | 2.887184 | 2.366751 | 1.613573 | 0.000000 |          |          |

Charge= 0, Number of electrons= 40

The basis set has now been re-ordered FDPs

The basis set has now been re-ordered FDPs

6-31G(d) Basis Set - Total number of basis functions: 85

Partitioning scheme set to: BECKE

Free format Z-Matrix for: 6(8(1)8(1)6(7,1,1))

|    |    |      |    |        |    |          |
|----|----|------|----|--------|----|----------|
| C1 |    |      |    |        |    |          |
| O2 | C1 | O2C1 |    |        |    |          |
| O3 | C1 | O3C1 | O2 | O3C1O2 |    |          |
| C4 | C1 | C4C1 | O2 | C4C1O2 | O3 | C4C1O2O3 |
| N5 | C4 | N5C4 | C1 | N5C4C1 | O2 | N5C4C1O2 |
| H6 | O2 | H6O2 | C1 | H6O2C1 | O3 | H6O2C1O3 |
| H7 | C4 | H7C4 | C1 | H7C4C1 | O2 | H7C4C1O2 |

|     |    |       |    |         |    |           |
|-----|----|-------|----|---------|----|-----------|
| H8  | C4 | H8C4  | C1 | H8C4C1  | O2 | H8C4C1O2  |
| H9  | N5 | H9N5  | C4 | H9N5C4  | C1 | H9N5C4C1  |
| H10 | N5 | H10N5 | C4 | H10N5C4 | C1 | H10N5C4C1 |

VARIABLES:

|          |   |            |          |   |            |           |   |            |
|----------|---|------------|----------|---|------------|-----------|---|------------|
| O2C1     | = | 1.33306652 | O3C1     | = | 1.18115749 | C4C1      | = | 1.51781924 |
| N5C4     | = | 1.44084286 | H6O2     | = | 0.94790958 | H7C4      | = | 1.08653973 |
| H8C4     | = | 1.09386650 | H9N5     | = | 1.00250002 | H10N5     | = | 1.00251139 |
| O3C1O2   | = | 120.87106  | C4C1O2   | = | 115.54740  | N5C4C1    | = | 110.31733  |
| H6O2C1   | = | 112.36145  | H7C4C1   | = | 109.48571  | H8C4C1    | = | 105.77163  |
| H9N5C4   | = | 107.17931  | H10N5C4  | = | 107.17911  | C4C1O2O3  | = | 176.91565  |
| N5C4C1O2 | = | 161.62252  | H6O2C1O3 | = | 177.57624  | H7C4C1O2  | = | 40.81887   |
| H8C4C1O2 | = | -73.79898  | H9N5C4C1 | = | -180.00000 | H10N5C4C1 | = | 65.22155   |

Cartesian coordinates for: 6(8(1)8(1)6(7,1,1))

| COORDINATES IN BOHR |     |             | COORDINATES IN ANGSTROMS |             |             |             |
|---------------------|-----|-------------|--------------------------|-------------|-------------|-------------|
| I                   | EL  | AN          | X                        | Y           | Z           | X           |
|                     |     | Y           | Z                        |             |             |             |
| 1                   | C1  | 6           | 0.00000000               | 0.00000000  | 0.00000000  | 0.00000000  |
|                     |     | 0.00000000  | 0.00000000               |             |             |             |
| 2                   | O2  | 8           | 0.00000000               | 0.00000000  | 1.33306652  | 0.00000000  |
|                     |     | 0.00000000  | 2.51913045               |             |             |             |
| 3                   | O3  | 8           | 1.01381600               | 0.00000000  | -0.60606116 | 1.91583445  |
|                     |     | 0.00000000  | -1.14528952              |             |             |             |
| 4                   | C4  | 6           | -1.36743652              | -0.07368323 | -0.65457115 | -2.58408033 |
|                     |     | -0.13924111 | -1.23696012              |             |             |             |
| 5                   | N5  | 7           | -1.28888329              | 0.35716835  | -2.02724171 | -2.43563626 |
|                     |     | 0.67495031  | -3.83093134              |             |             |             |
| 6                   | H6  | 1           | -0.87584463              | 0.03707255  | 1.69369704  | -1.65510637 |
|                     |     | 0.07005695  | 3.20062332               |             |             |             |
| 7                   | H7  | 1           | -2.06380818              | 0.55932392  | -0.11149008 | -3.90003195 |
|                     |     | 1.05696894  | -0.21068570              |             |             |             |
| 8                   | H8  | 1           | -1.70738451              | -1.10434945 | -0.51779972 | -3.22648888 |
|                     |     | -2.08691785 | -0.97849960              |             |             |             |
| 9                   | H9  | 1           | -2.21219942              | 0.29008929  | -2.41194164 | -4.18045073 |
|                     |     | 0.54818927  | -4.55790880              |             |             |             |
| 10                  | H10 | 1           | -0.71652891              | -0.30676556 | -2.51368937 | -1.35404331 |
|                     |     | -0.57970285 | -4.75018412              |             |             |             |

Nuclear repulsion energy: 181.310954311

The basis set has now been re-ordered FDPs

Projecting extended Huckel matrix (STO-3G) to 6-31G(d)

NOTE: INTEGRALS .LE. 1.00E-07 (I2EACC) WERE NOT KEPT

Exponent cutoff used: 2.00E+01 PQCUT2 cutoff used: 1.00E-16

4014630 TWO-ELECTRON INTEGRALS CALCULATED IN IDFCLC (RAW)

TOTAL NUMBER OF EACH OF THE 8 TYPES OF INTEGRALS SAVED:

|       |        |       |        |       |        |       |         |
|-------|--------|-------|--------|-------|--------|-------|---------|
| IIKL: | 111868 | IJKJ: | 112757 | IJJL: | 113371 | IIKK: | 3774    |
| IJJJ: | 1448   | IIIL: | 1461   | IIII: | 30     | IJKL: | 3669921 |

Number of integrals in INCORE buffers:

|       |       |       |         |       |       |
|-------|-------|-------|---------|-------|-------|
| IIKK: | 2085  | IJJL: | 63881   | IJKJ: | 63331 |
| IIKL: | 63943 | IJKL: | 1515284 |       |       |

918968 TWO-ELECTRON INTEGRALS CALCULATED IN ISPCLC (RAW)

TOTAL NUMBER OF EACH OF THE 8 TYPES OF INTEGRALS SAVED:

|       |       |       |       |       |       |       |      |
|-------|-------|-------|-------|-------|-------|-------|------|
| IIKL: | 42900 | IJKJ: | 43766 | IJJL: | 43075 | IIKK: | 2721 |
|-------|-------|-------|-------|-------|-------|-------|------|

```

IJJJ:      1151 IJIL:      1156 IIII:      55 IJKL:      784144
Number of integrals in INCORE buffers:
IIKK:      3570 IJJL:      87863 IJKJ:      87429
IIKL:      88004 IJKL:      247089
Number of buffers:
IJJL:      0 IJKJ:      0
IIKL:      0 IJKL:      1
TOTAL OF      4933598 TWO-ELECTRON INTEGRALS CALCULATED (RAW)

CLOSED SHELL SCF          Nuclear Repulsion Energy is      181.310954311 Hartrees
Convergence on Density Matrix Required to Exit is 5.0000E-06

      CYCLE      ELECTRONIC ENERGY      TOTAL ENERGY      CONVERGENCE      EXTRAPOLATION
SCF_CYCLE: 1      -463.088972091      -281.778017781
SCF_CYCLE: 2      -463.951585158      -282.640630848      2.82205E-02
SCF_CYCLE: 3      -464.049586122      -282.738631811      1.12631E-02
SCF_CYCLE: 4      -464.079700052      -282.768745742      8.95038E-03
SCF_CYCLE: 5      -464.098442900      -282.787488590      6.79317E-03
SCF_CYCLE: 6      -464.108208348      -282.797254037      5.37894E-03
SCF_CYCLE: 7      -464.123557409      -282.812603099
SCF_CYCLE: 8      -464.125156274      -282.814201963      2.45268E-03      4-POINT
SCF_CYCLE: 9      -464.125167209      -282.814212898      1.14106E-04
SCF_CYCLE: 10     -464.125171428      -282.814217117      7.40949E-05
SCF_CYCLE: 11     -464.125173119      -282.814218809      4.69002E-05
SCF_CYCLE: 12     -464.125174638      -282.814220328
SCF_CYCLE: 13     -464.125174309      -282.814219999      1.92028E-05      4-POINT
At termination total energy is      -282.814220 Hartrees

Energy components:
Kinetic =      282.320136597
Potential =     -1028.401570033
Kinetic + Potential =     -746.081433436
Coulomb repulsion =      317.284416897
Exchange =     -35.328157773
Coulomb+Exchange =      281.956259124
Nuclear =      181.310954311
Total electronic =     -464.125174312
Total energy =     -282.814220001

Virial =      2.001750082

Atom      J      K      Vee      JHF      KHF
VeeHF
1      44.469534      -0.959412      43.510122      48.885473      -5.375351
43.510122
Atom      Coulomb
1      53.046478
J_total      53.046478

Atom      Kinetic(x)      Kinetic(y)      Kinetic(z)      Total
1      12.923600      12.858647      12.914850      38.697097

Ttotal:      12.923600      12.858647      12.914850      38.697097

Atom      Vne
1      -159.522455
Vne_total:      -159.522455

*****
The fragment with symbol 6(8(1)8( )6(7,1,1))
has been added to the database with index #      117

```

```

*****

Atomic properties for atom #      1
+++++
Number of Electrons, N           =    6.1259220110
Pure Exchange, K ( 2K_ab)       =   -0.9594119713
HF Exchange, KHF ( 2K_ab+ Kaa)  =   -5.3753513417
Kinetic energy Numerical, T      =    38.6970966577
Potential Energy Analytical, Vne =  -156.5604713149
Potential Energy Numerical, Vne  = -159.5224545210
Coulomb Energy Anal/Num, Vee     =    53.0464780861
Pure Coulomb, J ( 4J_ab+ Jaa)   =    44.4695336088
HF Coulomb, JHF ( 4J_ab+ 2Jaa)  =    48.8854729793
Jaa = Kaa                        =    4.4159393704
Coulomb Numerically Over A      =    19.3435272796

PROGRAM> end of inputs

Program terminated normally

Job: RUN_FRAG_0117 ended on :24-Aug-18 at 15:38:24
User: ibrahim
Cpu    time: 00h00m39s51c on ibrahim-Lenovo-IdeaPad-P500
Elapsed time: 00h00m40s00c

*** RUNing the inputfile :: INPUT_0118.dat

*****
Working on symbol 8(6(8,6))
IndexNUM      118
*****

Welcome to MUNgauss - July 9, 2018 Version 7

N_molecules: 1
Molecule is an asymmetric top.
Point group: C1
Cartesian coordinates for: 8(6(8,6))

-----

```

| COORDINATES IN BOHR |    |             |             | COORDINATES IN ANGSTROMS |             |             |
|---------------------|----|-------------|-------------|--------------------------|-------------|-------------|
| I                   | EL | AN          | X           | Y                        | Z           | X           |
|                     |    | Y           | Z           |                          |             |             |
| 1                   | O  | 8           | -0.00000000 | 0.00000000               | -0.00000000 | -0.00000000 |
|                     |    | 0.00000000  | -0.00000000 |                          |             |             |
| 2                   | C  | 6           | -0.00000000 | -0.00000000              | 1.18115749  | -0.00000000 |
|                     |    | -0.00000000 | 2.23206401  |                          |             |             |
| 3                   | O  | 8           | -0.00000000 | 1.14420319               | 1.86516436  | -0.00000000 |
|                     |    | 2.16223051  | 3.52464956  |                          |             |             |
| 4                   | C  | 6           | 0.07368323  | -1.26347662              | 2.01899570  | 0.13924111  |
|                     |    | -2.38762460 | 3.81534865  |                          |             |             |
| 5                   | H  | 1           | 0.00000000  | 0.86533235               | 2.81445637  | 0.00000000  |
|                     |    | 1.63524104  | 5.31855135  |                          |             |             |

|     |             |             |             |            |             |
|-----|-------------|-------------|-------------|------------|-------------|
| 6 H | 1           | 0.05839270  | -1.00128368 | 3.07034682 | 0.11034620  |
|     | -1.89215179 | 5.80211417  |             |            |             |
| 7 H | 1           | 0.99092661  | -1.79429459 | 1.79245948 | 1.87257978  |
|     | -3.39072511 | 3.38725726  |             |            |             |
| 8 H | 1           | -0.77563740 | -1.89735718 | 1.79239856 | -1.46574214 |
|     | -3.58548516 | 3.38714215  |             |            |             |

Nuclear repulsion energy: 122.224021925

Distance Matrix for: 8(6(8,6))

|   | 1        | 2        | 3        | 4        | 5        | 6        |
|---|----------|----------|----------|----------|----------|----------|
| 7 |          |          |          |          |          |          |
| 8 |          |          |          |          |          |          |
| 1 | 0.000000 | 1.181157 | 2.188159 | 2.382886 | 2.944480 | 3.230017 |
| 2 | 2.722928 | 2.722916 |          |          |          |          |
| 2 | 1.181157 | 0.000000 | 1.333067 | 1.517819 | 1.848368 | 2.138928 |
| 2 | 2.138953 | 2.138970 |          |          |          |          |
| 3 | 2.188159 | 1.333067 | 0.000000 | 2.413714 | 0.989406 | 2.461501 |
| 3 | 3.101933 | 3.139745 |          |          |          |          |
| 4 | 2.382886 | 1.517819 | 2.413714 | 0.000000 | 2.273767 | 1.083660 |
| 4 | 1.083707 | 1.083742 |          |          |          |          |
| 5 | 2.944480 | 1.848368 | 0.989406 | 2.273767 | 0.000000 | 1.884979 |
| 5 | 3.016625 | 3.046091 |          |          |          |          |
| 6 | 3.230017 | 2.138928 | 2.461501 | 1.083660 | 1.884979 | 0.000000 |
| 6 | 1.769599 | 1.769663 |          |          |          |          |
| 7 | 2.722928 | 2.138953 | 3.101933 | 1.083707 | 3.016625 | 1.769599 |
| 7 | 0.000000 | 1.769568 |          |          |          |          |
| 8 | 2.722916 | 2.138970 | 3.139745 | 1.083742 | 3.046091 | 1.769663 |
| 8 | 1.769568 | 0.000000 |          |          |          |          |

Charge= 0, Number of electrons= 32

The basis set has now been re-ordered FDPS

The basis set has now been re-ordered FDPS

6-31G(d) Basis Set - Total number of basis functions: 68

Partitioning scheme set to: BECKE

Free format Z-Matrix for: 8(6(8,6))

|    |    |      |    |        |    |          |
|----|----|------|----|--------|----|----------|
| O1 |    |      |    |        |    |          |
| C2 | O1 | C2O1 |    |        |    |          |
| O3 | C2 | O3C2 | O1 | O3C2O1 |    |          |
| C4 | C2 | C4C2 | O1 | C4C2O1 | O3 | C4C2O1O3 |
| H5 | O3 | H5O3 | C2 | H5O3C2 | O1 | H5O3C2O1 |
| H6 | C4 | H6C4 | C2 | H6C4C2 | O1 | H6C4C2O1 |
| H7 | C4 | H7C4 | C2 | H7C4C2 | O1 | H7C4C2O1 |
| H8 | C4 | H8C4 | C2 | H8C4C2 | O1 | H8C4C2O1 |

VARIABLES:

|          |              |          |              |          |              |
|----------|--------------|----------|--------------|----------|--------------|
| C2O1     | = 1.18115749 | O3C2     | = 1.33306652 | C4C2     | = 1.51781924 |
| H5O3     | = 0.98940602 | H6C4     | = 1.08365959 | H7C4     | = 1.08370744 |
| H8C4     | = 1.08374183 | O3C2O1   | = 120.87106  | C4C2O1   | = 123.50442  |
| H5O3C2   | = 104.50001  | H6C4C2   | = 109.47840  | H7C4C2   | = 109.47751  |
| H8C4C2   | = 109.47686  | C4C2O1O3 | = -176.66241 | H5O3C2O1 | = -180.00000 |
| H6C4C2O1 | = -180.00000 | H7C4C2O1 | = 59.99718   | H8C4C2O1 | = -59.99363  |

Cartesian coordinates for: 8(6(8,6))

| COORDINATES IN BOHR |    |    | COORDINATES IN ANGSTROMS |   |   |
|---------------------|----|----|--------------------------|---|---|
| I                   | EL | AN | X                        | Y | Z |
|                     |    | Y  |                          |   | X |
|                     |    | Z  |                          |   |   |



|   |    |   |             |             |            |             |
|---|----|---|-------------|-------------|------------|-------------|
| 1 | O1 | 8 | 0.00000000  | 0.00000000  | 0.00000000 | 0.00000000  |
|   |    |   | 0.00000000  | 0.00000000  |            |             |
| 2 | C2 | 6 | 0.00000000  | 0.00000000  | 1.18115749 | 0.00000000  |
|   |    |   | 0.00000000  | 2.23206401  |            |             |
| 3 | O3 | 8 | 1.14420319  | 0.00000000  | 1.86516436 | 2.16223051  |
|   |    |   | 0.00000000  | 3.52464956  |            |             |
| 4 | C4 | 6 | -1.26347662 | -0.07368323 | 2.01899570 | -2.38762460 |
|   |    |   | -0.13924111 | 3.81534865  |            |             |
| 5 | H5 | 1 | 0.86533235  | 0.00000000  | 2.81445637 | 1.63524104  |
|   |    |   | 0.00000000  | 5.31855135  |            |             |
| 6 | H6 | 1 | -1.00128368 | -0.05839270 | 3.07034682 | -1.89215179 |
|   |    |   | -0.11034620 | 5.80211417  |            |             |
| 7 | H7 | 1 | -1.79429459 | -0.99092661 | 1.79245948 | -3.39072511 |
|   |    |   | -1.87257978 | 3.38725726  |            |             |
| 8 | H8 | 1 | -1.89735718 | 0.77563740  | 1.79239856 | -3.58548516 |
|   |    |   | 1.46574214  | 3.38714215  |            |             |

Nuclear repulsion energy: 122.224021925

The basis set has now been re-ordered FDPs  
 Projecting extended Huckel matrix (STO-3G) to 6-31G(d)  
 All integrals will be kept INCORE  
 NOTE: INTEGRALS .LE. 1.00E-07 (I2EACC) WERE NOT KEPT  
 Exponent cutoff used: 2.00E+01 PQUT2 cutoff used: 1.00E-16

1737317 TWO-ELECTRON INTEGRALS CALCULATED IN IDFCCL (RAW)  
 TOTAL NUMBER OF EACH OF THE 8 TYPES OF INTEGRALS SAVED:  
 IIKL: 55477 IJKJ: 57072 IJJL: 57795 IIKK: 2543  
 IJJJ: 863 IIIL: 872 IIIL: 24 IJKL: 1562671  
 Number of integrals in INCORE buffers:  
 IIKK: 1332 IJJL: 30556 IJKJ: 30159  
 IIKL: 29823 IJKL: 578843

389020 TWO-ELECTRON INTEGRALS CALCULATED IN ISPCCL (RAW)  
 TOTAL NUMBER OF EACH OF THE 8 TYPES OF INTEGRALS SAVED:  
 IIKL: 21017 IJKJ: 22145 IJJL: 21363 IIKK: 1797  
 IJJJ: 688 IIIL: 691 IIIL: 44 IJKL: 321275  
 Number of integrals in INCORE buffers:  
 IIKK: 2278 IJJL: 42002 IJKJ: 41919  
 IIKL: 41149 IJKL: 698709  
 TOTAL OF 2126337 TWO-ELECTRON INTEGRALS CALCULATED (RAW)

CLOSED SHELL SCF Nuclear Repulsion Energy is 122.224021925 Hartrees  
 Convergence on Density Matrix Required to Exit is 5.0000E-06

|                                | CYCLE | ELECTRONIC ENERGY | TOTAL ENERGY         | CONVERGENCE | EXTRAPOLATION |
|--------------------------------|-------|-------------------|----------------------|-------------|---------------|
| SCF_CYCLE:                     | 1     | -349.147748946    | -226.923727021       |             |               |
| SCF_CYCLE:                     | 2     | -349.864153613    | -227.640131688       | 3.23251E-02 |               |
| SCF_CYCLE:                     | 3     | -349.947256672    | -227.723234748       | 1.31011E-02 |               |
| SCF_CYCLE:                     | 4     | -349.974926317    | -227.750904392       | 1.04327E-02 |               |
| SCF_CYCLE:                     | 5     | -349.992426837    | -227.768404912       | 7.92034E-03 |               |
| SCF_CYCLE:                     | 6     | -350.001568398    | -227.777546473       | 6.22572E-03 |               |
| SCF_CYCLE:                     | 7     | -350.018394724    | -227.794372799       |             | 4-POINT       |
| SCF_CYCLE:                     | 8     | -350.016674930    | -227.792653005       | 2.80451E-03 |               |
| SCF_CYCLE:                     | 9     | -350.016704330    | -227.792682405       | 2.31882E-04 |               |
| SCF_CYCLE:                     | 10    | -350.016716162    | -227.792694238       | 1.55235E-04 |               |
| SCF_CYCLE:                     | 11    | -350.016725932    | -227.792704008       |             | 4-POINT       |
| SCF_CYCLE:                     | 12    | -350.016724621    | -227.792702696       | 6.16034E-05 |               |
| At termination total energy is |       |                   | -227.792703 Hartrees |             |               |

```

Energy components:
Kinetic =                227.331216809
Potential =             -780.930714443
Kinetic + Potential =   -553.599497634
Coulomb repulsion =     231.888100768
Exchange =              -28.305327756
Coulomb+Exchange =      203.582773012
Nuclear =               122.224021925
Total electronic =      -350.016724623
Total energy =          -227.792702698

Virial =                  2.002030015

Atom      J      K      Vee      JHF      KHF
VeeHF
  1      60.217314    -1.863484    58.353830    66.554675    -8.200845
58.353830
Atom      Coulomb
  1      63.929851
J_total    63.929851

Atom      Kinetic(x)    Kinetic(y)    Kinetic(z)    Total
  1      25.117564      24.651336      24.456601      74.225501
Ttotal:    25.117564      24.651336      24.456601      74.225501

Atom      Vne
  1      -231.797194
Vne_total: -231.797194

*****
The fragment with symbol 8(6(8,6))
has been added to the database with index #          118
*****

Atomic properties for atom #      1
+++++
Number of Electrons, N          =      8.1424852160
Pure Exchange, K ( 2K_ab)      =      -1.8634842591
HF Exchange, KHF ( 2K_ab+ Kaa) =      -8.2008448054
Kinetic energy Numerical, T     =      74.2255012751
Potential Energy Analytical, Vne =     -232.1835859918
Potential Energy Numerical, Vne =     -231.7971941706
Coulomb Energy Anal/Num, Vee    =      63.9298510309
Pure Coulomb, J ( 4J_ab+ Jaa)   =      60.2173142797
HF Coulomb, JHF ( 4J_ab+ 2Jaa)  =      66.5546748260
Jaa = Kaa                      =      6.3373605464
Coulomb Numerically Over A      =      37.9275965221

PROGRAM> end of inputs

Program terminated normally

Job: RUN_FRAG_0118 ended on :24-Aug-18 at 15:38:44
User: ibrahim
Cpu      time:  00h00m19s85c on ibrahim-Lenovo-IdeaPad-P500
Elapsed time:  00h00m20s00c

```

\*\*\* RUNing the inputfile :: INPUT\_0119.dat

\*\*\*\*\*  
 Working on symbol 1(6(7,6,1))  
 IndexNUM 119  
 \*\*\*\*\*

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N\_molecules: 1  
 Molecule is an asymmetric top.  
 Point group: C1  
 Cartesian coordinates for: 1(6(7,6,1))

| COORDINATES IN BOHR |    |             |             | COORDINATES IN ANGSTROMS |             |             |  |
|---------------------|----|-------------|-------------|--------------------------|-------------|-------------|--|
| I                   | EL | AN          | X           | Y                        | Z           | X           |  |
|                     |    | Y           | Z           |                          |             |             |  |
| 1                   | H  | 1           | -0.00000000 | 0.00000000               | -0.00000000 | -0.00000000 |  |
|                     |    | 0.00000000  | -0.00000000 |                          |             |             |  |
| 2                   | C  | 6           | -0.00000000 | -0.00000000              | 1.08653973  | -0.00000000 |  |
|                     |    | -0.00000000 | 2.05326237  |                          |             |             |  |
| 3                   | N  | 7           | 0.00000000  | 1.35660739               | 1.57197206  | 0.00000000  |  |
|                     |    | 2.56361623  | 2.97059646  |                          |             |             |  |
| 4                   | C  | 6           | 1.22412734  | -0.74090902              | 1.59284135  | 2.31326525  |  |
|                     |    | -1.40011502 | 3.01003370  |                          |             |             |  |
| 5                   | H  | 1           | -0.86045971 | -0.59784588              | 1.40075830  | -1.62603308 |  |
|                     |    | -1.12976490 | 2.64704936  |                          |             |             |  |
| 6                   | H  | 1           | 0.00000000  | 1.31271584               | 2.57351079  | 0.00000000  |  |
|                     |    | 2.48067324  | 4.86323023  |                          |             |             |  |
| 7                   | H  | 1           | -0.86960710 | 1.77063919               | 1.29378103  | -1.64331913 |  |
|                     |    | 3.34602290  | 2.44489164  |                          |             |             |  |
| 8                   | H  | 1           | 1.22400910  | -0.74083745              | 2.67650093  | 2.31304180  |  |
|                     |    | -1.39997979 | 5.05785337  |                          |             |             |  |
| 9                   | H  | 1           | 2.11948787  | -0.24860401              | 1.23174979  | 4.00525131  |  |
|                     |    | -0.46979346 | 2.32766958  |                          |             |             |  |
| 10                  | H  | 1           | 1.20323095  | -1.76248761              | 1.23168092  | 2.27377680  |  |
|                     |    | -3.33061865 | 2.32753944  |                          |             |             |  |

Nuclear repulsion energy: 83.935233098

Distance Matrix for: 1(6(7,6,1))

|   | 1        | 2        | 3        | 4        | 5        | 6        |
|---|----------|----------|----------|----------|----------|----------|
| 7 | 8        | 9        | 10       |          |          |          |
| 1 | 0.000000 | 1.086540 | 2.076410 | 2.141163 | 1.749267 | 2.888976 |
|   | 2.359078 | 3.034913 | 2.463989 | 2.463973 |          |          |
| 2 | 1.086540 | 0.000000 | 1.440843 | 1.517819 | 1.093867 | 1.983508 |
|   | 1.983514 | 2.138928 | 2.138953 | 2.138970 |          |          |
| 3 | 2.076410 | 1.440843 | 0.000000 | 2.428682 | 2.142333 | 1.002500 |
|   | 1.002511 | 2.667856 | 2.680426 | 3.360404 |          |          |
| 4 | 2.141163 | 1.517819 | 2.428682 | 0.000000 | 2.098301 | 2.584100 |
|   | 3.283449 | 1.083660 | 1.083707 | 1.083742 |          |          |
| 5 | 1.749267 | 1.093867 | 2.142333 | 2.098301 | 0.000000 | 2.401247 |
|   | 2.370917 | 2.448056 | 3.005099 | 2.375668 |          |          |
| 6 | 2.888976 | 1.983508 | 1.002500 | 2.584100 | 2.401247 | 0.000000 |

```

1.613573    2.392882    2.954703    3.564428
7    2.359078    1.983514    1.002511    3.283449    2.370917    1.613573
0.000000    3.550022    3.607753    4.096767
8    3.034913    2.138928    2.667856    1.083660    2.448056    2.392882
3.550022    0.000000    1.769599    1.769663
9    2.463989    2.138953    2.680426    1.083707    3.005099    2.954703
3.607753    1.769599    0.000000    1.769568
10   2.463973    2.138970    3.360404    1.083742    2.375668    3.564428
4.096767    1.769663    1.769568    0.000000

Charge=      0, Number of electrons=    26

The basis set has now been re-ordered FDPS
The basis set has now been re-ordered FDPS
6-31G(d) Basis Set - Total number of basis functions:      59

Partitioning scheme set to: BECKE
Free format Z-Matrix for: 1(6(7,6,1))
H1
C2      H1      C2H1
N3      C2      N3C2      H1      N3C2H1
C4      C2      C4C2      H1      C4C2H1      N3      C4C2H1N3
H5      C2      H5C2      H1      H5C2H1      N3      H5C2H1N3
H6      N3      H6N3      C2      H6N3C2      H1      H6N3C2H1
H7      N3      H7N3      C2      H7N3C2      H1      H7N3C2H1
H8      C4      H8C4      C2      H8C4C2      H1      H8C4C2H1
H9      C4      H9C4      C2      H9C4C2      H1      H9C4C2H1
H10     C4      H10C4     C2      H10C4C2     H1      H10C4C2H1

VARIABLES:
C2H1      = 1.08653973      N3C2      = 1.44084286      C4C2      = 1.51781924
H5C2      = 1.09386650      H6N3      = 1.00250002      H7N3      = 1.00251139
H8C4      = 1.08365959      H9C4      = 1.08370744      H10C4     = 1.08374183
N3C2H1    = 109.68864      C4C2H1    = 109.48571      H5C2H1    = 106.69369
H6N3C2    = 107.17931      H7N3C2    = 107.17911      H8C4C2    = 109.47840
H9C4C2    = 109.47751      H10C4C2   = 109.47686      C4C2H1N3  = -121.18463
H5C2H1N3  = 124.79150      H6N3C2H1  = -180.00000      H7N3C2H1  = 65.22155
H8C4C2H1  = 180.00000      H9C4C2H1  = 59.99718      H10C4C2H1 = -59.99363

Cartesian coordinates for: 1(6(7,6,1))
-----
COORDINATES IN BOHR      COORDINATES IN ANGSTROMS
I EL      AN      X      Y      Z      X
Y      Z
-----
1 H1      1      0.00000000      0.00000000      0.00000000      0.00000000
0.00000000      0.00000000
2 C2      6      0.00000000      0.00000000      1.08653973      0.00000000
0.00000000      2.05326237
3 N3      7      1.35660739      0.00000000      1.57197206      2.56361623
0.00000000      2.97059646
4 C4      6      -0.74090902      -1.22412734      1.59284135      -1.40011502
-2.31326525      3.01003370
5 H5      1      -0.59784588      0.86045971      1.40075830      -1.12976490
1.62603308      2.64704936
6 H6      1      1.31271584      0.00000000      2.57351079      2.48067324
0.00000000      4.86323023
7 H7      1      1.77063919      0.86960710      1.29378103      3.34602290
1.64331913      2.44489164
8 H8      1      -0.74083745      -1.22400910      2.67650093      -1.39997979

```

```

-2.31304180      5.05785337
 9 H9      1 -0.24860401 -2.11948787      1.23174979      -0.46979346
-4.00525131      2.32766958
10 H10      1 -1.76248761 -1.20323095      1.23168092      -3.33061865
-2.27377680      2.32753944
-----
Nuclear repulsion energy:      83.935233098

The basis set has now been re-ordered FDPS
Projecting extended Huckel matrix (STO-3G) to 6-31G(d)
All integrals will be kept INCORE
NOTE: INTEGRALS .LE. 1.00E-07 (I2EACC) WERE NOT KEPT
Exponent cutoff used: 2.00E+01 PQCUT2 cutoff used: 1.00E-16

1054921 TWO-ELECTRON INTEGRALS CALCULATED IN IDFCLC (RAW)
TOTAL NUMBER OF EACH OF THE 8 TYPES OF INTEGRALS SAVED:
IIKL:      36877 IJKJ:      37743 IJJL:      37996 IIKK:      1743
IJJJ:      634 IIIL:      637 IIII:      18 IJKL:      939273
Number of integrals in INCORE buffers:
IIKK:      891 IJJL:      19581 IJKJ:      19420
IIKL:      19181 IJKL:      331066

335016 TWO-ELECTRON INTEGRALS CALCULATED IN ISPCLC (RAW)
TOTAL NUMBER OF EACH OF THE 8 TYPES OF INTEGRALS SAVED:
IIKL:      18959 IJKJ:      19633 IJJL:      19108 IIKK:      1589
IJJJ:      670 IIIL:      674 IIII:      41 IJKL:      274342
Number of integrals in INCORE buffers:
IIKK:      1711 IJJL:      29511 IJKJ:      29541
IIKL:      29052 IJKL:      428347
TOTAL OF      1389937 TWO-ELECTRON INTEGRALS CALCULATED (RAW)

CLOSED SHELL SCF      Nuclear Repulsion Energy is      83.935233098 Hartrees
Convergence on Density Matrix Required to Exit is 5.0000E-06

      CYCLE      ELECTRONIC ENERGY      TOTAL ENERGY      CONVERGENCE      EXTRAPOLATION
SCF_CYCLE:      1      -217.901047167      -133.965814069
SCF_CYCLE:      2      -218.158366446      -134.223133348      1.52834E-02
SCF_CYCLE:      3      -218.177684138      -134.242451041      4.76679E-03
SCF_CYCLE:      4      -218.180330431      -134.245097333      2.22284E-03
SCF_CYCLE:      5      -218.180811678      -134.245578580      9.73653E-04
SCF_CYCLE:      6      -218.180916474      -134.245683376      5.26826E-04
SCF_CYCLE:      7      -218.180941559      -134.245708461      2.43426E-04
SCF_CYCLE:      8      -218.180962327      -134.245729229
SCF_CYCLE:      9      -218.180950251      -134.245717154      2.13781E-04      4-POINT
At termination total energy is      -134.245717 Hartrees

Energy components:
Kinetic =      134.198831358
Potential =      -481.028943959
Kinetic + Potential =      -346.830112601
Coulomb repulsion =      148.179763427
Exchange =      -19.530601084
Coulomb+Exchange =      128.649162343
Nuclear =      83.935233098
Total electronic =      -218.180950258
Total energy =      -134.245717160

Virial =      2.000349376

Atom      J      K      Vee      JHF      KHF
VeeHF

```

```

      1      4.777116      -0.173768      4.603348      4.935666      -0.332318
      4.603348
Atom      Coulomb
      1      3.666756
J_total      3.666756

Atom      Kinetic(x)      Kinetic(y)      Kinetic(z)      Total
      1      0.198498      0.199391      0.193105      0.590994

Ttotal:      0.198498      0.199391      0.193105      0.590994

Atom      Vne
      1      -7.870894
Vne_total:      -7.870894

```

```

*****
The fragment with symbol 1(6(7,6,1))
has been added to the database with index #      119
*****

```

```

      Atomic properties for atom #      1
      ++++++
Number of Electrons, N      =      0.9393195075
Pure Exchange, K ( 2K_ab)      =      -0.1737683920
HF Exchange, KHF ( 2K_ab+ Kaa)      =      -0.3323177771
Kinetic energy Numerical, T      =      0.5909938427
Potential Energy Analytical, Vne      =      -8.6399581266
Potential Energy Numerical, Vne      =      -7.8708936682
Coulomb Energy Anal/Num, Vee      =      3.6667562665
Pure Coulomb, J ( 4J_ab+ Jaa)      =      4.7771162122
HF Coulomb, JHF ( 4J_ab+ 2Jaa)      =      4.9356655972
Jaa = Kaa      =      0.1585493850
Coulomb Numerically Over A      =      0.3757362482

```

PROGRAM> end of inputs

Program terminated normally

```

Job: RUN_FRAG_0119 ended on :24-Aug-18 at 15:38:55
User: ibrahim
Cpu      time:  00h00m11s63c on ibrahim-Lenovo-IdeaPad-P500
Elapsed time:  00h00m11s00c

```

\*\*\* RUNing the inputfile :: INPUT\_0120.dat

```

*****
Working on symbol 1(7(6,1))
IndexNUM      120
*****

```

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```

N_molecules:  1
Molecule is an asymmetric top.
Point group: C1
Cartesian coordinates for: 1(7(6,1))

```

| COORDINATES IN BOHR                                      |          |            |             |             |            |          | COORDINATES IN ANGSTROMS |             |             |  |  |  |  |
|--|----------|------------|-------------|-------------|------------|----------|--------------------------|-------------|-------------|--|--|--|--|
| I  | EL       | AN         | X           | Y           | Z          |          | X                        | Y           | Z           |  |  |  |  |
| 1  | H        | 1          | 0.00000000  | 0.00000000  | 0.00000000 |          | 0.00000000               | 0.00000000  | 0.00000000  |  |  |  |  |
| 2  | N        | 7          | -0.00000000 | 0.00000000  | 0.00000000 |          | 1.00062635               | -0.00000000 | 0.00000000  |  |  |  |  |
| 3  | C        | 6          | 0.00000000  | 1.35162371  | 1.49976710 |          | 0.00000000               | 1.35162371  | 1.49976710  |  |  |  |  |
| 4  | H        | 1          | 0.79799306  | -0.50815411 | 1.32380172 |          | 1.50798822               | 0.79799306  | -0.50815411 |  |  |  |  |
| 5  | H        | 1          | 0.00000000  | 1.33667752  | 2.58332361 |          | 0.00000000               | 1.33667752  | 2.58332361  |  |  |  |  |
| 6  | H        | 1          | 0.88478395  | 1.86757977  | 1.14569291 |          | 1.67199922               | 0.88478395  | 1.86757977  |  |  |  |  |
| 7  | H        | 1          | -0.88478388 | 1.86760507  | 1.14562438 |          | -1.67199910              | -0.88478388 | 1.86760507  |  |  |  |  |
| Nuclear repulsion energy: 42.600440492                   |          |            |             |             |            |          |                          |             |             |  |  |  |  |
| Distance Matrix for: 1(7(6,1))                           |          |            |             |             |            |          |                          |             |             |  |  |  |  |
|  |          | 1          | 2           | 3           | 4          | 5        | 6                        |             |             |  |  |  |  |
| 1  | 0.000000 | 1.000626   | 2.018957    | 1.627103    | 2.908654   | 2.362903 |                          |             |             |  |  |  |  |
| 2  | 1.000626 | 0.000000   | 1.440843    | 0.999728    | 2.071627   | 2.071652 |                          |             |             |  |  |  |  |
| 3  | 2.018957 | 1.440843   | 0.000000    | 2.031386    | 1.083660   | 1.083707 |                          |             |             |  |  |  |  |
| 4  | 1.627103 | 0.999728   | 2.031386    | 0.000000    | 2.372044   | 2.383981 |                          |             |             |  |  |  |  |
| 5  | 2.908654 | 2.071627   | 1.083660    | 2.372044    | 0.000000   | 1.769599 |                          |             |             |  |  |  |  |
| 6  | 2.362903 | 2.071652   | 1.083707    | 2.383981    | 1.769599   | 0.000000 |                          |             |             |  |  |  |  |
| 7  | 2.362889 | 2.071670   | 1.083742    | 2.916799    | 1.769663   | 1.769568 |                          |             |             |  |  |  |  |
| Charge= 0, Number of electrons= 18                       |          |            |             |             |            |          |                          |             |             |  |  |  |  |
| The basis set has now been re-ordered FDPS               |          |            |             |             |            |          |                          |             |             |  |  |  |  |
| 6-31G(d) Basis Set - Total number of basis functions: 40 |          |            |             |             |            |          |                          |             |             |  |  |  |  |
| Partitioning scheme set to: BECKE                        |          |            |             |             |            |          |                          |             |             |  |  |  |  |
| Free format Z-Matrix for: 1(7(6,1))                      |          |            |             |             |            |          |                          |             |             |  |  |  |  |
| H1   |          |            |             |             |            |          |                          |             |             |  |  |  |  |
| N2   | H1       | N2H1       |             |             |            |          |                          |             |             |  |  |  |  |
| C3   | N2       | C3N2       | H1          | C3N2H1      |            |          |                          |             |             |  |  |  |  |
| H4   | N2       | H4N2       | H1          | H4N2H1      | C3         | H4N2H1C3 |                          |             |             |  |  |  |  |
| H5   | C3       | H5C3       | N2          | H5C3N2      | H1         | H5C3N2H1 |                          |             |             |  |  |  |  |
| H6   | C3       | H6C3       | N2          | H6C3N2      | H1         | H6C3N2H1 |                          |             |             |  |  |  |  |
| H7   | C3       | H7C3       | N2          | H7C3N2      | H1         | H7C3N2H1 |                          |             |             |  |  |  |  |
| VARIABLES:   |          |            |             |             |            |          |                          |             |             |  |  |  |  |
| N2H1   | =        | 1.00062635 | C3N2        | =           | 1.44084286 | H4N2     | =                        | 0.99972788  |             |  |  |  |  |

|          |   |            |          |   |            |          |   |            |
|----------|---|------------|----------|---|------------|----------|---|------------|
| H5C3     | = | 1.08365959 | H6C3     | = | 1.08370744 | H7C3     | = | 1.08374183 |
| C3N2H1   | = | 110.26867  | H4N2H1   | = | 108.86039  | H5C3N2   | = | 109.47840  |
| H6C3N2   | = | 109.47751  | H7C3N2   | = | 109.47686  | H4N2H1C3 | = | -122.48858 |
| H5C3N2H1 | = | -180.00000 | H6C3N2H1 | = | -59.99718  | H7C3N2H1 | = | 59.99363   |

Cartesian coordinates for: 1(7(6,1))

| COORDINATES IN BOHR |    |             |             | COORDINATES IN ANGSTROMS |            |             |  |
|---------------------|----|-------------|-------------|--------------------------|------------|-------------|--|
| I                   | EL | AN          | X           | Y                        | Z          | X           |  |
|                     |    | Y           | Z           |                          |            |             |  |
| 1                   | H1 | 1           | 0.00000000  | 0.00000000               | 0.00000000 | 0.00000000  |  |
|                     |    | 0.00000000  | 0.00000000  |                          |            |             |  |
| 2                   | N2 | 7           | 0.00000000  | 0.00000000               | 1.00062635 | 0.00000000  |  |
|                     |    | 0.00000000  | 1.89090962  |                          |            |             |  |
| 3                   | C3 | 6           | 1.35162371  | 0.00000000               | 1.49976710 | 2.55419846  |  |
|                     |    | 0.00000000  | 2.83414886  |                          |            |             |  |
| 4                   | H4 | 1           | -0.50815411 | -0.79799306              | 1.32380172 | -0.96027202 |  |
|                     |    | -1.50798822 | 2.50162252  |                          |            |             |  |
| 5                   | H5 | 1           | 1.33667752  | 0.00000000               | 2.58332361 | 2.52595426  |  |
|                     |    | 0.00000000  | 4.88177376  |                          |            |             |  |
| 6                   | H6 | 1           | 1.86757977  | -0.88478395              | 1.14569291 | 3.52921402  |  |
|                     |    | -1.67199922 | 2.16504566  |                          |            |             |  |
| 7                   | H7 | 1           | 1.86760507  | 0.88478388               | 1.14562438 | 3.52926185  |  |
|                     |    | 1.67199910  | 2.16491617  |                          |            |             |  |

Nuclear repulsion energy: 42.600440492

The basis set has now been re-ordered FDPS

Projecting extended Huckel matrix (STO-3G) to 6-31G(d)

All integrals will be kept INCORE

NOTE: INTEGRALS .LE. 1.00E-07 (I2EACC) WERE NOT KEPT

Exponent cutoff used: 2.00E+01 PQCUT2 cutoff used: 1.00E-16

180162 TWO-ELECTRON INTEGRALS CALCULATED IN IDFCLC (RAW)  
TOTAL NUMBER OF EACH OF THE 8 TYPES OF INTEGRALS SAVED:  
IIKL: 8554 IJKJ: 9223 IJJL: 9391 IIKK: 804  
IJJJ: 210 IIIL: 210 IIII: 12 IJKL: 151758  
Number of integrals in INCORE buffers:  
IIKK: 402 IJJL: 4705 IJKJ: 4616  
IIKL: 4293 IJKL: 50857

67855 TWO-ELECTRON INTEGRALS CALCULATED IN ISPCLC (RAW)  
TOTAL NUMBER OF EACH OF THE 8 TYPES OF INTEGRALS SAVED:  
IIKL: 5131 IJKJ: 5704 IJJL: 5281 IIKK: 748  
IJJJ: 269 IIIL: 270 IIII: 28 IJKL: 50424  
Number of integrals in INCORE buffers:  
IIKK: 780 IJJL: 7361 IJKJ: 7484  
IIKL: 6877 IJKL: 67853  
TOTAL OF 248017 TWO-ELECTRON INTEGRALS CALCULATED (RAW)

CLOSED SHELL SCF Nuclear Repulsion Energy is 42.600440492 Hartrees  
Convergence on Density Matrix Required to Exit is 5.0000E-06

|            | CYCLE | ELECTRONIC ENERGY | TOTAL ENERGY  | CONVERGENCE | EXTRAPOLATION |
|------------|-------|-------------------|---------------|-------------|---------------|
| SCF_CYCLE: | 1     | -137.590780390    | -94.990339899 |             |               |
| SCF_CYCLE: | 2     | -137.789680302    | -95.189239810 | 1.90611E-02 |               |



```

SCF_CYCLE: 3      -137.805890774      -95.205450282      6.03089E-03
SCF_CYCLE: 4      -137.808182440      -95.207741948      2.82661E-03
SCF_CYCLE: 5      -137.808604264      -95.208163772      1.31116E-03
SCF_CYCLE: 6      -137.808696267      -95.208255775      7.00462E-04
SCF_CYCLE: 7      -137.808718192      -95.208277700      3.41534E-04
SCF_CYCLE: 8      -137.808738184      -95.208297692
SCF_CYCLE: 9      -137.808725645      -95.208285153      3.23403E-04      4-POINT
At termination total energy is      -95.208285      Hartrees

Energy components:
Kinetic =      95.147962603
Potential =      -307.639621452
Kinetic + Potential =      -212.491658849
Coulomb repulsion =      88.293824920
Exchange =      -13.610891725
Coulomb+Exchange =      74.682933195
Nuclear =      42.600440492
Total electronic =      -137.808725654
Total energy =      -95.208285162

Virial =      2.000633987

Atom      J      K      Vee      JHF      KHF
VeeHF
1      3.283684      -0.158275      3.125409      3.464068      -0.338659
3.125409
Atom      Coulomb
1      3.003021
J_total      3.003021

Atom      Kinetic(x)      Kinetic(y)      Kinetic(z)      Total
1      0.222024      0.227975      0.237672      0.687671

Ttotal:      0.222024      0.227975      0.237672      0.687671

Atom      Vne
1      -6.583313
Vne_total:      -6.583313

*****
The fragment with symbol 1(7(6,1))
has been added to the database with index #      120
*****

Atomic properties for atom #      1
+++++
Number of Electrons, N      =      0.9122004836
Pure Exchange, K ( 2K_ab)      =      -0.1582751924
HF Exchange, KHF ( 2K_ab+ Kaa)      =      -0.3386585185
Kinetic energy Numerical, T      =      0.6876713434
Potential Energy Analytical, Vne      =      -7.3027568705
Potential Energy Numerical, Vne      =      -6.5833134021
Coulomb Energy Anal/Num, Vee      =      3.0030205592
Pure Coulomb, J ( 4J_ab+ Jaa)      =      3.2836843595
HF Coulomb, JHF ( 4J_ab+ 2Jaa)      =      3.4640676857
Jaa = Kaa      =      0.1803833262
Coulomb Numerically Over A      =      0.3613871992

PROGRAM> end of inputs

Program terminated normally

```

Job: RUN\_FRAG\_0120 ended on :24-Aug-18 at 15:39:00  
 User: ibrahim  
 Cpu time: 00h00m04s48c on ibrahim-Lenovo-IdeaPad-P500  
 Elapsed time: 00h00m05s00c

\*\*\* RUNing the inputfile :: INPUT\_0121.dat

\*\*\*\*\*  
 Working on symbol 8(6(8,6)1())  
 IndexNUM 121  
 \*\*\*\*\*

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N\_molecules: 1  
 Molecule is an asymmetric top.  
 Point group: C1  
 Cartesian coordinates for: 8(6(8,6)1())

| COORDINATES IN BOHR |    |             | COORDINATES IN ANGSTROMS |             |             |             |
|---------------------|----|-------------|--------------------------|-------------|-------------|-------------|
| I                   | EL | AN          | X                        | Y           | Z           | X           |
|                     |    | Y           | Z                        |             |             |             |
| 1                   | O  | 8           | -0.00000000              | -0.00000000 | 0.00000000  | -0.00000000 |
|                     |    | -0.00000000 | 0.00000000               |             |             |             |
| 2                   | C  | 6           | -0.00000000              | -0.00000000 | 1.33306652  | -0.00000000 |
|                     |    | -0.00000000 | 2.51913045               |             |             |             |
| 3                   | H  | 1           | -0.00000000              | 0.87662888  | -0.36063052 | -0.00000000 |
|                     |    | 1.65658838  | -0.68149287              |             |             |             |
| 4                   | O  | 8           | 0.04287417               | -1.01290903 | 1.93912768  | 0.08102044  |
|                     |    | -1.91412051 | 3.66441996               |             |             |             |
| 5                   | C  | 6           | -0.13144606              | 1.36309713  | 1.98763767  | -0.24839703 |
|                     |    | 2.57588007  | 3.75609056               |             |             |             |
| 6                   | H  | 1           | -0.12044877              | 1.24905513  | 3.06522367  | -0.22761517 |
|                     |    | 2.36037195  | 5.79243283               |             |             |             |
| 7                   | H  | 1           | -1.06458625              | 1.82199002  | 1.68253358  | -2.01177631 |
|                     |    | 3.44306189  | 3.17952743               |             |             |             |
| 8                   | H  | 1           | 0.69680779               | 1.99187626  | 1.68246767  | 1.31677578  |
|                     |    | 3.76410033  | 3.17940288               |             |             |             |

Nuclear repulsion energy: 122.214452707

Distance Matrix for: 8(6(8,6)1())

|   | 1        | 2        | 3        | 4        | 5        | 6        |
|---|----------|----------|----------|----------|----------|----------|
| 7 | 8        |          |          |          |          |          |
| 1 | 0.000000 | 1.333067 | 0.947910 | 2.188159 | 2.413714 | 3.312136 |
|   | 2.698872 | 2.698853 |          |          |          |          |
| 2 | 1.333067 | 0.000000 | 1.907115 | 1.181157 | 1.517819 | 2.138928 |
|   | 2.138953 | 2.138970 |          |          |          |          |
| 3 | 0.947910 | 1.907115 | 0.000000 | 2.976757 | 2.401727 | 3.448142 |
|   | 2.490295 | 2.429726 |          |          |          |          |
| 4 | 2.188159 | 1.181157 | 2.976757 | 0.000000 | 2.382886 | 2.532044 |

```

3.054335    3.085812
5      2.413714    1.517819    2.401727    2.382886    0.000000    1.083660
1.083707    1.083742
6      3.312136    2.138928    3.448142    2.532044    1.083660    0.000000
1.769599    1.769663
7      2.698872    2.138953    2.490295    3.054335    1.083707    1.769599
0.000000    1.769568
8      2.698853    2.138970    2.429726    3.085812    1.083742    1.769663
1.769568    0.000000

Charge=      0, Number of electrons=    32

The basis set has now been re-ordered FDPS
The basis set has now been re-ordered FDPS
6-31G(d) Basis Set - Total number of basis functions:      68

Partitioning scheme set to: BECKE
Free format Z-Matrix for: 8(6(8,6)1())
01
C2      01      C201
H3      01      H301      C2      H301C2
04      C2      04C2      01      04C201      H3      04C201H3
C5      C2      C5C2      01      C5C201      H3      C5C201H3
H6      C5      H6C5      C2      H6C5C2      01      H6C5C201
H7      C5      H7C5      C2      H7C5C2      01      H7C5C201
H8      C5      H8C5      C2      H8C5C2      01      H8C5C201

VARIABLES:
C201      = 1.33306652      H301      = 0.94790958      04C2      = 1.18115749
C5C2      = 1.51781924      H6C5      = 1.08365959      H7C5      = 1.08370744
H8C5      = 1.08374183      H301C2     = 112.36145      04C201     = 120.87106
C5C201     = 115.54740      H6C5C2     = 109.47840      H7C5C2     = 109.47751
H8C5C2     = 109.47686      04C201H3   = -177.57624      C5C201H3   = 5.50811
H6C5C201   = -180.00000      H7C5C201   = 59.99718      H8C5C201   = -59.99363

Cartesian coordinates for: 8(6(8,6)1())
-----
COORDINATES IN BOHR      COORDINATES IN ANGSTROMS
I EL      AN      X      Y      Z      X
      Y      Z
-----
1 01      8      0.00000000      0.00000000      0.00000000      0.00000000
      0.00000000      0.00000000
2 C2      6      0.00000000      0.00000000      1.33306652      0.00000000
      0.00000000      2.51913045
3 H3      1      0.87662888      0.00000000      -0.36063052      1.65658838
      0.00000000      -0.68149287
4 04      8      -1.01290903      -0.04287417      1.93912768      -1.91412051
      -0.08102044      3.66441996
5 C5      6      1.36309713      0.13144606      1.98763767      2.57588007
      0.24839703      3.75609056
6 H6      1      1.24905513      0.12044877      3.06522367      2.36037195
      0.22761517      5.79243283
7 H7      1      1.82199002      1.06458625      1.68253358      3.44306189
      2.01177631      3.17952743
8 H8      1      1.99187626      -0.69680779      1.68246767      3.76410033
      -1.31677578      3.17940288
-----

Nuclear repulsion energy:      122.214452707

```

```

The basis set has now been re-ordered FDPS
Projecting extended Huckel matrix (STO-3G) to 6-31G(d)
All integrals will be kept INCORE
NOTE: INTEGRALS .LE. 1.00E-07 (I2EACC) WERE NOT KEPT
Exponent cutoff used: 2.00E+01 PQCUT2 cutoff used: 1.00E-16

1941636 TWO-ELECTRON INTEGRALS CALCULATED IN IDFCLC (RAW)
TOTAL NUMBER OF EACH OF THE 8 TYPES OF INTEGRALS SAVED:
IIKL: 62701 IJKJ: 63397 IJLJ: 64227 IIKK: 2542
IJJJ: 988 IIIL: 996 IIII: 24 IJKL: 1746761
Number of integrals in INCORE buffers:
IIKK: 1332 IJLJ: 33817 IJKJ: 33328
IIKL: 33529 IJKL: 640583

420642 TWO-ELECTRON INTEGRALS CALCULATED IN ISPCLC (RAW)
TOTAL NUMBER OF EACH OF THE 8 TYPES OF INTEGRALS SAVED:
IIKL: 22933 IJKJ: 23388 IJLJ: 22937 IIKK: 1798
IJJJ: 744 IIIL: 747 IIII: 44 IJKL: 348051
Number of integrals in INCORE buffers:
IIKK: 2278 IJLJ: 46089 IJKJ: 45739
IIKL: 45855 IJKL: 769781
TOTAL OF 2362278 TWO-ELECTRON INTEGRALS CALCULATED (RAW)

CLOSED SHELL SCF Nuclear Repulsion Energy is 122.214452707 Hartrees
Convergence on Density Matrix Required to Exit is 5.0000E-06

CYCLE ELECTRONIC ENERGY TOTAL ENERGY CONVERGENCE EXTRAPOLATION
SCF_CYCLE: 1 -349.154741428 -226.940288720
SCF_CYCLE: 2 -349.862343589 -227.647890882 3.25587E-02
SCF_CYCLE: 3 -349.944259438 -227.729806730 1.31475E-02
SCF_CYCLE: 4 -349.971468508 -227.757015801 1.06269E-02
SCF_CYCLE: 5 -349.988791077 -227.774338370 8.06760E-03
SCF_CYCLE: 6 -349.997923915 -227.783471208 6.36459E-03
SCF_CYCLE: 7 -350.013104804 -227.798652097 4-POINT
SCF_CYCLE: 8 -350.013249551 -227.798796844 2.86969E-03
SCF_CYCLE: 9 -350.013263275 -227.798810568 1.55019E-04
SCF_CYCLE: 10 -350.013268589 -227.798815882 1.01277E-04
SCF_CYCLE: 11 -350.013272533 -227.798819825 4-POINT
SCF_CYCLE: 12 -350.013272150 -227.798819443 3.91761E-05
At termination total energy is -227.798819 Hartrees

Energy components:
Kinetic = 227.373772738
Potential = -780.977542957
Kinetic + Potential = -553.603770218
Coulomb repulsion = 231.913768217
Exchange = -28.323270150
Coulomb+Exchange = 203.590498067
Nuclear = 122.214452707
Total electronic = -350.013272152
Total energy = -227.798819445

Virial = 2.001869374

Atom J K Vee JHF KHF
VeeHF
1 54.706818 -2.023621 52.683198 60.872873 -8.189676
52.683198
Atom Coulomb
1 63.516302
J_total 63.516302

```

| Atom    | Kinetic(x) | Kinetic(y) | Kinetic(z) | Total     |
|---------|------------|------------|------------|-----------|
| 1       | 24.631878  | 25.067331  | 24.421903  | 74.121111 |
| Ttotal: | 24.631878  | 25.067331  | 24.421903  | 74.121111 |

| Atom       | Vne         |
|------------|-------------|
| 1          | -231.651040 |
| Vne_total: | -231.651040 |

```
*****
The fragment with symbol 8(6(8,6)1())
has been added to the database with index #      121
*****
```

```
Atomic properties for atom #      1
+++++
Number of Electrons, N      =      8.0097633529
Pure Exchange, K ( 2K_ab)  =     -2.0236205654
HF Exchange, KHF ( 2K_ab+ Kaa) =    -8.1896757065
Kinetic energy Numerical, T =     74.1211113231
Potential Energy Analytical, Vne = -231.9860332794
Potential Energy Numerical, Vne = -231.6510404857
Coulomb Energy Anal/Num, Vee =     63.5163024543
Pure Coulomb, J ( 4J_ab+ Jaa) =     54.7068180655
HF Coulomb, JHF ( 4J_ab+ 2Jaa) =     60.8728732066
Jaa = Kaa                  =     6.1660551412
Coulomb Numerically Over A =     37.6296518663
```

PROGRAM> end of inputs

Program terminated normally

```
Job: RUN_FRAG_0121 ended on :24-Aug-18 at 15:39:20
User: ibrahim
Cpu   time: 00h00m19s55c on ibrahim-Lenovo-IdeaPad-P500
Elapsed time: 00h00m20s00c
```

\*\*\* RUNing the inputfile :: INPUT\_0122.dat

```
*****
Working on symbol 1(8(6))
IndexNUM      122
*****
```

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```
N_molecules: 1
Molecule is an asymmetric top.
Point group: C1
Cartesian coordinates for: 1(8(6))
```

---

COORDINATES IN BOHR

| I | EL | AN          | X           | Y           | Z           | X           |
|---|----|-------------|-------------|-------------|-------------|-------------|
|   |    | Y           | Z           |             |             |             |
| 1 | H  | 1           | 0.00000000  | -0.00000000 | -0.00000000 | 0.00000000  |
|   |    | -0.00000000 | -0.00000000 |             |             |             |
| 2 | O  | 8           | 0.00000000  | -0.00000000 | 0.94790958  | 0.00000000  |
|   |    | -0.00000000 | 1.79128937  |             |             |             |
| 3 | C  | 6           | 0.00000000  | 1.23282287  | 1.45507238  | 0.00000000  |
|   |    | 2.32969742  | 2.74968810  |             |             |             |
| 4 | H  | 1           | -0.00000000 | 1.17831758  | 2.53736036  | -0.00000000 |
|   |    | 2.22669735  | 4.79491582  |             |             |             |
| 5 | H  | 1           | 0.88478395  | 1.76136479  | 1.12007593  | 1.67199922  |
|   |    | 3.32849682  | 2.11663659  |             |             |             |
| 6 | H  | 1           | -0.88478388 | 1.76139258  | 1.12000837  | -1.67199910 |
|   |    | 3.32854934  | 2.11650893  |             |             |             |

Nuclear repulsion energy: 41.946468146

Distance Matrix for: 1(8(6))

|   | 1        | 2        | 3        | 4        | 5        | 6        |
|---|----------|----------|----------|----------|----------|----------|
| 1 | 0.000000 | 0.947910 | 1.907115 | 2.797611 | 2.267117 | 2.267105 |
| 2 | 0.947910 | 0.000000 | 1.333067 | 1.978582 | 1.978608 | 1.978627 |
| 3 | 1.907115 | 1.333067 | 0.000000 | 1.083660 | 1.083707 | 1.083742 |
| 4 | 2.797611 | 1.978582 | 1.083660 | 0.000000 | 1.769599 | 1.769663 |
| 5 | 2.267117 | 1.978608 | 1.083707 | 1.769599 | 0.000000 | 1.769568 |
| 6 | 2.267105 | 1.978627 | 1.083742 | 1.769663 | 1.769568 | 0.000000 |

Charge= 0, Number of electrons= 18

The basis set has now been re-ordered FDPs

The basis set has now been re-ordered FDPs

6-31G(d) Basis Set - Total number of basis functions: 38

Partitioning scheme set to: BECKE

Free format Z-Matrix for: 1(8(6))

H1

O2 H1 O2H1

C3 O2 C3O2 H1 C3O2H1

H4 C3 H4C3 O2 H4C3O2 H1 H4C3O2H1

H5 C3 H5C3 O2 H5C3O2 H1 H5C3O2H1

H6 C3 H6C3 O2 H6C3O2 H1 H6C3O2H1

VARIABLES:

O2H1 = 0.94790958 C3O2 = 1.33306652 H4C3 = 1.08365959

H5C3 = 1.08370744 H6C3 = 1.08374183 C3O2H1 = 112.36145

H4C3O2 = 109.47840 H5C3O2 = 109.47751 H6C3O2 = 109.47686

H4C3O2H1 = 180.00000 H5C3O2H1 = -59.99718 H6C3O2H1 = 59.99363

Cartesian coordinates for: 1(8(6))

| COORDINATES IN BOHR |    |            |            |            |            |            |
|---------------------|----|------------|------------|------------|------------|------------|
| I                   | EL | AN         | X          | Y          | Z          | X          |
|                     |    | Y          | Z          |            |            |            |
| 1                   | H1 | 1          | 0.00000000 | 0.00000000 | 0.00000000 | 0.00000000 |
|                     |    | 0.00000000 | 0.00000000 |            |            |            |
| 2                   | O2 | 8          | 0.00000000 | 0.00000000 | 0.94790958 | 0.00000000 |

|   |    |             |            |             |            |            |
|---|----|-------------|------------|-------------|------------|------------|
|   |    | 0.00000000  | 1.79128937 |             |            |            |
| 3 | C3 | 6           | 1.23282287 | 0.00000000  | 1.45507238 | 2.32969742 |
|   |    | 0.00000000  | 2.74968810 |             |            |            |
| 4 | H4 | 1           | 1.17831758 | 0.00000000  | 2.53736036 | 2.22669735 |
|   |    | 0.00000000  | 4.79491582 |             |            |            |
| 5 | H5 | 1           | 1.76136479 | -0.88478395 | 1.12007593 | 3.32849682 |
|   |    | -1.67199922 | 2.11663659 |             |            |            |
| 6 | H6 | 1           | 1.76139258 | 0.88478388  | 1.12000837 | 3.32854934 |
|   |    | 1.67199910  | 2.11650893 |             |            |            |

Nuclear repulsion energy: 41.946468146

The basis set has now been re-ordered FDPS  
 Projecting extended Huckel matrix (STO-3G) to 6-31G(d)  
 All integrals will be kept INCORE  
 NOTE: INTEGRALS .LE. 1.00E-07 (I2EACC) WERE NOT KEPT  
 Exponent cutoff used: 2.00E+01 PQCUT2 cutoff used: 1.00E-16

139570 TWO-ELECTRON INTEGRALS CALCULATED IN IDFCLC (RAW)  
 TOTAL NUMBER OF EACH OF THE 8 TYPES OF INTEGRALS SAVED:  
 IIKL: 7031 IJKJ: 7696 IJJL: 7864 IIKK: 756  
 IJJJ: 186 IIIL: 186 IIII: 12 IJKL: 115839  
 Number of integrals in INCORE buffers:  
 IIKK: 378 IJJL: 3937 IJKJ: 3848  
 IIKL: 3525 IJKL: 38807

47218 TWO-ELECTRON INTEGRALS CALCULATED IN ISPCLC (RAW)  
 TOTAL NUMBER OF EACH OF THE 8 TYPES OF INTEGRALS SAVED:  
 IIKL: 3789 IJKJ: 4371 IJJL: 3937 IIKK: 645  
 IJJJ: 217 IIIL: 217 IIII: 26 IJKL: 34016  
 Number of integrals in INCORE buffers:  
 IIKK: 703 IJJL: 5917 IJKJ: 6040  
 IIKL: 5433 IJKL: 50279  
 TOTAL OF 186788 TWO-ELECTRON INTEGRALS CALCULATED (RAW)

CLOSED SHELL SCF Nuclear Repulsion Energy is 41.946468146 Hartrees  
 Convergence on Density Matrix Required to Exit is 5.0000E-06

|            | CYCLE | ELECTRONIC ENERGY | TOTAL ENERGY   | CONVERGENCE | EXTRAPOLATION |
|------------|-------|-------------------|----------------|-------------|---------------|
| SCF_CYCLE: | 1     | -156.599321648    | -114.652853501 |             |               |
| SCF_CYCLE: | 2     | -156.936416209    | -114.989948063 | 3.11167E-02 |               |
| SCF_CYCLE: | 3     | -156.968403650    | -115.021935504 | 9.05693E-03 |               |
| SCF_CYCLE: | 4     | -156.974430396    | -115.027962249 | 5.35023E-03 |               |
| SCF_CYCLE: | 5     | -156.976003655    | -115.029535509 | 2.57563E-03 |               |
| SCF_CYCLE: | 6     | -156.976488501    | -115.030020355 | 1.63447E-03 |               |
| SCF_CYCLE: | 7     | -156.976798256    | -115.030330110 |             | 4-POINT       |
| SCF_CYCLE: | 8     | -156.976733196    | -115.030265049 | 6.30655E-04 |               |
| SCF_CYCLE: | 9     | -156.976733226    | -115.030265080 | 1.01698E-05 |               |

At termination total energy is -115.030265 Hartrees

Energy components:  
 Kinetic = 115.033978657  
 Potential = -355.000327289  
 Kinetic + Potential = -239.966348632  
 Coulomb repulsion = 97.903875440  
 Exchange = -14.914260040  
 Coulomb+Exchange = 82.989615401  
 Nuclear = 41.946468146  
 Total electronic = -156.976733232  
 Total energy = -115.030265085

```

Virial = 1.999967718

Atom      J      K      Vee      JHF      KHF
  VeeHF
    1      3.189304    -0.148166    3.041138    3.381697    -0.340559
  3.041138
Atom      Coulomb
    1      3.134308
J_total    3.134308

Atom      Kinetic(x)    Kinetic(y)    Kinetic(z)    Total
    1      0.247152      0.256589      0.288540      0.792281

Ttotal:    0.247152      0.256589      0.288540      0.792281

Atom      Vne
    1      -6.886960
Vne_total:    -6.886960

```

```

*****
The fragment with symbol 1(8(6))
has been added to the database with index # 122
*****

```

```

Atomic properties for atom # 1
+++++
Number of Electrons, N = 0.8692169593
Pure Exchange, K ( 2K_ab) = -0.1481660000
HF Exchange, KHF ( 2K_ab+ Kaa) = -0.3405589381
Kinetic energy Numerical, T = 0.7922814061
Potential Energy Analytical, Vne = -7.7815163959
Potential Energy Numerical, Vne = -6.8869597878
Coulomb Energy Anal/Num, Vee = 3.1343081800
Pure Coulomb, J ( 4J_ab+ Jaa) = 3.1893037531
HF Coulomb, JHF ( 4J_ab+ 2Jaa) = 3.3816966912
Jaa = Kaa = 0.1923929381
Coulomb Numerically Over A = 0.3393770945

```

PROGRAM> end of inputs

Program terminated normally

```

Job: RUN_FRAG_0122 ended on :24-Aug-18 at 15:39:23
User: ibrahim
Cpu time: 00h00m03s92c on ibrahim-Lenovo-IdeaPad-P500
Elapsed time: 00h00m03s00c

```

PROGRAM> end of inputs

Program terminated normally

```

Job: RUN_Gly_ ended on :24-Aug-18 at 15:39:24
User: ibrahim
Cpu time: 00h00m04s72c on ibrahim-Lenovo-IdeaPad-P500
Elapsed time: 00h02m36s00c

```



### B.1.3 Building the Molecular Properties Using AIMD

Here is example of input file to calculate the molecular properties of Glycine from the database and direct methods,

```
MOLECULE
  UNITS=ANGSTROM !(default)
  CHARGE=0
  MULTIPLICITY=1
FREEZ
N
C   1 B1
C   2 B2 1 A2
O   3 B3 2 A3 1 D3
H   2 B4 1 A4 3 D4
H   2 B5 1 A5 3 D5
H   1 B6 2 A6 3 D6
H   1 B7 2 A7 3 D7
O   3 B8 2 A8 1 D8
H   9 B9 3 A9 2 D9

END
DEFINE
B1 = 1.44084286          B2 = 1.51781924          B3 = 1.18115749
B4 = 1.08653973          B5 = 1.09386650          B6 = 1.00062635
B7 = 0.99972788          B8 = 1.33306652          B9 = 0.94790958
A2 = 110.31733           A3 = 123.50442           A4 = 109.68864
A5 = 114.69657           A6 = 110.26867           A7 = 111.39727
A8 = 115.54740           A9 = 112.36145           D3 = -21.55248
D4 = 120.68224           D5 = -119.29373          D6 = 38.90011
D7 = 159.88520           D8 = 161.62252          D9 = -5.50811
END
end !molecule

BASIS name=6-31G(d) end

SET RUN NAME = "RUN_Gly_" end end

AIMDFT
  Level=2
  Direct ! Building of the Molecule (Direct or Database) (default is Database)
end !AIMDFT

output object=AIMDFT:MOLECULE%BUILD end
stop
```

Here is the results using the database,

```
Welcome to MUNgauss - July 9, 2018 Version 7

N_molecules: 1
Molecule is an asymmetric top.
Point group: C1
Free format Z-Matrix for: C2H5NO2, (C1)
N
C      N      B1
C      C      B2 N      A2
O      C      B3 C      A3 N      D3
H      C      B4 N      A4 C      D4
```

|   |   |    |   |    |   |    |
|---|---|----|---|----|---|----|
| H | C | B5 | N | A5 | C | D5 |
| H | N | B6 | C | A6 | C | D6 |
| H | N | B7 | C | A7 | C | D7 |
| O | C | B8 | C | A8 | N | D8 |
| H | O | B9 | C | A9 | C | D9 |

VARIABLES :

|                 |                 |                 |
|-----------------|-----------------|-----------------|
| B1 = 1.44084286 | B2 = 1.51781924 | B3 = 1.18115749 |
| B4 = 1.08653973 | B5 = 1.09386650 | B6 = 1.00062635 |
| B7 = 0.99972788 | B8 = 1.33306652 | B9 = 0.94790958 |
| A2 = 110.31733  | A3 = 123.50442  | A4 = 109.68864  |
| A5 = 114.69657  | A6 = 110.26867  | A7 = 111.39727  |
| A8 = 115.54740  | A9 = 112.36145  | D3 = -21.55248  |
| D4 = 120.68224  | D5 = -119.29373 | D6 = 38.90011   |
| D7 = 159.88520  | D8 = 161.62252  | D9 = -5.50811   |

Z MATRIX FOR: C2H5NO2, (C1)

| I  | AN  | Z1 | BL            | Z2 | ALPHA          | Z3 | BETA        |
|----|-----|----|---------------|----|----------------|----|-------------|
|    |     | Z4 |               |    |                |    |             |
| 1  | 7   |    |               |    |                |    |             |
| 2  | 6   | 1  | 1.440843 ( 1) |    |                |    |             |
| 3  | 6   | 2  | 1.517819 ( 2) | 1  | 110.3173 ( 10) |    |             |
| 4  | 8   | 3  | 1.181157 ( 3) | 2  | 123.5044 ( 11) | 1  | -21.5525 (  |
|    | 18) | 0  |               |    |                |    |             |
| 5  | 1   | 2  | 1.086540 ( 4) | 1  | 109.6886 ( 12) | 3  | 120.6822 (  |
|    | 19) | 0  |               |    |                |    |             |
| 6  | 1   | 2  | 1.093866 ( 5) | 1  | 114.6966 ( 13) | 3  | -119.2937 ( |
|    | 20) | 0  |               |    |                |    |             |
| 7  | 1   | 1  | 1.000626 ( 6) | 2  | 110.2687 ( 14) | 3  | 38.9001 (   |
|    | 21) | 0  |               |    |                |    |             |
| 8  | 1   | 1  | 0.999728 ( 7) | 2  | 111.3973 ( 15) | 3  | 159.8852 (  |
|    | 22) | 0  |               |    |                |    |             |
| 9  | 8   | 3  | 1.333067 ( 8) | 2  | 115.5474 ( 16) | 1  | 161.6225 (  |
|    | 23) | 0  |               |    |                |    |             |
| 10 | 1   | 9  | 0.947910 ( 9) | 3  | 112.3615 ( 17) | 2  | -5.5081 (   |
|    | 24) | 0  |               |    |                |    |             |

Cartesian coordinates for: C2H5NO2, (C1)

| COORDINATES IN BOHR |    |             |             | COORDINATES IN ANGSTROMS |            |             |  |
|---------------------|----|-------------|-------------|--------------------------|------------|-------------|--|
| I                   | EL | AN          | X           | Y                        | Z          | X           |  |
|                     |    | Y           | Z           |                          |            |             |  |
| 1                   | N  | 7           | 0.00000000  | 0.00000000               | 0.00000000 | 0.00000000  |  |
|                     |    | 0.00000000  | 0.00000000  |                          |            |             |  |
| 2                   | C  | 6           | 0.00000000  | 0.00000000               | 1.44084286 | 0.00000000  |  |
|                     |    | 0.00000000  | 2.72279820  |                          |            |             |  |
| 3                   | C  | 6           | 1.42338653  | 0.00000000               | 1.96785902 | 2.68981052  |  |
|                     |    | 0.00000000  | 3.71871433  |                          |            |             |  |
| 4                   | O  | 8           | 2.35288801  | -0.36180632              | 1.33520098 | 4.44631363  |  |
|                     |    | -0.68371481 | 2.52316398  |                          |            |             |  |
| 5                   | H  | 1           | -0.52202178 | 0.87980599               | 1.80690743 | -0.98647812 |  |
|                     |    | 1.66259225  | 3.41455992  |                          |            |             |  |
| 6                   | H  | 1           | -0.48626030 | -0.86672789              | 1.89787416 | -0.91889872 |  |

|    |   |             |             |             |             |             |
|----|---|-------------|-------------|-------------|-------------|-------------|
|    |   | -1.63787822 | 3.58646212  |             |             |             |
| 7  | H | 1           | 0.73050930  | -0.58944902 | -0.34663973 | 1.38046242  |
|    |   | -1.11389713 | -0.65505411 |             |             |             |
| 8  | H | 1           | -0.87404490 | -0.32011104 | -0.36473314 | -1.65170536 |
|    |   | -0.60492216 | -0.68924570 |             |             |             |
| 9  | O | 8           | 1.56620070  | 0.37919230  | 3.23785258  | 2.95969016  |
|    |   | 0.71656954  | 6.11865417  |             |             |             |
| 10 | H | 1           | 0.75176248  | 0.68026803  | 3.61808458  | 1.42062510  |
|    |   | 1.28552018  | 6.83718846  |             |             |             |

Nuclear repulsion energy: 181.510743561

Distance Matrix for: C2H5NO2, (C1)

|          | 1        | 2        | 3        | 4        | 5        | 6        |
|----------|----------|----------|----------|----------|----------|----------|
| 7        | 8        | 9        | 10       |          |          |          |
| 1        | 0.000000 | 1.440843 | 2.428682 | 2.729423 | 2.076410 | 2.142333 |
| 1.000626 | 0.999728 | 3.616692 | 3.757452 |          |          |          |
| 2        | 1.440843 | 0.000000 | 1.517819 | 2.382886 | 1.086540 | 1.093866 |
| 2.018957 | 2.031386 | 2.413714 | 2.401727 |          |          |          |
| 3        | 2.428682 | 1.517819 | 0.000000 | 1.181157 | 2.141163 | 2.098301 |
| 2.486852 | 3.289627 | 1.333067 | 1.907115 |          |          |          |
| 4        | 2.729423 | 2.382886 | 1.181157 | 0.000000 | 3.166893 | 2.938079 |
| 2.347876 | 3.647549 | 2.188159 | 2.976757 |          |          |          |
| 5        | 2.076410 | 1.086540 | 2.141163 | 3.166893 | 0.000000 | 1.749267 |
| 2.892285 | 2.505942 | 2.580483 | 2.223219 |          |          |          |
| 6        | 2.142333 | 1.093866 | 2.098301 | 2.938079 | 1.749267 | 0.000000 |
| 2.568123 | 2.359779 | 2.749628 | 2.623932 |          |          |          |
| 7        | 1.000626 | 2.018957 | 2.486852 | 2.347876 | 2.892285 | 2.568123 |
| 0.000000 | 1.627103 | 3.805947 | 4.163132 |          |          |          |
| 8        | 0.999728 | 2.031386 | 3.289627 | 3.647549 | 2.505942 | 2.359779 |
| 1.627103 | 0.000000 | 4.407091 | 4.416655 |          |          |          |
| 9        | 3.616692 | 2.413714 | 1.333067 | 2.188159 | 2.580483 | 2.749628 |
| 3.805947 | 4.407091 | 0.000000 | 0.947910 |          |          |          |
| 10       | 3.757452 | 2.401727 | 1.907115 | 2.976757 | 2.223219 | 2.623932 |
| 4.163132 | 4.416655 | 0.947910 | 0.000000 |          |          |          |

Charge= 0, Number of electrons= 40

The basis set has now been re-ordered FDPS

The basis set has now been re-ordered FDPS

6-31G(d) Basis Set - Total number of basis functions: 85

The basis set has now been re-ordered FDPS

Projecting extended Huckel matrix (STO-3G) to 6-31G(d)

NOTE: INTEGRALS .LE. 1.00E-07 (I2EACC) WERE NOT KEPT

Exponent cutoff used: 2.00E+01 PQCUT2 cutoff used: 1.00E-16

4124847 TWO-ELECTRON INTEGRALS CALCULATED IN IDFCLC (COMBINATIONS)

TOTAL NUMBER OF EACH OF THE 8 TYPES OF INTEGRALS SAVED:

IIKL: 113538 IJKJ: 113807 IJJL: 114903 IIKK: 3799

IJJJ: 1462 IIIL: 1470 IIII: 30 IJKL: 3775838

Number of integrals in INCORE buffers:

IIKK: 2085 IJJL: 64223 IJKJ: 63543

IIKL: 64237 IJKL: 1535810

937565 TWO-ELECTRON INTEGRALS CALCULATED IN ISPCLC (COMBINATIONS)

TOTAL NUMBER OF EACH OF THE 8 TYPES OF INTEGRALS SAVED:

IIKL: 43244 IJKJ: 44015 IJJL: 43286 IIKK: 2740

IJJJ: 1152 IIIL: 1158 IIII: 55 IJKL: 801915

Number of integrals in INCORE buffers:

IIKK: 3570 IJJL: 88212 IJKJ: 87691

IIKL: 88349 IJKL: 271361

```

Number of buffers:
IJJL:          0 IJKJ:          0
IIKL:          0 IJKL:          1
TOTAL OF      5062412 TWO-ELECTRON INTEGRALS CALCULATED (COMBINATIONS)

CLOSED SHELL SCF              Nuclear Repulsion Energy is      181.510743561 Hartrees
Convergence on Density Matrix Required to Exit is  5.0000E-06

      CYCLE      ELECTRONIC ENERGY      TOTAL ENERGY      CONVERGENCE      EXTRAPOLATION
SCF_CYCLE:   1      -463.303052879      -281.792309318
SCF_CYCLE:   2      -464.156082549      -282.645338988      2.78967E-02
SCF_CYCLE:   3      -464.252967556      -282.742223996      1.11306E-02
SCF_CYCLE:   4      -464.282711412      -282.771967851      8.78492E-03
SCF_CYCLE:   5      -464.301085806      -282.790342245      6.67293E-03
SCF_CYCLE:   6      -464.310646467      -282.799902906      5.26956E-03
SCF_CYCLE:   7      -464.325561119      -282.814817558
SCF_CYCLE:   8      -464.326970023      -282.816226462      2.39444E-03      4-POINT
SCF_CYCLE:   9      -464.326980754      -282.816237193      1.12426E-04
SCF_CYCLE:  10      -464.326984887      -282.816241326      7.27041E-05
SCF_CYCLE:  11      -464.326986541      -282.816242980      4.60760E-05
SCF_CYCLE:  12      -464.326987992      -282.816244431      4-POINT
SCF_CYCLE:  13      -464.326987698      -282.816244138      1.87641E-05
At termination total energy is      -282.816244 Hartrees

Number of indexes available in the database      =      8
Number of indexes NOT available in the database =      0
*****
The unique Symbols in the molecule
*****
SYM#   1 is 7(6(6,1,1)1()1())
SYM#   2 is 6(7(1,1)6(8,8)1()1())
SYM#   3 is 6(8(1)8()6(7,1,1))
SYM#   4 is 8(6(8,6))
SYM#   5 is 1(6(7,6,1))
SYM#   6 is 1(7(6,1))
SYM#   7 is 8(6(8,6)1())
SYM#   8 is 1(8(6))
*****
      CART OF INDEX
*****
index #      115 For Symbol 7(6(6,1,1)1()1())
  1  N      -0.00000000      0.00000000      0.00000000
  2  C      -0.00000000      -0.00000000      1.44084286
  3  H       0.00000000      0.93866607      -0.34663973
  4  H       0.79799306      -0.47920001      -0.36473314
  5  C      -0.89383628      1.10773910      1.96785902
  6  H      -0.35689075      -0.95874622      1.80690743
  7  H       0.97987823      0.16584623      1.89787416
  8  H      -0.88387273      1.09539118      3.05140244
  9  H      -1.90660309      0.95389867      1.61422314
 10  H      -0.52946711      2.06514427      1.61415464
index #      116 For Symbol 6(7(1,1)6(8,8)1()1())
  1  C      -0.00000000      -0.00000000      -0.00000000
  2  N       0.00000000      0.00000000      1.44084286
  3  C      -0.00000000      1.42338653      -0.52701616
  4  H       0.87980599      -0.52202178      -0.36606457
  5  H      -0.86672789      -0.48626030      -0.45703130
  6  O       0.37919230      1.56620070      -1.79700972
  7  O      -0.36180632      2.35288801      0.10564188
  8  H      -0.58944902      0.73050930      1.78748259
  9  H      -0.32011104      -0.87404490      1.80557600

```

```

10 H 0.31941883 2.53932814 -1.96545672
index # 117 For Symbol 6(8(1)8(6(7,1,1)))
1 C 0.00000000 -0.00000000 0.00000000
2 O -0.00000000 0.00000000 1.33306652
3 O 0.00000000 1.01381600 -0.60606116
4 C 0.07368323 -1.36743652 -0.65457115
5 N -0.35716835 -1.28888329 -2.02724171
6 H -0.03707255 -0.87584463 1.69369704
7 H -0.55932392 -2.06380818 -0.11149008
8 H 1.10434945 -1.70738451 -0.51779972
9 H -0.29008929 -2.21219942 -2.41194164
10 H 0.30676556 -0.71652891 -2.51368937
index # 118 For Symbol 8(6(8,6))
1 O 0.00000000 0.00000000 0.00000000
2 C 0.00000000 -0.00000000 1.18115749
3 O 0.00000000 1.14420319 1.86516436
4 C 0.07368323 -1.26347662 2.01899570
5 H 0.00000000 0.86533235 2.81445637
6 H 0.05839270 -1.00128368 3.07034682
7 H 0.99092661 -1.79429459 1.79245948
8 H -0.77563740 -1.89735718 1.79239856
index # 119 For Symbol 1(6(7,6,1))
1 H -0.00000000 -0.00000000 -0.00000000
2 C 0.00000000 0.00000000 1.08653973
3 N 0.00000000 1.35660739 1.57197206
4 C 1.22412734 -0.74090902 1.59284135
5 H -0.86045971 -0.59784588 1.40075830
6 H 0.00000000 1.31271584 2.57351079
7 H -0.86960710 1.77063919 1.29378103
8 H 1.22400910 -0.74083745 2.67650093
9 H 2.11948787 -0.24860401 1.23174979
10 H 1.20323095 -1.76248761 1.23168092
index # 120 For Symbol 1(7(6,1))
1 H -0.00000000 0.00000000 0.00000000
2 N 0.00000000 0.00000000 1.00062635
3 C -0.00000000 1.35162371 1.49976710
4 H 0.79799306 -0.50815411 1.32380172
5 H -0.00000000 1.33667752 2.58332361
6 H 0.88478395 1.86757977 1.14569291
7 H -0.88478388 1.86760507 1.14562438
index # 121 For Symbol 8(6(8,6)1())
1 O 0.00000000 -0.00000000 -0.00000000
2 C -0.00000000 -0.00000000 1.33306652
3 H 0.00000000 0.87662888 -0.36063052
4 O 0.04287417 -1.01290903 1.93912768
5 C -0.13144606 1.36309713 1.98763767
6 H -0.12044877 1.24905513 3.06522367
7 H -1.06458625 1.82199002 1.68253358
8 H 0.69680779 1.99187626 1.68246767
index # 122 For Symbol 1(8(6))
1 H 0.00000000 -0.00000000 -0.00000000
2 O 0.00000000 0.00000000 0.94790958
3 C 0.00000000 1.23282287 1.45507238
4 H 0.00000000 1.17831758 2.53736036
5 H -0.88478395 1.76136479 1.12007593
6 H 0.88478388 1.76139258 1.1200837
*****
NEW FRAG CART (FROM DATABASE)
*****
Atom# 1
Atom X Y Z
N -0.00000000 0.00000000 0.00000000
C -0.00000000 -0.00000000 1.44084286
H 0.00000000 0.93866607 -0.34663973

```

|       |             |             |             |
|-------|-------------|-------------|-------------|
| H     | 0.79799306  | -0.47920001 | -0.36473314 |
| C     | -0.89383628 | 1.10773910  | 1.96785902  |
| H     | -0.35689075 | -0.95874622 | 1.80690743  |
| H     | 0.97987823  | 0.16584623  | 1.89787416  |
| H     | -0.88387273 | 1.09539118  | 3.05140244  |
| H     | -1.90660309 | 0.95389867  | 1.61422314  |
| H     | -0.52946711 | 2.06514427  | 1.61415464  |
| Atom# | 2           |             |             |
| Atom  | X           | Y           | Z           |
| C     | -0.00000000 | -0.00000000 | -0.00000000 |
| N     | 0.00000000  | 0.00000000  | 1.44084286  |
| C     | -0.00000000 | 1.42338653  | -0.52701616 |
| H     | 0.87980599  | -0.52202178 | -0.36606457 |
| H     | -0.86672789 | -0.48626030 | -0.45703130 |
| O     | 0.37919230  | 1.56620070  | -1.79700972 |
| O     | -0.36180632 | 2.35288801  | 0.10564188  |
| H     | -0.58944902 | 0.73050930  | 1.78748259  |
| H     | -0.32011104 | -0.87404490 | 1.80557600  |
| H     | 0.31941883  | 2.53932814  | -1.96545672 |
| Atom# | 3           |             |             |
| Atom  | X           | Y           | Z           |
| C     | 0.00000000  | -0.00000000 | 0.00000000  |
| O     | -0.00000000 | 0.00000000  | 1.33306652  |
| O     | 0.00000000  | 1.01381600  | -0.60606116 |
| C     | 0.07368323  | -1.36743652 | -0.65457115 |
| N     | -0.35716835 | -1.28888329 | -2.02724171 |
| H     | -0.03707255 | -0.87584463 | 1.69369704  |
| H     | -0.55932392 | -2.06380818 | -0.11149008 |
| H     | 1.10434945  | -1.70738451 | -0.51779972 |
| H     | -0.29008929 | -2.21219942 | -2.41194164 |
| H     | 0.30676556  | -0.71652891 | -2.51368937 |
| Atom# | 4           |             |             |
| Atom  | X           | Y           | Z           |
| O     | 0.00000000  | 0.00000000  | 0.00000000  |
| C     | 0.00000000  | -0.00000000 | 1.18115749  |
| O     | 0.00000000  | 1.14420319  | 1.86516436  |
| C     | 0.07368323  | -1.26347662 | 2.01899570  |
| H     | 0.00000000  | 0.86533235  | 2.81445637  |
| H     | 0.05839270  | -1.00128368 | 3.07034682  |
| H     | 0.99092661  | -1.79429459 | 1.79245948  |
| H     | -0.77563740 | -1.89735718 | 1.79239856  |
| Atom# | 5           |             |             |
| Atom  | X           | Y           | Z           |
| H     | -0.00000000 | -0.00000000 | -0.00000000 |
| C     | 0.00000000  | 0.00000000  | 1.08653973  |
| N     | 0.00000000  | 1.35660739  | 1.57197206  |
| C     | 1.22412734  | -0.74090902 | 1.59284135  |
| H     | -0.86045971 | -0.59784588 | 1.40075830  |
| H     | 0.00000000  | 1.31271584  | 2.57351079  |
| H     | -0.86960710 | 1.77063919  | 1.29378103  |
| H     | 1.22400910  | -0.74083745 | 2.67650093  |
| H     | 2.11948787  | -0.24860401 | 1.23174979  |
| H     | 1.20323095  | -1.76248761 | 1.23168092  |
| Atom# | 6           |             |             |
| Atom  | X           | Y           | Z           |
| H     | -0.00000000 | -0.00000000 | -0.00000000 |
| C     | 0.00000000  | 0.00000000  | 1.08653973  |
| N     | 0.00000000  | 1.35660739  | 1.57197206  |
| C     | 1.22412734  | -0.74090902 | 1.59284135  |
| H     | -0.86045971 | -0.59784588 | 1.40075830  |
| H     | 0.00000000  | 1.31271584  | 2.57351079  |
| H     | -0.86960710 | 1.77063919  | 1.29378103  |
| H     | 1.22400910  | -0.74083745 | 2.67650093  |
| H     | 2.11948787  | -0.24860401 | 1.23174979  |

|  |             |             |             |
|--|-------------|-------------|-------------|
| H  | 1.20323095  | -1.76248761 | 1.23168092  |
| Atom#  | 7           |             |             |
| Atom   | X           | Y           | Z           |
| H  | -0.00000000 | 0.00000000  | 0.00000000  |
| N  | 0.00000000  | 0.00000000  | 1.00062635  |
| C  | -0.00000000 | 1.35162371  | 1.49976710  |
| H  | 0.79799306  | -0.50815411 | 1.32380172  |
| H  | -0.00000000 | 1.33667752  | 2.58332361  |
| H  | 0.88478395  | 1.86757977  | 1.14569291  |
| H  | -0.88478388 | 1.86760507  | 1.14562438  |
| Atom#  | 8           |             |             |
| Atom   | X           | Y           | Z           |
| H  | -0.00000000 | 0.00000000  | 0.00000000  |
| N  | 0.00000000  | 0.00000000  | 1.00062635  |
| C  | -0.00000000 | 1.35162371  | 1.49976710  |
| H  | 0.79799306  | -0.50815411 | 1.32380172  |
| H  | -0.00000000 | 1.33667752  | 2.58332361  |
| H  | 0.88478395  | 1.86757977  | 1.14569291  |
| H  | -0.88478388 | 1.86760507  | 1.14562438  |
| Atom#  | 9           |             |             |
| Atom   | X           | Y           | Z           |
| O  | 0.00000000  | -0.00000000 | -0.00000000 |
| C  | -0.00000000 | -0.00000000 | 1.33306652  |
| H  | 0.00000000  | 0.87662888  | -0.36063052 |
| O  | 0.04287417  | -1.01290903 | 1.93912768  |
| C  | -0.13144606 | 1.36309713  | 1.98763767  |
| H  | -0.12044877 | 1.24905513  | 3.06522367  |
| H  | -1.06458625 | 1.82199002  | 1.68253358  |
| H  | 0.69680779  | 1.99187626  | 1.68246767  |
| Atom#  | 10          |             |             |
| Atom   | X           | Y           | Z           |
| H  | 0.00000000  | -0.00000000 | -0.00000000 |
| O  | 0.00000000  | 0.00000000  | 0.94790958  |
| C  | 0.00000000  | 1.23282287  | 1.45507238  |
| H  | 0.00000000  | 1.17831758  | 2.53736036  |
| H  | -0.88478395 | 1.76136479  | 1.12007593  |
| H  | 0.88478388  | 1.76139258  | 1.12000837  |
| *****  |             |             |             |
| Where the fragment should be (database form) |             |             |             |
| *****  |             |             |             |
| Atom#  | 1           |             |             |
| Atom   | X           | Y           | Z           |
| N  | 0.00000000  | -0.00000000 | 0.00000000  |
| C  | -0.00000000 | -0.00000000 | 1.44084286  |
| H  | 0.00000000  | 0.93866607  | -0.34663973 |
| H  | 0.79799306  | -0.47920001 | -0.36473314 |
| C  | -0.89383628 | 1.10773910  | 1.96785902  |
| H  | -0.35689075 | -0.95874622 | 1.80690743  |
| H  | 0.97987823  | 0.16584623  | 1.89787416  |
| H  | -0.88387273 | 1.09539118  | 3.05140244  |
| H  | -1.90660309 | 0.95389867  | 1.61422314  |
| H  | -0.52946711 | 2.06514427  | 1.61415464  |
| Atom#  | 2           |             |             |
| Atom   | X           | Y           | Z           |
| C  | -0.00000000 | 0.00000000  | 0.00000000  |
| N  | 0.00000000  | -0.00000000 | 1.44084286  |
| C  | 0.00000000  | 1.42338653  | -0.52701616 |
| H  | 0.87980599  | -0.52202178 | -0.36606457 |
| H  | -0.86672789 | -0.48626030 | -0.45703130 |
| O  | 0.37919230  | 1.56620070  | -1.79700972 |
| O  | -0.36180632 | 2.35288801  | 0.10564188  |
| H  | -0.58944902 | 0.73050930  | 1.78748259  |
| H  | -0.32011104 | -0.87404490 | 1.80557600  |
| H  | 0.31941883  | 2.53932814  | -1.96545672 |

| Atom# | 3           |             |             |
|-------|-------------|-------------|-------------|
| Atom  | X           | Y           | Z           |
| C     | -0.00000000 | -0.00000000 | 0.00000000  |
| O     | -0.00000000 | 0.00000000  | 1.33306652  |
| O     | 0.00000000  | 1.01381600  | -0.60606116 |
| C     | 0.07368323  | -1.36743652 | -0.65457115 |
| N     | -0.35716835 | -1.28888329 | -2.02724171 |
| H     | -0.03707255 | -0.87584463 | 1.69369704  |
| H     | -0.55932392 | -2.06380818 | -0.11149008 |
| H     | 1.10434945  | -1.70738451 | -0.51779972 |
| H     | -0.29008929 | -2.21219942 | -2.41194164 |
| H     | 0.30676556  | -0.71652891 | -2.51368937 |
| Atom# | 4           |             |             |
| Atom  | X           | Y           | Z           |
| O     | -0.00000000 | 0.00000000  | -0.00000000 |
| C     | -0.00000000 | -0.00000000 | 1.18115749  |
| O     | -0.00000000 | 1.14420319  | 1.86516436  |
| C     | 0.07368323  | -1.26347662 | 2.01899570  |
| H     | 0.00000000  | 0.86533235  | 2.81445637  |
| H     | 0.05839270  | -1.00128368 | 3.07034682  |
| H     | 0.99092661  | -1.79429459 | 1.79245948  |
| H     | -0.77563740 | -1.89735718 | 1.79239856  |
| Atom# | 5           |             |             |
| Atom  | X           | Y           | Z           |
| H     | -0.00000000 | 0.00000000  | -0.00000000 |
| C     | -0.00000000 | -0.00000000 | 1.08653973  |
| N     | 0.00000000  | 1.35660739  | 1.57197206  |
| C     | 1.22412734  | -0.74090902 | 1.59284135  |
| H     | -0.86045971 | -0.59784588 | 1.40075830  |
| H     | 0.00000000  | 1.31271584  | 2.57351079  |
| H     | -0.86960710 | 1.77063919  | 1.29378103  |
| H     | 1.22400910  | -0.74083745 | 2.67650093  |
| H     | 2.11948787  | -0.24860401 | 1.23174979  |
| H     | 1.20323095  | -1.76248761 | 1.23168092  |
| Atom# | 6           |             |             |
| Atom  | X           | Y           | Z           |
| H     | -0.00000000 | 0.00000000  | 0.00000000  |
| C     | 0.00000000  | 0.00000000  | 1.09386650  |
| N     | 0.00000000  | 1.30905356  | 1.69586892  |
| C     | 1.24136788  | -0.76979504 | 1.50641545  |
| H     | -0.88574475 | -0.54645193 | 1.40598042  |
| H     | 0.00000000  | 1.17790163  | 2.68975295  |
| H     | -0.86960710 | 1.74578916  | 1.45488219  |
| H     | 1.30090798  | -0.80671695 | 2.58780800  |
| H     | 2.12120534  | -0.27430176 | 1.11297643  |
| H     | 1.18863813  | -1.77819294 | 1.11290613  |
| Atom# | 7           |             |             |
| Atom  | X           | Y           | Z           |
| H     | 0.00000000  | 0.00000000  | 0.00000000  |
| N     | -0.00000000 | 0.00000000  | 1.00062635  |
| C     | 0.00000000  | 1.35162371  | 1.49976710  |
| H     | 0.79799306  | -0.50815411 | 1.32380172  |
| H     | 0.00000000  | 1.33667752  | 2.58332361  |
| H     | 0.88478395  | 1.86757977  | 1.14569291  |
| H     | -0.88478388 | 1.86760507  | 1.14562438  |
| Atom# | 8           |             |             |
| Atom  | X           | Y           | Z           |
| H     | 0.00000000  | -0.00000000 | -0.00000000 |
| N     | -0.00000000 | -0.00000000 | 0.99972788  |
| C     | 0.00000000  | 1.34153017  | 1.52539407  |
| H     | 0.80471964  | -0.49904836 | 1.32319369  |
| H     | 0.00000000  | 1.30524460  | 2.60844599  |
| H     | 0.88478395  | 1.86436016  | 1.18155110  |
| H     | -0.88478388 | 1.86438681  | 1.18148309  |



| Atom#                                  | 9           |             |             |
|--|-------------|-------------|-------------|
| Atom                                   | X           | Y           | Z           |
| O                                      | -0.00000000 | -0.00000000 | 0.00000000  |
| C                                      | -0.00000000 | -0.00000000 | 1.33306652  |
| H                                      | -0.00000000 | 0.87662888  | -0.36063052 |
| O                                      | 0.04287417  | -1.01290903 | 1.93912768  |
| C                                      | -0.13144606 | 1.36309713  | 1.98763767  |
| H                                      | -0.12044877 | 1.24905513  | 3.06522367  |
| H                                      | -1.06458625 | 1.82199002  | 1.68253358  |
| H                                      | 0.69680779  | 1.99187626  | 1.68246767  |
| Atom#                                  | 10          |             |             |
| Atom                                   | X           | Y           | Z           |
| H                                      | 0.00000000  | -0.00000000 | -0.00000000 |
| O                                      | 0.00000000  | -0.00000000 | 0.94790958  |
| C                                      | 0.00000000  | 1.23282287  | 1.45507238  |
| H                                      | -0.00000000 | 1.17831758  | 2.53736036  |
| H                                      | 0.88478395  | 1.76136479  | 1.12007593  |
| H                                      | -0.88478388 | 1.76139258  | 1.12000837  |
| *****                                  |             |             |             |
| The error: (FROM DATABASE)- (Original) |             |             |             |
| *****                                  |             |             |             |
| Atom#                                  | 1           |             |             |
| Atom                                   | X           | Y           | Z           |
| N                                      | -0.00000000 | 0.00000000  | 0.00000000  |
| C                                      | 0.00000000  | -0.00000000 | 0.00000000  |
| H                                      | 0.00000000  | 0.00000000  | 0.00000000  |
| H                                      | 0.00000000  | 0.00000000  | 0.00000000  |
| C                                      | 0.00000000  | 0.00000000  | -0.00000000 |
| H                                      | -0.00000000 | -0.00000000 | -0.00000000 |
| H                                      | 0.00000000  | -0.00000000 | -0.00000000 |
| H                                      | 0.00000000  | 0.00000000  | -0.00000000 |
| H                                      | 0.00000000  | -0.00000000 | -0.00000000 |
| H                                      | 0.00000000  | -0.00000000 | -0.00000000 |
| Atom#                                  | 2           |             |             |
| Atom                                   | X           | Y           | Z           |
| C                                      | -0.00000000 | -0.00000000 | -0.00000000 |
| N                                      | 0.00000000  | 0.00000000  | -0.00000000 |
| C                                      | -0.00000000 | 0.00000000  | 0.00000000  |
| H                                      | 0.00000000  | 0.00000000  | 0.00000000  |
| H                                      | -0.00000000 | 0.00000000  | 0.00000000  |
| O                                      | 0.00000000  | 0.00000000  | 0.00000000  |
| O                                      | -0.00000000 | -0.00000000 | 0.00000000  |
| H                                      | -0.00000000 | 0.00000000  | -0.00000000 |
| H                                      | -0.00000000 | 0.00000000  | -0.00000000 |
| H                                      | 0.00000000  | 0.00000000  | 0.00000000  |
| Atom#                                  | 3           |             |             |
| Atom                                   | X           | Y           | Z           |
| C                                      | 0.00000000  | -0.00000000 | 0.00000000  |
| O                                      | -0.00000000 | 0.00000000  | 0.00000000  |
| O                                      | 0.00000000  | 0.00000000  | 0.00000000  |
| C                                      | 0.00000000  | -0.00000000 | 0.00000000  |
| N                                      | -0.00000000 | -0.00000000 | 0.00000000  |
| H                                      | -0.00000000 | -0.00000000 | -0.00000000 |
| H                                      | 0.00000000  | 0.00000000  | 0.00000000  |
| H                                      | 0.00000000  | 0.00000000  | 0.00000000  |
| H                                      | 0.00000000  | -0.00000000 | 0.00000000  |
| H                                      | -0.00000000 | -0.00000000 | 0.00000000  |
| Atom#                                  | 4           |             |             |
| Atom                                   | X           | Y           | Z           |
| O                                      | 0.00000000  | 0.00000000  | 0.00000000  |
| C                                      | 0.00000000  | -0.00000000 | 0.00000000  |
| O                                      | 0.00000000  | 0.00000000  | -0.00000000 |
| C                                      | 0.00000000  | -0.00000000 | -0.00000000 |
| H                                      | -0.00000000 | 0.00000000  | -0.00000000 |

|       |             |             |             |
|-------|-------------|-------------|-------------|
| H     | 0.00000000  | -0.00000000 | -0.00000000 |
| H     | 0.00000000  | 0.00000000  | -0.00000000 |
| H     | 0.00000000  | -0.00000000 | -0.00000000 |
| Atom# | 5           |             |             |
| Atom  | X           | Y           | Z           |
| H     | -0.00000000 | -0.00000000 | -0.00000000 |
| C     | 0.00000000  | 0.00000000  | 0.00000000  |
| N     | -0.00000000 | 0.00000000  | -0.00000000 |
| C     | 0.00000000  | 0.00000000  | -0.00000000 |
| H     | -0.00000000 | 0.00000000  | -0.00000000 |
| H     | -0.00000000 | 0.00000000  | -0.00000000 |
| H     | -0.00000000 | -0.00000000 | -0.00000000 |
| H     | 0.00000000  | 0.00000000  | -0.00000000 |
| H     | -0.00000000 | 0.00000000  | -0.00000000 |
| H     | 0.00000000  | 0.00000000  | -0.00000000 |
| Atom# | 6           |             |             |
| Atom  | X           | Y           | Z           |
| H     | -0.00000000 | -0.00000000 | -0.00000000 |
| C     | 0.00000000  | 0.00000000  | -0.00732677 |
| N     | 0.00000000  | 0.04755382  | -0.12389686 |
| C     | -0.01724054 | 0.02888602  | 0.08642590  |
| H     | 0.02528504  | -0.05139395 | -0.00522212 |
| H     | 0.00000000  | 0.13481421  | -0.11624216 |
| H     | -0.00000000 | 0.02485004  | -0.16110116 |
| H     | -0.07689888 | 0.06587950  | 0.08869293  |
| H     | -0.00171747 | 0.02569774  | 0.11877335  |
| H     | 0.01459281  | 0.01570533  | 0.11877479  |
| Atom# | 7           |             |             |
| Atom  | X           | Y           | Z           |
| H     | -0.00000000 | 0.00000000  | 0.00000000  |
| N     | 0.00000000  | 0.00000000  | 0.00000000  |
| C     | -0.00000000 | 0.00000000  | -0.00000000 |
| H     | 0.00000000  | 0.00000000  | -0.00000000 |
| H     | -0.00000000 | 0.00000000  | -0.00000000 |
| H     | -0.00000000 | -0.00000000 | -0.00000000 |
| H     | -0.00000000 | -0.00000000 | -0.00000000 |
| Atom# | 8           |             |             |
| Atom  | X           | Y           | Z           |
| H     | -0.00000000 | 0.00000000  | 0.00000000  |
| N     | 0.00000000  | 0.00000000  | 0.00089847  |
| C     | -0.00000000 | 0.01009354  | -0.02562697 |
| H     | -0.00672658 | -0.00910575 | 0.00060803  |
| H     | -0.00000000 | 0.03143292  | -0.02512238 |
| H     | -0.00000000 | 0.00321961  | -0.03585820 |
| H     | -0.00000000 | 0.00321826  | -0.03585871 |
| Atom# | 9           |             |             |
| Atom  | X           | Y           | Z           |
| O     | 0.00000000  | 0.00000000  | -0.00000000 |
| C     | 0.00000000  | -0.00000000 | -0.00000000 |
| H     | 0.00000000  | 0.00000000  | 0.00000000  |
| O     | 0.00000000  | -0.00000000 | -0.00000000 |
| C     | 0.00000000  | 0.00000000  | -0.00000000 |
| H     | 0.00000000  | 0.00000000  | -0.00000000 |
| H     | -0.00000000 | -0.00000000 | -0.00000000 |
| H     | 0.00000000  | -0.00000000 | -0.00000000 |
| Atom# | 10          |             |             |
| Atom  | X           | Y           | Z           |
| H     | -0.00000000 | -0.00000000 | 0.00000000  |
| O     | 0.00000000  | 0.00000000  | 0.00000000  |
| C     | 0.00000000  | 0.00000000  | -0.00000000 |
| H     | 0.00000000  | 0.00000000  | -0.00000000 |
| H     | -1.76956789 | -0.00000000 | -0.00000000 |
| H     | 1.76956776  | -0.00000000 | -0.00000000 |
| ***** |             |             |             |

| CART WANTED (CART OF MOLECULE) |             |             |             |
|--------------------------------|-------------|-------------|-------------|
| *****                          |             |             |             |
| Atom#                          | 1           |             |             |
| Atom                           | X           | Y           | Z           |
| N                              | 0.00000000  | 0.00000000  | 0.00000000  |
| C                              | 0.00000000  | 0.00000000  | 1.44084286  |
| H                              | 0.73050930  | -0.58944902 | -0.34663973 |
| H                              | -0.87404490 | -0.32011104 | -0.36473314 |
| C                              | 1.42338653  | 0.00000000  | 1.96785902  |
| H                              | -0.52202178 | 0.87980599  | 1.80690743  |
| H                              | -0.48626030 | -0.86672789 | 1.89787416  |
| H                              | 1.40752011  | 0.00000000  | 3.05140244  |
| H                              | 1.93964311  | 0.88478395  | 1.61422314  |
| H                              | 1.93966847  | -0.88478388 | 1.61415464  |
| Atom#                          | 2           |             |             |
| Atom                           | X           | Y           | Z           |
| C                              | 0.00000000  | 0.00000000  | 1.44084286  |
| N                              | 0.00000000  | 0.00000000  | 0.00000000  |
| C                              | 1.42338653  | 0.00000000  | 1.96785902  |
| H                              | -0.52202178 | 0.87980599  | 1.80690743  |
| H                              | -0.48626030 | -0.86672789 | 1.89787416  |
| O                              | 1.56620070  | 0.37919230  | 3.23785258  |
| O                              | 2.35288801  | -0.36180632 | 1.33520098  |
| H                              | 0.73050930  | -0.58944902 | -0.34663973 |
| H                              | -0.87404490 | -0.32011104 | -0.36473314 |
| H                              | 2.53932814  | 0.31941883  | 3.40629958  |
| Atom#                          | 3           |             |             |
| Atom                           | X           | Y           | Z           |
| C                              | 1.42338653  | 0.00000000  | 1.96785902  |
| O                              | 1.56620070  | 0.37919230  | 3.23785258  |
| O                              | 2.35288801  | -0.36180632 | 1.33520098  |
| C                              | 0.00000000  | 0.00000000  | 1.44084286  |
| N                              | 0.00000000  | 0.00000000  | 0.00000000  |
| H                              | 0.75176248  | 0.68026803  | 3.61808458  |
| H                              | -0.52202178 | 0.87980599  | 1.80690743  |
| H                              | -0.48626030 | -0.86672789 | 1.89787416  |
| H                              | -0.95777357 | 0.00000000  | -0.29610147 |
| H                              | 0.40141812  | -0.86960710 | -0.29610147 |
| Atom#                          | 4           |             |             |
| Atom                           | X           | Y           | Z           |
| O                              | 2.35288801  | -0.36180632 | 1.33520098  |
| C                              | 1.42338653  | 0.00000000  | 1.96785902  |
| O                              | 1.56620070  | 0.37919230  | 3.23785258  |
| C                              | 0.00000000  | 0.00000000  | 1.44084286  |
| H                              | 0.65316574  | 0.62862176  | 3.52608235  |
| H                              | -0.66879662 | 0.37530236  | 2.20646504  |
| H                              | -0.28673497 | -1.01059766 | 1.17457750  |
| H                              | -0.06099919 | 0.63521777  | 0.56490118  |
| Atom#                          | 5           |             |             |
| Atom                           | X           | Y           | Z           |
| H                              | -0.52202178 | 0.87980599  | 1.80690743  |
| C                              | 0.00000000  | 0.00000000  | 1.44084286  |
| N                              | 0.00000000  | 0.00000000  | 0.00000000  |
| C                              | 1.42338653  | 0.00000000  | 1.96785902  |
| H                              | -0.48626030 | -0.86672789 | 1.89787416  |
| H                              | 0.48872922  | -0.82369531 | -0.29610147 |
| H                              | -0.95270540 | -0.09851616 | -0.29610147 |
| H                              | 1.94391057  | -0.87751344 | 1.60269742  |
| H                              | 1.93528625  | 0.89191159  | 1.62599031  |
| H                              | 1.40752416  | -0.01421698 | 3.05139149  |
| Atom#                          | 6           |             |             |
| Atom                           | X           | Y           | Z           |
| H                              | -0.48626030 | -0.86672789 | 1.89787416  |
| C                              | 0.00000000  | 0.00000000  | 1.44084286  |

|   |             |             |             |
|---|-------------|-------------|-------------|
| N                                       | 0.00000000  | 0.00000000  | 0.00000000  |
| C                                       | 1.42338653  | 0.00000000  | 1.96785902  |
| H                                       | -0.52202178 | 0.87980599  | 1.80690743  |
| H                                       | 0.46862617  | 0.83529619  | -0.29610147 |
| H                                       | -0.95481287 | 0.07540156  | -0.29610147 |
| H                                       | 1.96357598  | 0.84116568  | 1.54958427  |
| H                                       | 1.91452219  | -0.92278526 | 1.68207074  |
| H                                       | 1.40861876  | 0.08144592  | 3.04843517  |
| Atom#                                   | 7           |             |             |
| Atom                                    | X           | Y           | Z           |
| H                                       | 0.73050930  | -0.58944902 | -0.34663973 |
| N                                       | 0.00000000  | 0.00000000  | 0.00000000  |
| C                                       | 0.00000000  | 0.00000000  | 1.44084286  |
| H                                       | -0.87404490 | -0.32011104 | -0.36473314 |
| H                                       | -0.79508214 | 0.64155293  | 1.80219079  |
| H                                       | -0.15801825 | -1.00939572 | 1.80219079  |
| H                                       | 0.95326447  | 0.36771031  | 1.80219079  |
| Atom#                                   | 8           |             |             |
| Atom                                    | X           | Y           | Z           |
| H                                       | -0.87404490 | -0.32011104 | -0.36473314 |
| N                                       | 0.00000000  | 0.00000000  | 0.00000000  |
| C                                       | 0.00000000  | 0.00000000  | 1.44084286  |
| H                                       | 0.73050930  | -0.58944902 | -0.34663973 |
| H                                       | 0.95932437  | 0.35134388  | 1.80219079  |
| H                                       | -0.17544770 | -1.00651259 | 1.80219079  |
| H                                       | -0.78407467 | 0.65509612  | 1.80219079  |
| Atom#                                   | 9           |             |             |
| Atom                                    | X           | Y           | Z           |
| O                                       | 1.56620070  | 0.37919230  | 3.23785258  |
| C                                       | 1.42338653  | 0.00000000  | 1.96785902  |
| H                                       | 0.75176248  | 0.68026803  | 3.61808458  |
| O                                       | 2.35288801  | -0.36180632 | 1.33520098  |
| C                                       | 0.00000000  | 0.00000000  | 1.44084286  |
| H                                       | -0.00222494 | -0.32209824 | 0.40616133  |
| H                                       | -0.60406692 | -0.67858912 | 2.03164044  |
| H                                       | -0.41037651 | 1.00075379  | 1.50851356  |
| Atom#                                   | 10          |             |             |
| Atom                                    | X           | Y           | Z           |
| H                                       | 0.75176248  | 0.68026803  | 3.61808458  |
| O                                       | 1.56620070  | 0.37919230  | 3.23785258  |
| C                                       | 1.42338653  | 0.00000000  | 1.96785902  |
| H                                       | 2.37886126  | -0.33411421 | 1.58087862  |
| H                                       | 0.70717887  | -0.81134180 | 1.91135348  |
| H                                       | 1.06777838  | 0.83714620  | 1.37860065  |
| *****                                   |             |             |             |
| The DatabaseInform Matrix for each atom |             |             |             |
| *****                                   |             |             |             |
| Atom#                                   | 1           |             |             |
|   | 1           | 2           | 3           |
| 1                                       | -0.627965   | 0.778242    | 0.000000    |
| 2                                       | -0.778242   | -0.627965   | -0.000000   |
| 3                                       | -0.000000   | -0.000000   | 1.000000    |
| Atom#                                   | 2           |             |             |
|   | 1           | 2           | 3           |
| 1                                       | 0.000000    | 1.000000    | 0.000000    |
| 2                                       | 1.000000    | -0.000000   | 0.000000    |
| 3                                       | 0.000000    | 0.000000    | -1.000000   |
| Atom#                                   | 3           |             |             |
|   | 1           | 2           | 3           |
| 1                                       | -0.162483   | 0.980878    | 0.107132    |
| 2                                       | -0.940310   | -0.186830   | 0.284451    |
| 3                                       | 0.299027    | -0.054519   | 0.952686    |

|                |             |             |             |
|----------------|-------------|-------------|-------------|
| Atom#          | 4           |             |             |
|                | 1           | 2           | 3           |
| 1              | -0.162483   | 0.595250    | -0.786941   |
| 2              | -0.940310   | 0.148287    | 0.306315    |
| 3              | 0.299027    | 0.789739    | 0.535625    |
| Atom#          | 5           |             |             |
|                | 1           | 2           | 3           |
| 1              | 0.860010    | -0.171916   | 0.480444    |
| 2              | 0.510276    | 0.289745    | -0.809732   |
| 3              | -0.000000   | -0.941537   | -0.336909   |
| Atom#          | 6           |             |             |
|                | 1           | 2           | 3           |
| 1              | 0.866107    | -0.199226   | 0.458441    |
| 2              | -0.499835   | -0.336286   | 0.798171    |
| 3              | -0.004849   | -0.920446   | -0.390840   |
| Atom#          | 7           |             |             |
|                | 1           | 2           | 3           |
| 1              | -0.627965   | 0.269601    | -0.730052   |
| 2              | -0.778242   | -0.217541   | 0.589080    |
| 3              | 0.000000    | 0.938079    | 0.346423    |
| Atom#          | 8           |             |             |
|                | 1           | 2           | 3           |
| 1              | 0.343739    | -0.339163   | 0.875678    |
| 2              | -0.939065   | -0.123900   | 0.320632    |
| 3              | 0.000250    | 0.932533    | 0.361086    |
| Atom#          | 9           |             |             |
|                | 1           | 2           | 3           |
| 1              | 0.203818    | -0.973129   | -0.107132   |
| 2              | 0.931567    | 0.226429    | -0.284451   |
| 3              | -0.301066   | 0.041824    | -0.952686   |
| Atom#          | 10          |             |             |
|                | 1           | 2           | 3           |
| 1              | -0.203818   | -0.469301   | 0.859194    |
| 2              | -0.931567   | -0.176917   | -0.317621   |
| 3              | 0.301066    | -0.865134   | -0.401127   |
| *****          |             |             |             |
| CART After ROT |             |             |             |
| *****          |             |             |             |
| Atom#          | 1           |             |             |
| Atom           | X           | Y           | Z           |
| N              | -0.00000000 | 0.00000000  | 0.00000000  |
| C              | -0.00000000 | -0.00000000 | 1.44084286  |
| H              | 0.73050930  | -0.58944902 | -0.34663973 |
| H              | -0.87404490 | -0.32011104 | -0.36473314 |
| C              | 1.42338653  | -0.00000000 | 1.96785902  |
| H              | -0.52202178 | 0.87980599  | 1.80690743  |
| H              | -0.48626030 | -0.86672789 | 1.89787416  |
| H              | 1.40752011  | -0.00000000 | 3.05140244  |
| H              | 1.93964311  | 0.88478395  | 1.61422314  |
| H              | 1.93966847  | -0.88478388 | 1.61415464  |
| Atom#          | 2           |             |             |
| Atom           | X           | Y           | Z           |
| C              | -0.00000000 | -0.00000000 | 1.44084286  |
| N              | 0.00000000  | 0.00000000  | -0.00000000 |
| C              | 1.42338653  | -0.00000000 | 1.96785902  |
| H              | -0.52202178 | 0.87980599  | 1.80690743  |

|       |             |             |             |
|-------|-------------|-------------|-------------|
| H     | -0.48626030 | -0.86672789 | 1.89787416  |
| O     | 1.56620070  | 0.37919230  | 3.23785258  |
| O     | 2.35288801  | -0.36180632 | 1.33520098  |
| H     | 0.73050930  | -0.58944902 | -0.34663973 |
| H     | -0.87404490 | -0.32011104 | -0.36473314 |
| H     | 2.53932814  | 0.31941883  | 3.40629958  |
| Atom# | 3           |             |             |
| Atom  | X           | Y           | Z           |
| C     | 1.42338653  | -0.00000000 | 1.96785902  |
| O     | 1.56620070  | 0.37919230  | 3.23785258  |
| O     | 2.35288801  | -0.36180632 | 1.33520098  |
| C     | -0.00000000 | 0.00000000  | 1.44084286  |
| N     | -0.00000000 | 0.00000000  | -0.00000000 |
| H     | 0.75176248  | 0.68026803  | 3.61808458  |
| H     | -0.52202178 | 0.87980599  | 1.80690743  |
| H     | -0.48626030 | -0.86672789 | 1.89787416  |
| H     | -0.95777357 | 0.00000000  | -0.29610147 |
| H     | 0.40141812  | -0.86960710 | -0.29610147 |
| Atom# | 4           |             |             |
| Atom  | X           | Y           | Z           |
| O     | 2.35288801  | -0.36180632 | 1.33520098  |
| C     | 1.42338653  | 0.00000000  | 1.96785902  |
| O     | 1.56620070  | 0.37919230  | 3.23785258  |
| C     | 0.00000000  | 0.00000000  | 1.44084286  |
| H     | 0.65316574  | 0.62862176  | 3.52608235  |
| H     | -0.66879662 | 0.37530236  | 2.20646504  |
| H     | -0.28673497 | -1.01059766 | 1.17457750  |
| H     | -0.06099919 | 0.63521777  | 0.56490118  |
| Atom# | 5           |             |             |
| Atom  | X           | Y           | Z           |
| H     | -0.52202178 | 0.87980599  | 1.80690743  |
| C     | -0.00000000 | 0.00000000  | 1.44084286  |
| N     | -0.00000000 | 0.00000000  | -0.00000000 |
| C     | 1.42338653  | 0.00000000  | 1.96785902  |
| H     | -0.48626030 | -0.86672789 | 1.89787416  |
| H     | 0.48872922  | -0.82369531 | -0.29610147 |
| H     | -0.95270540 | -0.09851616 | -0.29610147 |
| H     | 1.94391057  | -0.87751344 | 1.60269742  |
| H     | 1.93528625  | 0.89191159  | 1.62599031  |
| H     | 1.40752416  | -0.01421698 | 3.05139149  |
| Atom# | 6           |             |             |
| Atom  | X           | Y           | Z           |
| H     | -0.48707570 | -0.86598074 | 1.89431168  |
| C     | 0.01103837  | 0.00126345  | 1.46964890  |
| N     | -0.03669094 | -0.06748657 | 0.03123884  |
| C     | 1.45098101  | 0.04267375  | 1.94779700  |
| H     | -0.47105478 | 0.88319877  | 1.90129715  |
| H     | 0.43119953  | 0.74667241  | -0.31980238 |
| H     | -0.99988359 | 0.00589677  | -0.23691020 |
| H     | 1.94765803  | 0.90765413  | 1.52419460  |
| H     | 1.96283998  | -0.85862697 | 1.63144402  |
| H     | 1.47083672  | 0.10839327  | 3.02936209  |
| Atom# | 7           |             |             |
| Atom  | X           | Y           | Z           |
| H     | 0.73050930  | -0.58944902 | -0.34663973 |
| N     | 0.00000000  | 0.00000000  | 0.00000000  |
| C     | 0.00000000  | 0.00000000  | 1.44084286  |
| H     | -0.87404490 | -0.32011104 | -0.36473314 |
| H     | -0.79508214 | 0.64155293  | 1.80219079  |
| H     | -0.15801825 | -1.00939572 | 1.80219079  |
| H     | 0.95326447  | 0.36771031  | 1.80219079  |
| Atom# | 8           |             |             |
| Atom  | X           | Y           | Z           |
| H     | -0.87407016 | -0.32011678 | -0.36417988 |

|                 |             |             |             |
|-----------------|-------------|-------------|-------------|
| N               | 0.00215639  | 0.00071617  | -0.00286791 |
| C               | -0.01917809 | -0.00670999 | 1.43779785  |
| H               | 0.73180217  | -0.58207056 | -0.35984482 |
| H               | 0.93473778  | 0.34256487  | 1.81511691  |
| H               | -0.20009206 | -1.01503466 | 1.79131350  |
| H               | -0.80842936 | 0.64668007  | 1.79087037  |
| Atom#           | 9           |             |             |
| Atom            | X           | Y           | Z           |
| O               | 1.56620070  | 0.37919230  | 3.23785258  |
| C               | 1.42338653  | 0.00000000  | 1.96785902  |
| H               | 0.75176248  | 0.68026803  | 3.61808458  |
| O               | 2.35288801  | -0.36180632 | 1.33520098  |
| C               | -0.00000000 | 0.00000000  | 1.44084286  |
| H               | -0.00222494 | -0.32209824 | 0.40616133  |
| H               | -0.60406692 | -0.67858912 | 2.03164044  |
| H               | -0.41037651 | 1.00075379  | 1.50851356  |
| Atom#           | 10          |             |             |
| Atom            | X           | Y           | Z           |
| H               | 0.75176248  | 0.68026803  | 3.61808458  |
| O               | 1.56620070  | 0.37919230  | 3.23785258  |
| C               | 1.42338653  | -0.00000000 | 1.96785902  |
| H               | 2.37886126  | -0.33411421 | 1.58087862  |
| H               | 1.06784948  | 0.83712972  | 1.37859757  |
| H               | 0.70710779  | -0.81132520 | 1.91135652  |
| *****           |             |             |             |
| CART Diff error |             |             |             |
| *****           |             |             |             |
| Atom#           | 1           |             |             |
| Atom            | X           | Y           | Z           |
| N               | -0.00000000 | 0.00000000  | 0.00000000  |
| C               | -0.00000000 | -0.00000000 | 0.00000000  |
| H               | 0.00000000  | -0.00000000 | 0.00000000  |
| H               | -0.00000000 | -0.00000000 | 0.00000000  |
| C               | 0.00000000  | -0.00000000 | -0.00000000 |
| H               | 0.00000000  | 0.00000000  | -0.00000000 |
| H               | -0.00000000 | -0.00000000 | -0.00000000 |
| H               | 0.00000000  | -0.00000000 | -0.00000000 |
| H               | -0.00000000 | 0.00000000  | -0.00000000 |
| H               | 0.00000000  | -0.00000000 | -0.00000000 |
| Atom#           | 2           |             |             |
| Atom            | X           | Y           | Z           |
| C               | -0.00000000 | -0.00000000 | -0.00000000 |
| N               | 0.00000000  | 0.00000000  | -0.00000000 |
| C               | -0.00000000 | -0.00000000 | -0.00000000 |
| H               | 0.00000000  | 0.00000000  | -0.00000000 |
| H               | -0.00000000 | -0.00000000 | -0.00000000 |
| O               | -0.00000000 | 0.00000000  | -0.00000000 |
| O               | -0.00000000 | -0.00000000 | -0.00000000 |
| H               | 0.00000000  | 0.00000000  | 0.00000000  |
| H               | 0.00000000  | -0.00000000 | -0.00000000 |
| H               | -0.00000000 | 0.00000000  | -0.00000000 |
| Atom#           | 3           |             |             |
| Atom            | X           | Y           | Z           |
| C               | -0.00000000 | -0.00000000 | -0.00000000 |
| O               | -0.00000000 | -0.00000000 | -0.00000000 |
| O               | -0.00000000 | 0.00000000  | -0.00000000 |
| C               | -0.00000000 | 0.00000000  | -0.00000000 |
| N               | -0.00000000 | 0.00000000  | -0.00000000 |
| H               | -0.00000000 | -0.00000000 | -0.00000000 |
| H               | -0.00000000 | 0.00000000  | -0.00000000 |
| H               | -0.00000000 | 0.00000000  | -0.00000000 |
| H               | -0.00000000 | 0.00000000  | -0.00000000 |
| H               | -0.00000000 | 0.00000000  | -0.00000000 |
| Atom#           | 4           |             |             |

| Atom  | X           | Y           | Z           |
|-------|-------------|-------------|-------------|
| O     | 0.00000000  | 0.00000000  | 0.00000000  |
| C     | 0.00000000  | 0.00000000  | 0.00000000  |
| O     | 0.00000000  | 0.00000000  | 0.00000000  |
| C     | 0.00000000  | 0.00000000  | 0.00000000  |
| H     | 0.00000000  | 0.00000000  | 0.00000000  |
| H     | 0.00000000  | 0.00000000  | 0.00000000  |
| H     | 0.00000000  | -0.00000000 | 0.00000000  |
| H     | 0.00000000  | -0.00000000 | -0.00000000 |
| Atom# | 5           |             |             |
| Atom  | X           | Y           | Z           |
| H     | -0.00000000 | 0.00000000  | -0.00000000 |
| C     | -0.00000000 | 0.00000000  | -0.00000000 |
| N     | -0.00000000 | 0.00000000  | -0.00000000 |
| C     | -0.00000000 | 0.00000000  | -0.00000000 |
| H     | -0.00000000 | 0.00000000  | -0.00000000 |
| H     | -0.00000000 | 0.00000000  | -0.00000000 |
| H     | -0.00000000 | 0.00000000  | -0.00000000 |
| H     | -0.00000000 | 0.00000000  | -0.00000000 |
| H     | -0.00000000 | 0.00000000  | -0.00000000 |
| H     | -0.00000000 | 0.00000000  | -0.00000000 |
| Atom# | 6           |             |             |
| Atom  | X           | Y           | Z           |
| H     | -0.00081540 | 0.00074714  | -0.00356248 |
| C     | 0.01103837  | 0.00126345  | 0.02880604  |
| N     | -0.03669094 | -0.06748657 | 0.03123884  |
| C     | 0.02759448  | 0.04267375  | -0.02006202 |
| H     | 0.05096700  | 0.00339277  | 0.09438972  |
| H     | -0.03742664 | -0.08862377 | -0.02370091 |
| H     | -0.04507072 | -0.06950479 | 0.05919127  |
| H     | -0.01591795 | 0.06648845  | -0.02538967 |
| H     | 0.04831779  | 0.06415829  | -0.05062672 |
| H     | 0.06221796  | 0.02694736  | -0.01907308 |
| Atom# | 7           |             |             |
| Atom  | X           | Y           | Z           |
| H     | -0.00000000 | 0.00000000  | -0.00000000 |
| N     | 0.00000000  | 0.00000000  | 0.00000000  |
| C     | 0.00000000  | 0.00000000  | -0.00000000 |
| H     | 0.00000000  | 0.00000000  | 0.00000000  |
| H     | 0.00000000  | 0.00000000  | 0.00000000  |
| H     | 0.00000000  | 0.00000000  | -0.00000000 |
| H     | 0.00000000  | -0.00000000 | -0.00000000 |
| Atom# | 8           |             |             |
| Atom  | X           | Y           | Z           |
| H     | -0.00002526 | -0.00000573 | 0.00055326  |
| N     | 0.00215639  | 0.00071617  | -0.00286791 |
| C     | -0.01917809 | -0.00670999 | -0.00304501 |
| H     | 0.00129286  | 0.00737846  | -0.01320508 |
| H     | -0.02458659 | -0.00877901 | 0.01292612  |
| H     | -0.02464436 | -0.00852207 | -0.01087729 |
| H     | -0.02435469 | -0.00841606 | -0.01132042 |
| Atom# | 9           |             |             |
| Atom  | X           | Y           | Z           |
| O     | -0.00000000 | 0.00000000  | 0.00000000  |
| C     | -0.00000000 | 0.00000000  | 0.00000000  |
| H     | -0.00000000 | 0.00000000  | 0.00000000  |
| O     | -0.00000000 | 0.00000000  | 0.00000000  |
| C     | -0.00000000 | 0.00000000  | 0.00000000  |
| H     | -0.00000000 | 0.00000000  | 0.00000000  |
| H     | -0.00000000 | 0.00000000  | 0.00000000  |
| H     | -0.00000000 | 0.00000000  | 0.00000000  |
| Atom# | 10          |             |             |
| Atom  | X           | Y           | Z           |
| H     | 0.00000000  | -0.00000000 | -0.00000000 |



|                   |             |             |             |
|-------------------|-------------|-------------|-------------|
| O                 | 0.00000000  | -0.00000000 | -0.00000000 |
| C                 | 0.00000000  | -0.00000000 | -0.00000000 |
| H                 | 0.00000000  | -0.00000000 | -0.00000000 |
| H                 | 0.36067062  | 1.64847152  | -0.53275591 |
| H                 | -0.36067059 | -1.64847141 | 0.53275587  |
| *****             |             |             |             |
| CART after TTCORR |             |             |             |
| *****             |             |             |             |
| Atom#             | 1           |             |             |
| Atom              | X           | Y           | Z           |
| N                 | 0.00000000  | 0.00000000  | 0.00000000  |
| C                 | -0.00000000 | -0.00000000 | 1.44084286  |
| H                 | 0.73050930  | -0.58944902 | -0.34663973 |
| H                 | -0.87404490 | -0.32011104 | -0.36473314 |
| C                 | 1.42338653  | -0.00000000 | 1.96785902  |
| H                 | -0.52202178 | 0.87980599  | 1.80690743  |
| H                 | -0.48626030 | -0.86672789 | 1.89787416  |
| H                 | 1.40752011  | -0.00000000 | 3.05140244  |
| H                 | 1.93964311  | 0.88478395  | 1.61422314  |
| H                 | 1.93966847  | -0.88478388 | 1.61415464  |
| Atom#             | 2           |             |             |
| Atom              | X           | Y           | Z           |
| C                 | 0.00000000  | 0.00000000  | 1.44084286  |
| N                 | 0.00000000  | 0.00000000  | -0.00000000 |
| C                 | 1.42338653  | -0.00000000 | 1.96785902  |
| H                 | -0.52202178 | 0.87980599  | 1.80690743  |
| H                 | -0.48626030 | -0.86672789 | 1.89787416  |
| O                 | 1.56620070  | 0.37919230  | 3.23785258  |
| O                 | 2.35288801  | -0.36180632 | 1.33520098  |
| H                 | 0.73050930  | -0.58944902 | -0.34663973 |
| H                 | -0.87404490 | -0.32011104 | -0.36473314 |
| H                 | 2.53932814  | 0.31941883  | 3.40629958  |
| Atom#             | 3           |             |             |
| Atom              | X           | Y           | Z           |
| C                 | 1.42338653  | 0.00000000  | 1.96785902  |
| O                 | 1.56620070  | 0.37919230  | 3.23785258  |
| O                 | 2.35288801  | -0.36180632 | 1.33520098  |
| C                 | 0.00000000  | 0.00000000  | 1.44084286  |
| N                 | 0.00000000  | 0.00000000  | 0.00000000  |
| H                 | 0.75176248  | 0.68026803  | 3.61808458  |
| H                 | -0.52202178 | 0.87980599  | 1.80690743  |
| H                 | -0.48626030 | -0.86672789 | 1.89787416  |
| H                 | -0.95777357 | 0.00000000  | -0.29610147 |
| H                 | 0.40141812  | -0.86960710 | -0.29610147 |
| Atom#             | 4           |             |             |
| Atom              | X           | Y           | Z           |
| O                 | 2.35288801  | -0.36180632 | 1.33520098  |
| C                 | 1.42338653  | -0.00000000 | 1.96785902  |
| O                 | 1.56620070  | 0.37919230  | 3.23785258  |
| C                 | 0.00000000  | -0.00000000 | 1.44084286  |
| H                 | 0.65316574  | 0.62862176  | 3.52608235  |
| H                 | -0.66879662 | 0.37530236  | 2.20646504  |
| H                 | -0.28673497 | -1.01059766 | 1.17457750  |
| H                 | -0.06099919 | 0.63521777  | 0.56490118  |
| Atom#             | 5           |             |             |
| Atom              | X           | Y           | Z           |
| H                 | -0.52202178 | 0.87980599  | 1.80690743  |
| C                 | 0.00000000  | -0.00000000 | 1.44084286  |
| N                 | -0.00000000 | 0.00000000  | -0.00000000 |
| C                 | 1.42338653  | 0.00000000  | 1.96785902  |
| H                 | -0.48626030 | -0.86672789 | 1.89787416  |
| H                 | 0.48872922  | -0.82369531 | -0.29610147 |
| H                 | -0.95270540 | -0.09851616 | -0.29610147 |
| H                 | 1.94391057  | -0.87751344 | 1.60269742  |

|                              |             |             |             |
|------------------------------|-------------|-------------|-------------|
| H                            | 1.93528625  | 0.89191159  | 1.62599031  |
| H                            | 1.40752416  | -0.01421698 | 3.05139149  |
| Atom#                        | 6           |             |             |
| Atom                         | X           | Y           | Z           |
| H                            | -0.48626030 | -0.86672789 | 1.89787416  |
| C                            | 0.01185377  | 0.00051630  | 1.47321138  |
| N                            | -0.03587554 | -0.06823372 | 0.03480132  |
| C                            | 1.45179641  | 0.04192661  | 1.95135948  |
| H                            | -0.47023938 | 0.88245162  | 1.90485963  |
| H                            | 0.43201493  | 0.74592527  | -0.31623990 |
| H                            | -0.99906819 | 0.00514962  | -0.23334772 |
| H                            | 1.94847343  | 0.90690698  | 1.52775708  |
| H                            | 1.96365538  | -0.85937412 | 1.63500650  |
| H                            | 1.47165212  | 0.10764613  | 3.03292457  |
| Atom#                        | 7           |             |             |
| Atom                         | X           | Y           | Z           |
| H                            | 0.73050930  | -0.58944902 | -0.34663973 |
| N                            | 0.00000000  | 0.00000000  | 0.00000000  |
| C                            | 0.00000000  | -0.00000000 | 1.44084286  |
| H                            | -0.87404490 | -0.32011104 | -0.36473314 |
| H                            | -0.79508214 | 0.64155293  | 1.80219079  |
| H                            | -0.15801825 | -1.00939572 | 1.80219079  |
| H                            | 0.95326447  | 0.36771031  | 1.80219079  |
| Atom#                        | 8           |             |             |
| Atom                         | X           | Y           | Z           |
| H                            | -0.87404490 | -0.32011104 | -0.36473314 |
| N                            | 0.00218165  | 0.00072190  | -0.00342117 |
| C                            | -0.01915283 | -0.00670426 | 1.43724459  |
| H                            | 0.73182743  | -0.58206482 | -0.36039808 |
| H                            | 0.93476304  | 0.34257060  | 1.81456365  |
| H                            | -0.20006680 | -1.01502893 | 1.79076024  |
| H                            | -0.80840410 | 0.64668580  | 1.79031711  |
| Atom#                        | 9           |             |             |
| Atom                         | X           | Y           | Z           |
| O                            | 1.56620070  | 0.37919230  | 3.23785258  |
| C                            | 1.42338653  | 0.00000000  | 1.96785902  |
| H                            | 0.75176248  | 0.68026803  | 3.61808458  |
| O                            | 2.35288801  | -0.36180632 | 1.33520098  |
| C                            | -0.00000000 | 0.00000000  | 1.44084286  |
| H                            | -0.00222494 | -0.32209824 | 0.40616133  |
| H                            | -0.60406692 | -0.67858912 | 2.03164044  |
| H                            | -0.41037651 | 1.00075379  | 1.50851356  |
| Atom#                        | 10          |             |             |
| Atom                         | X           | Y           | Z           |
| H                            | 0.75176248  | 0.68026803  | 3.61808458  |
| O                            | 1.56620070  | 0.37919230  | 3.23785258  |
| C                            | 1.42338653  | 0.00000000  | 1.96785902  |
| H                            | 2.37886126  | -0.33411421 | 1.58087862  |
| H                            | 1.06784948  | 0.83712972  | 1.37859757  |
| H                            | 0.70710779  | -0.81132520 | 1.91135652  |
| *****                        |             |             |             |
| CART Diff error after TTCORR |             |             |             |
| *****                        |             |             |             |
| Atom#                        | 1           |             |             |
| Atom                         | X           | Y           | Z           |
| N                            | 0.00000000  | 0.00000000  | 0.00000000  |
| C                            | -0.00000000 | -0.00000000 | 0.00000000  |
| H                            | 0.00000000  | -0.00000000 | 0.00000000  |
| H                            | -0.00000000 | -0.00000000 | 0.00000000  |
| C                            | 0.00000000  | -0.00000000 | -0.00000000 |
| H                            | 0.00000000  | 0.00000000  | -0.00000000 |
| H                            | -0.00000000 | -0.00000000 | -0.00000000 |
| H                            | 0.00000000  | -0.00000000 | -0.00000000 |
| H                            | -0.00000000 | 0.00000000  | -0.00000000 |

|       |             |             |             |
|-------|-------------|-------------|-------------|
| H     | 0.00000000  | -0.00000000 | -0.00000000 |
| Atom# | 2           |             |             |
| Atom  | X           | Y           | Z           |
| C     | 0.00000000  | 0.00000000  | 0.00000000  |
| N     | 0.00000000  | 0.00000000  | -0.00000000 |
| C     | 0.00000000  | -0.00000000 | -0.00000000 |
| H     | 0.00000000  | 0.00000000  | -0.00000000 |
| H     | 0.00000000  | -0.00000000 | -0.00000000 |
| O     | -0.00000000 | 0.00000000  | 0.00000000  |
| O     | -0.00000000 | -0.00000000 | -0.00000000 |
| H     | 0.00000000  | 0.00000000  | 0.00000000  |
| H     | 0.00000000  | -0.00000000 | -0.00000000 |
| H     | -0.00000000 | 0.00000000  | 0.00000000  |
| Atom# | 3           |             |             |
| Atom  | X           | Y           | Z           |
| C     | 0.00000000  | 0.00000000  | 0.00000000  |
| O     | -0.00000000 | -0.00000000 | 0.00000000  |
| O     | 0.00000000  | 0.00000000  | 0.00000000  |
| C     | 0.00000000  | 0.00000000  | 0.00000000  |
| N     | 0.00000000  | 0.00000000  | 0.00000000  |
| H     | -0.00000000 | -0.00000000 | 0.00000000  |
| H     | 0.00000000  | 0.00000000  | 0.00000000  |
| H     | 0.00000000  | 0.00000000  | -0.00000000 |
| H     | 0.00000000  | 0.00000000  | -0.00000000 |
| H     | 0.00000000  | 0.00000000  | -0.00000000 |
| Atom# | 4           |             |             |
| Atom  | X           | Y           | Z           |
| O     | 0.00000000  | 0.00000000  | 0.00000000  |
| C     | -0.00000000 | -0.00000000 | -0.00000000 |
| O     | 0.00000000  | 0.00000000  | -0.00000000 |
| C     | 0.00000000  | -0.00000000 | -0.00000000 |
| H     | 0.00000000  | -0.00000000 | -0.00000000 |
| H     | -0.00000000 | -0.00000000 | -0.00000000 |
| H     | 0.00000000  | -0.00000000 | 0.00000000  |
| H     | 0.00000000  | -0.00000000 | -0.00000000 |
| Atom# | 5           |             |             |
| Atom  | X           | Y           | Z           |
| H     | 0.00000000  | 0.00000000  | 0.00000000  |
| C     | 0.00000000  | -0.00000000 | 0.00000000  |
| N     | -0.00000000 | 0.00000000  | -0.00000000 |
| C     | 0.00000000  | 0.00000000  | -0.00000000 |
| H     | -0.00000000 | 0.00000000  | -0.00000000 |
| H     | 0.00000000  | 0.00000000  | -0.00000000 |
| H     | -0.00000000 | 0.00000000  | 0.00000000  |
| H     | 0.00000000  | 0.00000000  | 0.00000000  |
| H     | -0.00000000 | 0.00000000  | -0.00000000 |
| H     | -0.00000000 | 0.00000000  | 0.00000000  |
| Atom# | 6           |             |             |
| Atom  | X           | Y           | Z           |
| H     | 0.00000000  | 0.00000000  | 0.00000000  |
| C     | 0.01185377  | 0.00051630  | 0.03236852  |
| N     | -0.03587554 | -0.06823372 | 0.03480132  |
| C     | 0.02840988  | 0.04192661  | -0.01649953 |
| H     | 0.05178240  | 0.00264563  | 0.09795221  |
| H     | -0.03661124 | -0.08937092 | -0.02013843 |
| H     | -0.04425532 | -0.07025193 | 0.06275375  |
| H     | -0.01510255 | 0.06574131  | -0.02182719 |
| H     | 0.04913319  | 0.06341114  | -0.04706424 |
| H     | 0.06303336  | 0.02620021  | -0.01551060 |
| Atom# | 7           |             |             |
| Atom  | X           | Y           | Z           |
| H     | 0.00000000  | 0.00000000  | 0.00000000  |
| N     | 0.00000000  | 0.00000000  | 0.00000000  |
| C     | 0.00000000  | -0.00000000 | 0.00000000  |

|       |             |             |             |
|-------|-------------|-------------|-------------|
| H     | 0.00000000  | -0.00000000 | 0.00000000  |
| H     | 0.00000000  | -0.00000000 | 0.00000000  |
| H     | 0.00000000  | -0.00000000 | -0.00000000 |
| H     | 0.00000000  | -0.00000000 | -0.00000000 |
| Atom# | 8           |             |             |
| Atom  | X           | Y           | Z           |
| H     | 0.00000000  | 0.00000000  | 0.00000000  |
| N     | 0.00218165  | 0.00072190  | -0.00342117 |
| C     | -0.01915283 | -0.00670426 | -0.00359827 |
| H     | 0.00131812  | 0.00738419  | -0.01375834 |
| H     | -0.02456133 | -0.00877328 | 0.01237286  |
| H     | -0.02461910 | -0.00851634 | -0.01143055 |
| H     | -0.02432943 | -0.00841033 | -0.01187368 |
| Atom# | 9           |             |             |
| Atom  | X           | Y           | Z           |
| O     | 0.00000000  | 0.00000000  | 0.00000000  |
| C     | 0.00000000  | 0.00000000  | -0.00000000 |
| H     | -0.00000000 | -0.00000000 | -0.00000000 |
| O     | 0.00000000  | 0.00000000  | 0.00000000  |
| C     | -0.00000000 | 0.00000000  | 0.00000000  |
| H     | 0.00000000  | 0.00000000  | 0.00000000  |
| H     | 0.00000000  | 0.00000000  | 0.00000000  |
| H     | 0.00000000  | 0.00000000  | 0.00000000  |
| Atom# | 10          |             |             |
| Atom  | X           | Y           | Z           |
| H     | 0.00000000  | 0.00000000  | 0.00000000  |
| O     | -0.00000000 | -0.00000000 | -0.00000000 |
| C     | -0.00000000 | 0.00000000  | -0.00000000 |
| H     | -0.00000000 | 0.00000000  | -0.00000000 |
| H     | 0.36067062  | 1.64847152  | -0.53275591 |
| H     | -0.36067059 | -1.64847141 | 0.53275587  |

```
|   Calculated molecule properties using the stored fragments properties
|                                     |
|   (already calculated, using fragment weights and geometry)
|                                     |
```

| Atom#                                 | Electrons   | Potential Vne | Vee Ana/Num   |             | HF Coulomb J | HF Exchange K |
|---------------------------------------|-------------|---------------|---------------|-------------|--------------|---------------|
|                                       | Coulomb J   | Exchange K    | Kinetic T     |             |              |               |
| +-----+-----+-----+-----+-----+-----+ |             |               |               |             |              |               |
|                                       | 1           | 7.16495286    | -167.95397335 | 46.80154137 | 44.37764402  | -6.64742071   |
|                                       | 38.96605300 | -1.23582969   | 54.00960395   |             |              |               |
|                                       | 2           | 6.10120522    | -147.17544847 | 48.23267128 | 47.81924437  | -5.27389078   |
|                                       | 43.47856678 | -0.93321319   | 37.96874603   |             |              |               |
|                                       | 3           | 6.12592201    | -156.56047131 | 53.04647809 | 48.88547298  | -5.37535134   |
|                                       | 44.46953361 | -0.95941197   | 38.69709666   |             |              |               |
|                                       | 4           | 8.14248522    | -232.18358599 | 63.92985103 | 66.55467483  | -8.20084481   |
|                                       | 60.21731428 | -1.86348426   | 74.22550128   |             |              |               |
|                                       | 5           | 0.93931951    | -8.63995813   | 3.66675627  | 4.93566560   | -0.33231778   |
|                                       | 4.77711621  | -0.17376839   | 0.59099384    |             |              |               |
|                                       | 6           | 0.93931951    | -8.63995813   | 3.66675627  | 4.93566560   | -0.33231778   |
|                                       | 4.77711621  | -0.17376839   | 0.59099384    |             |              |               |
|                                       | 7           | 0.91220048    | -7.30275687   | 3.00302056  | 3.46406769   | -0.33865852   |
|                                       | 3.28368436  | -0.15827519   | 0.68767134    |             |              |               |
|                                       | 8           | 0.91220048    | -7.30275687   | 3.00302056  | 3.46406769   | -0.33865852   |
|                                       | 3.28368436  | -0.15827519   | 0.68767134    |             |              |               |



```

    FD|    |%Error CW|
  1   7.16495286    7.15774153    7.14764497    0.01730789    0.01009656
    0.24214814    0.14125709
  2   6.10120522    6.10204839    6.11995184   -0.01874662   -0.01790345
    -0.30631971   -0.29254238
  3   6.12592201    6.12571670    6.12367153    0.00225048    0.00204517
    0.03675049    0.03339780
  4   8.14248522    8.12706805    8.13922047    0.00326475   -0.01215242
    0.04011128   -0.14930694
  5   0.93931951    0.93062947    0.92410217    0.01521734    0.00652730
    1.64671636    0.70634003
  6   0.93931951    0.93645619    0.92516674    0.01415277    0.01128945
    1.52975359    1.22026147
  7   0.91220048    0.88913919    0.87210384    0.04009664    0.01703535
    4.59769143    1.95336267
  8   0.91220048    0.91007803    0.90122937    0.01097112    0.00884867
    1.21735013    0.98184418
  9   8.00976335    8.00787444    8.00382871    0.00593465    0.00404574
    0.07414761    0.05054752
 10   0.86921696    0.85461607    0.84267594    0.02654102    0.01194012
    3.14961138    1.41692949
Sum=  40.11658560    40.04136806    39.99959557    0.11699003    0.04177249
    0.29247804    0.10443228

PROGRAM> end of inputs

Program terminated normally

Job: RUN_Gly_ ended on :24-Aug-18 at 16:01:05
User: ibrahim
Cpu    time:  00h01m10s23c on ibrahim-Lenovo-IdeaPad-P500
Elapsed time: 00h01m11s00c

```

Here is the results using the direct method,

```

Welcome to MUNgauss - July 9, 2018 Version 7

N_molecules: 1
Molecule is an asymmetric top.
Point group: C1
Free format Z-Matrix for: C2H5NO2, (C1)
N
C      N      B1
C      C      B2 N      A2
O      C      B3 C      A3 N      D3
H      C      B4 N      A4 C      D4
H      C      B5 N      A5 C      D5
H      N      B6 C      A6 C      D6
H      N      B7 C      A7 C      D7
O      C      B8 C      A8 N      D8
H      O      B9 C      A9 C      D9

VARIABLES:
B1 = 1.44084286      B2 = 1.51781924      B3 = 1.18115749
B4 = 1.08653973      B5 = 1.09386650      B6 = 1.00062635
B7 = 0.99972788      B8 = 1.33306652      B9 = 0.94790958
A2 = 110.31733       A3 = 123.50442       A4 = 109.68864
A5 = 114.69657       A6 = 110.26867       A7 = 111.39727
A8 = 115.54740       A9 = 112.36145       D3 = -21.55248
D4 = 120.68224       D5 = -119.29373     D6 = 38.90011

```

D7 = 159.88520                      D8 = 161.62252                      D9 = -5.50811

Z MATRIX FOR: C2H5NO2, (C1)

| I   | AN | Z1<br>Z4 | BL       |      | Z2 | ALPHA    |       | Z3 | BETA      |   |
|-----|----|----------|----------|------|----|----------|-------|----|-----------|---|
| 1   | 7  |          |          |      |    |          |       |    |           |   |
| 2   | 6  | 1        | 1.440843 | ( 1) |    |          |       |    |           |   |
| 3   | 6  | 2        | 1.517819 | ( 2) | 1  | 110.3173 | ( 10) |    |           |   |
| 4   | 8  | 3        | 1.181157 | ( 3) | 2  | 123.5044 | ( 11) | 1  | -21.5525  | ( |
| 18) |    | 0        |          |      |    |          |       |    |           |   |
| 5   | 1  | 2        | 1.086540 | ( 4) | 1  | 109.6886 | ( 12) | 3  | 120.6822  | ( |
| 19) |    | 0        |          |      |    |          |       |    |           |   |
| 6   | 1  | 2        | 1.093866 | ( 5) | 1  | 114.6966 | ( 13) | 3  | -119.2937 | ( |
| 20) |    | 0        |          |      |    |          |       |    |           |   |
| 7   | 1  | 1        | 1.000626 | ( 6) | 2  | 110.2687 | ( 14) | 3  | 38.9001   | ( |
| 21) |    | 0        |          |      |    |          |       |    |           |   |
| 8   | 1  | 1        | 0.999728 | ( 7) | 2  | 111.3973 | ( 15) | 3  | 159.8852  | ( |
| 22) |    | 0        |          |      |    |          |       |    |           |   |
| 9   | 8  | 3        | 1.333067 | ( 8) | 2  | 115.5474 | ( 16) | 1  | 161.6225  | ( |
| 23) |    | 0        |          |      |    |          |       |    |           |   |
| 10  | 1  | 9        | 0.947910 | ( 9) | 3  | 112.3615 | ( 17) | 2  | -5.5081   | ( |
| 24) |    | 0        |          |      |    |          |       |    |           |   |

Cartesian coordinates for: C2H5NO2, (C1)

| COORDINATES IN BOHR |    |             |             | COORDINATES IN ANGSTROMS |             |             |  |
|---------------------|----|-------------|-------------|--------------------------|-------------|-------------|--|
| I                   | EL | AN          | X           | Y                        | Z           | X           |  |
|                     |    | Y           | Z           |                          |             |             |  |
| 1                   | N  | 7           | 0.00000000  | 0.00000000               | 0.00000000  | 0.00000000  |  |
|                     |    | 0.00000000  | 0.00000000  |                          |             |             |  |
| 2                   | C  | 6           | 0.00000000  | 0.00000000               | 1.44084286  | 0.00000000  |  |
|                     |    | 0.00000000  | 2.72279820  |                          |             |             |  |
| 3                   | C  | 6           | 1.42338653  | 0.00000000               | 1.96785902  | 2.68981052  |  |
|                     |    | 0.00000000  | 3.71871433  |                          |             |             |  |
| 4                   | O  | 8           | 2.35288801  | -0.36180632              | 1.33520098  | 4.44631363  |  |
|                     |    | -0.68371481 | 2.52316398  |                          |             |             |  |
| 5                   | H  | 1           | -0.52202178 | 0.87980599               | 1.80690743  | -0.98647812 |  |
|                     |    | 1.66259225  | 3.41455992  |                          |             |             |  |
| 6                   | H  | 1           | -0.48626030 | -0.86672789              | 1.89787416  | -0.91889872 |  |
|                     |    | -1.63787822 | 3.58646212  |                          |             |             |  |
| 7                   | H  | 1           | 0.73050930  | -0.58944902              | -0.34663973 | 1.38046242  |  |
|                     |    | -1.11389713 | -0.65505411 |                          |             |             |  |
| 8                   | H  | 1           | -0.87404490 | -0.32011104              | -0.36473314 | -1.65170536 |  |
|                     |    | -0.60492216 | -0.68924570 |                          |             |             |  |
| 9                   | O  | 8           | 1.56620070  | 0.37919230               | 3.23785258  | 2.95969016  |  |
|                     |    | 0.71656954  | 6.11865417  |                          |             |             |  |
| 10                  | H  | 1           | 0.75176248  | 0.68026803               | 3.61808458  | 1.42062510  |  |
|                     |    | 1.28552018  | 6.83718846  |                          |             |             |  |

Nuclear repulsion energy: 181.510743561

```

Distance Matrix for: C2H5NO2, (C1)
      1      2      3      4      5      6
    7      8      9     10
  1  0.000000  1.440843  2.428682  2.729423  2.076410  2.142333
    1.000626  0.999728  3.616692  3.757452
  2  1.440843  0.000000  1.517819  2.382886  1.086540  1.093866
    2.018957  2.031386  2.413714  2.401727
  3  2.428682  1.517819  0.000000  1.181157  2.141163  2.098301
    2.486852  3.289627  1.333067  1.907115
  4  2.729423  2.382886  1.181157  0.000000  3.166893  2.938079
    2.347876  3.647549  2.188159  2.976757
  5  2.076410  1.086540  2.141163  3.166893  0.000000  1.749267
    2.892285  2.505942  2.580483  2.223219
  6  2.142333  1.093866  2.098301  2.938079  1.749267  0.000000
    2.568123  2.359779  2.749628  2.623932
  7  1.000626  2.018957  2.486852  2.347876  2.892285  2.568123
    0.000000  1.627103  3.805947  4.163132
  8  0.999728  2.031386  3.289627  3.647549  2.505942  2.359779
    1.627103  0.000000  4.407091  4.416655
  9  3.616692  2.413714  1.333067  2.188159  2.580483  2.749628
    3.805947  4.407091  0.000000  0.947910
 10  3.757452  2.401727  1.907115  2.976757  2.223219  2.623932
    4.163132  4.416655  0.947910  0.000000

Charge=      0, Number of electrons=      40

The basis set has now been re-ordered FDPS
The basis set has now been re-ordered FDPS
6-31G(d) Basis Set - Total number of basis functions:      85

The basis set has now been re-ordered FDPS
Projecting extended Huckel matrix (STO-3G) to 6-31G(d)
NOTE: INTEGRALS .LE. 1.00E-07 (I2EACC) WERE NOT KEPT
Exponent cutoff used: 2.00E+01 PQCUT2 cutoff used: 1.00E-16

    4124847 TWO-ELECTRON INTEGRALS CALCULATED IN IDFCLC (COMBINATIONS)
TOTAL NUMBER OF EACH OF THE 8 TYPES OF INTEGRALS SAVED:
IIKL:      113538 IJKJ:      113807 IJJL:      114903 IIKK:      3799
IJJJ:      1462 IIIL:      1470 IIII:      30 IJKL:      3775838
Number of integrals in INCORE buffers:
IIKK:      2085 IJJL:      64223 IJKJ:      63543
IIKL:      64237 IJKL:      1535810

    937565 TWO-ELECTRON INTEGRALS CALCULATED IN ISPCLC (COMBINATIONS)
TOTAL NUMBER OF EACH OF THE 8 TYPES OF INTEGRALS SAVED:
IIKL:      43244 IJKJ:      44015 IJJL:      43286 IIKK:      2740
IJJJ:      1152 IIIL:      1158 IIII:      55 IJKL:      801915
Number of integrals in INCORE buffers:
IIKK:      3570 IJJL:      88212 IJKJ:      87691
IIKL:      88349 IJKL:      271361
Number of buffers:
IJJL:      0 IJKJ:      0
IIKL:      0 IJKL:      1
TOTAL OF      5062412 TWO-ELECTRON INTEGRALS CALCULATED (COMBINATIONS)

CLOSED SHELL SCF                      Nuclear Repulsion Energy is      181.510743561 Hartrees
Convergence on Density Matrix Required to Exit is 5.0000E-06

      CYCLE      ELECTRONIC ENERGY      TOTAL ENERGY      CONVERGENCE      EXTRAPOLATION
SCF_CYCLE:      1      -463.303052879      -281.792309318
SCF_CYCLE:      2      -464.156082549      -282.645338988      2.78967E-02
SCF_CYCLE:      3      -464.252967556      -282.742223996      1.11306E-02

```



```

SCF_CYCLE: 4 -464.282711412 -282.771967851 8.78492E-03
SCF_CYCLE: 5 -464.301085806 -282.790342245 6.67293E-03
SCF_CYCLE: 6 -464.310646467 -282.799902906 5.26956E-03
SCF_CYCLE: 7 -464.325561119 -282.814817558 4-POINT
SCF_CYCLE: 8 -464.326970023 -282.816226462 2.39444E-03
SCF_CYCLE: 9 -464.326980754 -282.816237193 1.12426E-04
SCF_CYCLE: 10 -464.326984887 -282.816241326 7.27041E-05
SCF_CYCLE: 11 -464.326986541 -282.816242980 4.60760E-05
SCF_CYCLE: 12 -464.326987992 -282.816244431 4-POINT
SCF_CYCLE: 13 -464.326987698 -282.816244138 1.87641E-05
At termination total energy is -282.816244 Hartrees

```

\*\*\* RUNing the inputfile :: INPUT\_ALL\_0001.dat

Welcome to MUNgauss - July 9, 2018 Version 7

N\_molecules: 1  
 Molecule is an asymmetric top.  
 Point group: C1  
 Cartesian coordinates for: TEST

|    |    | COORDINATES IN BOHR |             |             |             |  | COORDINATES IN ANGSTROMS |   |   |  |  |
|----|----|---------------------|-------------|-------------|-------------|--|--------------------------|---|---|--|--|
| I  | EL | AN                  | X           | Y           | Z           |  | X                        | Y | Z |  |  |
| 1  | N  | 7                   | 0.00000000  | 0.00000000  | 0.00000000  |  | 0.00000000               |   |   |  |  |
|    |    |                     | 0.00000000  | 0.00000000  |             |  |                          |   |   |  |  |
| 2  | C  | 6                   | 0.00000000  | 0.00000000  | 1.44084286  |  | 0.00000000               |   |   |  |  |
|    |    |                     | 0.00000000  | 2.72279820  |             |  |                          |   |   |  |  |
| 3  | H  | 1                   | 0.73050930  | -0.58944902 | -0.34663973 |  | 1.38046242               |   |   |  |  |
|    |    |                     | -1.11389713 | -0.65505411 |             |  |                          |   |   |  |  |
| 4  | H  | 1                   | -0.87404490 | -0.32011104 | -0.36473314 |  | -1.65170536              |   |   |  |  |
|    |    |                     | -0.60492216 | -0.68924570 |             |  |                          |   |   |  |  |
| 5  | C  | 6                   | 1.42338653  | 0.00000000  | 1.96785902  |  | 2.68981052               |   |   |  |  |
|    |    |                     | 0.00000000  | 3.71871433  |             |  |                          |   |   |  |  |
| 6  | H  | 1                   | -0.52202178 | 0.87980599  | 1.80690743  |  | -0.98647812              |   |   |  |  |
|    |    |                     | 1.66259225  | 3.41455992  |             |  |                          |   |   |  |  |
| 7  | H  | 1                   | -0.48626030 | -0.86672789 | 1.89787416  |  | -0.91889872              |   |   |  |  |
|    |    |                     | -1.63787822 | 3.58646212  |             |  |                          |   |   |  |  |
| 8  | H  | 1                   | 1.40752011  | 0.00000000  | 3.05140244  |  | 2.65982733               |   |   |  |  |
|    |    |                     | 0.00000000  | 5.76631450  |             |  |                          |   |   |  |  |
| 9  | H  | 1                   | 1.93964311  | 0.88478395  | 1.61422314  |  | 3.66539399               |   |   |  |  |
|    |    |                     | 1.67199922  | 3.05043943  |             |  |                          |   |   |  |  |
| 10 | H  | 1                   | 1.93966847  | -0.88478388 | 1.61415464  |  | 3.66544192               |   |   |  |  |
|    |    |                     | -1.67199910 | 3.05030997  |             |  |                          |   |   |  |  |

Nuclear repulsion energy: 83.927218609

Distance Matrix for: TEST

|    | 1        | 2        | 3        | 4        | 5        | 6        |
|----|----------|----------|----------|----------|----------|----------|
| 7  |          |          |          |          |          |          |
| 8  |          |          |          |          |          |          |
| 9  |          |          |          |          |          |          |
| 10 |          |          |          |          |          |          |
| 1  | 0.000000 | 1.440843 | 1.000626 | 0.999728 | 2.428682 | 2.076410 |
| 2  | 2.142333 | 3.360382 | 2.674093 | 2.674070 |          |          |
| 3  | 1.440843 | 0.000000 | 2.018957 | 2.031386 | 1.517819 | 1.086540 |

```

1.093866    2.138928    2.138953    2.138970
3    1.000626    2.018957    0.000000    1.627103    2.486852    2.892285
2.568123    3.514610    2.735023    2.322499
4    0.999728    2.031386    1.627103    0.000000    3.289627    2.505942
2.359779    4.120436    3.644843    3.485948
5    2.428682    1.517819    2.486852    3.289627    0.000000    2.141163
2.098301    1.083660    1.083707    1.083742
6    2.076410    1.086540    2.892285    2.505942    2.141163    0.000000
1.749267    2.458853    2.469199    3.034938
7    2.142333    1.093866    2.568123    2.359779    2.098301    1.749267
0.000000    2.380808    3.005538    2.442530
8    3.360382    2.138928    3.514610    4.120436    1.083660    2.458853
2.380808    0.000000    1.769599    1.769663
9    2.674093    2.138953    2.735023    3.644843    1.083707    2.469199
3.005538    1.769599    0.000000    1.769568
10   2.674070    2.138970    2.322499    3.485948    1.083742    3.034938
2.442530    1.769663    1.769568    0.000000

Charge=      0, Number of electrons=      26

The basis set has now been re-ordered FDPS
The basis set has now been re-ordered FDPS
6-31G(d) Basis Set - Total number of basis functions:      59

Partitioning scheme set to: BECKE
The basis set has now been re-ordered FDPS
Projecting extended Huckel matrix (STO-3G) to 6-31G(d)
All integrals will be kept INCORE
NOTE: INTEGRALS .LE. 1.00E-07 (I2EACC) WERE NOT KEPT
Exponent cutoff used: 2.00E+01 PQCUT2 cutoff used: 1.00E-16

      864066 TWO-ELECTRON INTEGRALS CALCULATED IN IDFCLC (COMBINATIONS)
TOTAL NUMBER OF EACH OF THE 8 TYPES OF INTEGRALS SAVED:
IIKL:      29276 IJKJ:      30830 IJJL:      31306 IIKK:      1744
IJJJ:      491  IIIL:      491  IIII:      18  IJKL:      769910
Number of integrals in INCORE buffers:
IIKK:      891  IJJL:      16092 IJKJ:      15832
IIKL:      15164 IJKL:      270736

      306612 TWO-ELECTRON INTEGRALS CALCULATED IN ISPCLC (COMBINATIONS)
TOTAL NUMBER OF EACH OF THE 8 TYPES OF INTEGRALS SAVED:
IIKL:      17091 IJKJ:      18220 IJJL:      17413 IIKK:      1588
IJJJ:      606  IIIL:      609  IIII:      41  IJKL:      251044
Number of integrals in INCORE buffers:
IIKK:      1711 IJJL:      25122 IJKJ:      25236
IIKL:      24040 IJKL:      359668
TOTAL OF      1170678 TWO-ELECTRON INTEGRALS CALCULATED (COMBINATIONS)

CLOSED SHELL SCF                      Nuclear Repulsion Energy is      83.927218609 Hartrees
Convergence on Density Matrix Required to Exit is 5.0000E-06

      CYCLE    ELECTRONIC ENERGY    TOTAL ENERGY    CONVERGENCE    EXTRAPOLATION
SCF_CYCLE:    1      -217.903421538      -133.976202929
SCF_CYCLE:    2      -218.150803414      -134.223584806      1.47304E-02
SCF_CYCLE:    3      -218.169358592      -134.242139983      4.55696E-03
SCF_CYCLE:    4      -218.171840190      -134.244621581      2.08161E-03
SCF_CYCLE:    5      -218.172279297      -134.245060688      9.12731E-04
SCF_CYCLE:    6      -218.172372515      -134.245153906      4.87522E-04
SCF_CYCLE:    7      -218.172394384      -134.245175775      2.28158E-04
SCF_CYCLE:    8      -218.172399849      -134.245181240      1.25079E-04
SCF_CYCLE:    9      -218.172404403      -134.245185794
SCF_CYCLE:   10      -218.172401795      -134.245183186      9.79975E-05      4-POINT

```

```

At termination total energy is      -134.245183  Hartrees

Atom      J      K      Vee      JHF      KHF
VeeHF
  1      38.966420      -1.235821      37.730599      44.378032      -6.647433
37.730599

Atom      Coulomb
  1      46.801953
J_total      46.801953

Atom      Kinetic(x)      Kinetic(y)      Kinetic(z)      Total
  1      17.838931      18.270367      17.899836      54.009134

Ttotal:      17.838931      18.270367      17.899836      54.009134

Atom      Vne
  1      -169.307253
Vne_total:      -169.307253

  Atomic properties for atom #      1
  ++++++
Number of Electrons, N      =      7.1650510475
Pure Exchange, K ( 2K_ab)      =      -1.2358207734
HF Exchange, KHF ( 2K_ab+ Kaa)      =      -6.6474328040
Kinetic energy Numerical, T      =      54.0091341428
Potential Energy Analytical, Vne = -167.9539736333
Potential Energy Numerical, Vne = -169.3072525485
Coulomb Energy Anal/Num, Vee      =      46.8019525071
Pure Coulomb, J ( 4J_ab+ Jaa)      =      38.9664198835
HF Coulomb, JHF ( 4J_ab+ 2Jaa)      =      44.3780319141
Jaa = Kaa      =      5.4116120306
Coulomb Numerically Over A      =      27.8336653055

PROGRAM> end of inputs

Program terminated normally

Job: C2H7N_C1_RHF_631Gd ended on :24-Aug-18 at 19:43:35
User: ibrahim
Cpu      time: 00h00m13s00c on ibrahim-Lenovo-IdeaPad-P500
Elapsed time: 00h00m13s00c

*** RUNing the inputfile :: INPUT_ALL_0002.dat

Welcome to MUNgauss - July 9, 2018 Version 7

N_molecules: 1
Molecule is an asymmetric top.
Point group: C1
Cartesian coordinates for: TEST

-----
                                COORDINATES IN ANGSTROMS
COORDINATES IN BOHR
I EL      AN      X      Y      Z      X

```

|    |   | Y           | Z           |             |             |
|----|---|-------------|-------------|-------------|-------------|
| 1  | C | 6           | 0.00000000  | 0.00000000  | 1.44084286  |
|    |   | 0.00000000  | 2.72279820  |             | 0.00000000  |
| 2  | N | 7           | 0.00000000  | 0.00000000  | 0.00000000  |
|    |   | 0.00000000  | 0.00000000  |             | 0.00000000  |
| 3  | C | 6           | 1.42338653  | 0.00000000  | 1.96785902  |
|    |   | 0.00000000  | 3.71871433  |             | 2.68981052  |
| 4  | H | 1           | -0.52202178 | 0.87980599  | 1.80690743  |
|    |   | 1.66259225  | 3.41455992  |             | -0.98647812 |
| 5  | H | 1           | -0.48626030 | -0.86672789 | 1.89787416  |
|    |   | -1.63787822 | 3.58646212  |             | -0.91889872 |
| 6  | O | 8           | 1.56620070  | 0.37919230  | 3.23785258  |
|    |   | 0.71656954  | 6.11865417  |             | 2.95969016  |
| 7  | O | 8           | 2.35288801  | -0.36180632 | 1.33520098  |
|    |   | -0.68371481 | 2.52316398  |             | 4.44631363  |
| 8  | H | 1           | 0.73050930  | -0.58944902 | -0.34663973 |
|    |   | -1.11389713 | -0.65505411 |             | 1.38046242  |
| 9  | H | 1           | -0.87404490 | -0.32011104 | -0.36473314 |
|    |   | -0.60492216 | -0.68924570 |             | -1.65170536 |
| 10 | H | 1           | 2.53932814  | 0.31941883  | 3.40629958  |
|    |   | 0.60361407  | 6.43697285  |             | 4.79863437  |

Nuclear repulsion energy: 181.273968331

Distance Matrix for: TEST

|    | 1        | 2        | 3        | 4        | 5        | 6        |
|----|----------|----------|----------|----------|----------|----------|
| 7  | 8        | 9        | 10       |          |          |          |
| 1  | 0.000000 | 1.440843 | 1.517819 | 1.086540 | 1.093866 | 2.413714 |
| 2  | 2.382886 | 2.018957 | 2.031386 | 3.226955 |          |          |
| 3  | 1.440843 | 0.000000 | 2.428682 | 2.076410 | 2.142333 | 3.616692 |
| 4  | 2.729423 | 1.000626 | 0.999728 | 4.260645 |          |          |
| 5  | 1.517819 | 2.428682 | 0.000000 | 2.141163 | 2.098301 | 1.333067 |
| 6  | 1.181157 | 2.486852 | 3.289627 | 1.848368 |          |          |
| 7  | 1.086540 | 2.076410 | 2.141163 | 0.000000 | 1.749267 | 2.580483 |
| 8  | 3.166893 | 2.892285 | 2.505942 | 3.499136 |          |          |
| 9  | 1.093866 | 2.142333 | 2.098301 | 1.749267 | 0.000000 | 2.749628 |
| 10 | 2.938079 | 2.568123 | 2.359779 | 3.582803 |          |          |
| 11 | 2.413714 | 3.616692 | 1.333067 | 2.580483 | 2.749628 | 0.000000 |
| 12 | 2.188159 | 3.805947 | 4.407091 | 0.989406 |          |          |
| 13 | 2.382886 | 2.729423 | 1.181157 | 3.166893 | 2.938079 | 2.188159 |
| 14 | 0.000000 | 2.347876 | 3.647549 | 2.188213 |          |          |
| 15 | 2.018957 | 1.000626 | 2.486852 | 2.892285 | 2.568123 | 3.805947 |
| 16 | 2.347876 | 0.000000 | 1.627103 | 4.264085 |          |          |
| 17 | 2.031386 | 0.999728 | 3.289627 | 2.505942 | 2.359779 | 4.407091 |
| 18 | 3.647549 | 1.627103 | 0.000000 | 5.126480 |          |          |
| 19 | 3.226955 | 4.260645 | 1.848368 | 3.499136 | 3.582803 | 0.989406 |
| 20 | 2.188213 | 4.264085 | 5.126480 | 0.000000 |          |          |

Charge= 0, Number of electrons= 40

The basis set has now been re-ordered FDPS

The basis set has now been re-ordered FDPS

6-31G(d) Basis Set - Total number of basis functions: 85

Partitioning scheme set to: BECKE

The basis set has now been re-ordered FDPS

Projecting extended Huckel matrix (STO-3G) to 6-31G(d)

NOTE: INTEGRALS .LE. 1.00E-07 (I2EACC) WERE NOT KEPT

Exponent cutoff used: 2.00E+01 PQCUT2 cutoff used: 1.00E-16

```

4104066 TWO-ELECTRON INTEGRALS CALCULATED IN IDFCLC (COMBINATIONS)
TOTAL NUMBER OF EACH OF THE 8 TYPES OF INTEGRALS SAVED:
IIKL:      113140 IJKJ:      113572 IJJL:      114756 IIKK:      3793
IJJJ:      1462 IIIL:      1470 IIII:      30 IJKL:      3755843
Number of integrals in INCORE buffers:
IIKK:      2085 IJJL:      64260 IJKJ:      63526
IIKL:      64095 IJKL:      1534076

930964 TWO-ELECTRON INTEGRALS CALCULATED IN ISPCLC (COMBINATIONS)
TOTAL NUMBER OF EACH OF THE 8 TYPES OF INTEGRALS SAVED:
IIKL:      43052 IJKJ:      43862 IJJL:      43150 IIKK:      2734
IJJJ:      1151 IIIL:      1157 IIII:      55 IJKL:      795803
Number of integrals in INCORE buffers:
IIKK:      3570 IJJL:      88256 IJKJ:      87663
IIKL:      88149 IJKL:      269361
Number of buffers:
IJJJ:      0 IJKJ:      0
IIKL:      0 IJKL:      1
TOTAL OF 5035030 TWO-ELECTRON INTEGRALS CALCULATED (COMBINATIONS)

CLOSED SHELL SCF Nuclear Repulsion Energy is 181.273968331 Hartrees
Convergence on Density Matrix Required to Exit is 5.0000E-06

CYCLE ELECTRONIC ENERGY TOTAL ENERGY CONVERGENCE EXTRAPOLATION
SCF_CYCLE: 1 -463.075382663 -281.801414332
SCF_CYCLE: 2 -463.938846750 -282.664878419 2.63503E-02
SCF_CYCLE: 3 -464.033491357 -282.759523025 1.04825E-02
SCF_CYCLE: 4 -464.061188697 -282.787220365 8.09175E-03
SCF_CYCLE: 5 -464.077105997 -282.803137666 6.06597E-03
SCF_CYCLE: 6 -464.085279580 -282.811311249 4.81216E-03
SCF_CYCLE: 7 -464.098916367 -282.824948035 4-POINT
SCF_CYCLE: 8 -464.098969593 -282.825001262 2.15105E-03
SCF_CYCLE: 9 -464.098999780 -282.825031449 1.85118E-04
SCF_CYCLE: 10 -464.099011460 -282.825043129 1.17512E-04
SCF_CYCLE: 11 -464.099021079 -282.825052747 4-POINT
SCF_CYCLE: 12 -464.099019232 -282.825050900 4.77928E-05
At termination total energy is -282.825051 Hartrees

Atom J K Vee JHF KHF
VeeHF
1 43.478567 -0.933213 42.545354 47.819244 -5.273891
42.545354

Atom Coulomb
1 48.232671
J_total 48.232671

Atom Kinetic(x) Kinetic(y) Kinetic(z) Total
1 12.644472 12.655040 12.669348 37.968860

Ttotal: 12.644472 12.655040 12.669348 37.968860

Atom Vne
1 -149.312739
Vne_total: -149.312739

Atomic properties for atom # 1
++++
Number of Electrons, N = 6.1012052215
Pure Exchange, K ( 2K_ab) = -0.9332131891
HF Exchange, KHF ( 2K_ab+ Kaa) = -5.2738907810

```

```

Kinetic energy Numerical, T      = 37.9688599979
Potential Energy Analytical, Vne = -147.1754484683
Potential Energy Numerical, Vne  = -149.3127391196
Coulomb Energy Anal/Num, Vee     = 48.2326712819
Pure Coulomb, J ( 4J_ab+ Jaa)    = 43.4785667800
HF Coulomb, JHF ( 4J_ab+ 2Jaa)   = 47.8192443719
Jaa = Kaa                        = 4.3406775919
Coulomb Numerically Over A       = 19.3994839860

```

PROGRAM> end of inputs

Program terminated normally

```

Job: C2H5NO2_C1_RHF_631Gd ended on :24-Aug-18 at 19:44:14
User: ibrahim
Cpu    time: 00h00m38s67c on ibrahim-Lenovo-IdeaPad-P500
Elapsed time: 00h00m39s00c

```

\*\*\* RUNing the inputfile :: INPUT\_ALL\_0003.dat

Welcome to MUNgauss - July 9, 2018 Version 7

```

N_molecules: 1
Molecule is an asymmetric top.
Point group: C1
Cartesian coordinates for: TEST

```

| COORDINATES IN BOHR |    |             | COORDINATES IN ANGSTROMS |             |             |             |
|---------------------|----|-------------|--------------------------|-------------|-------------|-------------|
| I                   | EL | AN          | X                        | Y           | Z           | X           |
|                     |    | Y           | Z                        |             |             |             |
| 1                   | C  | 6           | 1.42338653               | 0.00000000  | 1.96785902  | 2.68981052  |
|                     |    | 0.00000000  | 3.71871433               |             |             |             |
| 2                   | O  | 8           | 1.56620070               | 0.37919230  | 3.23785258  | 2.95969016  |
|                     |    | 0.71656954  | 6.11865417               |             |             |             |
| 3                   | O  | 8           | 2.35288801               | -0.36180632 | 1.33520098  | 4.44631363  |
|                     |    | -0.68371481 | 2.52316398               |             |             |             |
| 4                   | C  | 6           | 0.00000000               | 0.00000000  | 1.44084286  | 0.00000000  |
|                     |    | 0.00000000  | 2.72279820               |             |             |             |
| 5                   | N  | 7           | 0.00000000               | 0.00000000  | 0.00000000  | 0.00000000  |
|                     |    | 0.00000000  | 0.00000000               |             |             |             |
| 6                   | H  | 1           | 0.75176248               | 0.68026803  | 3.61808458  | 1.42062510  |
|                     |    | 1.28552018  | 6.83718846               |             |             |             |
| 7                   | H  | 1           | -0.52202178              | 0.87980599  | 1.80690743  | -0.98647812 |
|                     |    | 1.66259225  | 3.41455992               |             |             |             |
| 8                   | H  | 1           | -0.48626030              | -0.86672789 | 1.89787416  | -0.91889872 |
|                     |    | -1.63787822 | 3.58646212               |             |             |             |
| 9                   | H  | 1           | -0.95777357              | 0.00000000  | -0.29610147 | -1.80992961 |
|                     |    | 0.00000000  | -0.55955064              |             |             |             |
| 10                  | H  | 1           | 0.40141812               | -0.86960710 | -0.29610147 | 0.75857025  |
|                     |    | -1.64331913 | -0.55955064              |             |             |             |

Nuclear repulsion energy: 181.310954311

Distance Matrix for: TEST

|    | 1        | 2        | 3        | 4        | 5        | 6        |
|----|----------|----------|----------|----------|----------|----------|
| 7  | 8        | 9        | 10       |          |          |          |
| 1  | 0.000000 | 1.333067 | 1.181157 | 1.517819 | 2.428682 | 1.907115 |
| 2  | 2.141163 | 2.098301 | 3.285642 | 2.631759 |          |          |
| 3  | 1.333067 | 0.000000 | 2.188159 | 2.413714 | 3.616692 | 0.947910 |
| 4  | 2.580483 | 2.749628 | 4.359250 | 3.924927 |          |          |
| 5  | 1.181157 | 2.188159 | 0.000000 | 2.382886 | 2.729423 | 2.976757 |
| 6  | 3.166893 | 2.938079 | 3.708441 | 2.593693 |          |          |
| 7  | 1.517819 | 2.413714 | 2.382886 | 0.000000 | 1.440843 | 2.401727 |
| 8  | 1.086540 | 1.093866 | 1.983508 | 1.983514 |          |          |
| 9  | 2.428682 | 3.616692 | 2.729423 | 1.440843 | 0.000000 | 3.757452 |
| 10 | 2.076410 | 2.142333 | 1.002500 | 1.002511 |          |          |
| 11 | 1.907115 | 0.947910 | 2.976757 | 2.401727 | 3.757452 | 0.000000 |
| 12 | 2.223219 | 2.623932 | 4.325058 | 4.224418 |          |          |
| 13 | 2.141163 | 2.580483 | 3.166893 | 1.086540 | 2.076410 | 2.223219 |
| 14 | 0.000000 | 1.749267 | 2.320902 | 2.887184 |          |          |
| 15 | 2.098301 | 2.749628 | 2.938079 | 1.093866 | 2.142333 | 2.623932 |
| 16 | 1.749267 | 0.000000 | 2.405633 | 2.366751 |          |          |
| 17 | 3.285642 | 4.359250 | 3.708441 | 1.983508 | 1.002500 | 4.325058 |
| 18 | 2.320902 | 2.405633 | 0.000000 | 1.613573 |          |          |
| 19 | 2.631759 | 3.924927 | 2.593693 | 1.983514 | 1.002511 | 4.224418 |
| 20 | 2.887184 | 2.366751 | 1.613573 | 0.000000 |          |          |

Charge= 0, Number of electrons= 40

The basis set has now been re-ordered FDPS

The basis set has now been re-ordered FDPS

6-31G(d) Basis Set - Total number of basis functions: 85

Partitioning scheme set to: BECKE

The basis set has now been re-ordered FDPS

Projecting extended Huckel matrix (STO-3G) to 6-31G(d)

NOTE: INTEGRALS .LE. 1.00E-07 (I2EACC) WERE NOT KEPT

Exponent cutoff used: 2.00E+01 PQCUT2 cutoff used: 1.00E-16

4086004 TWO-ELECTRON INTEGRALS CALCULATED IN IDFCLC (COMBINATIONS)  
TOTAL NUMBER OF EACH OF THE 8 TYPES OF INTEGRALS SAVED:  
IIKL: 112311 IJKJ: 113197 IJJL: 113940 IIKK: 3794  
IJJJ: 1449 IIIL: 1456 IIII: 30 IJKL: 3739827  
Number of integrals in INCORE buffers:  
IIKK: 2085 IJJL: 63789 IJKJ: 63182  
IIKL: 63550 IJKL: 1523366

926721 TWO-ELECTRON INTEGRALS CALCULATED IN ISPCLC (COMBINATIONS)  
TOTAL NUMBER OF EACH OF THE 8 TYPES OF INTEGRALS SAVED:  
IIKL: 42686 IJKJ: 43768 IJJL: 42806 IIKK: 2735  
IJJJ: 1138 IIIL: 1146 IIII: 55 IJKL: 792387  
Number of integrals in INCORE buffers:  
IIKK: 3570 IJJL: 87544 IJKJ: 87219  
IIKL: 87364 IJKL: 255867  
Number of buffers:  
IJJJ: 0 IJKJ: 0  
IIKL: 0 IJKL: 1  
TOTAL OF 5012725 TWO-ELECTRON INTEGRALS CALCULATED (COMBINATIONS)

CLOSED SHELL SCF Nuclear Repulsion Energy is 181.310954311 Hartrees  
Convergence on Density Matrix Required to Exit is 5.0000E-06

CYCLE ELECTRONIC ENERGY TOTAL ENERGY CONVERGENCE EXTRAPOLATION

```

SCF_CYCLE: 1 -463.088972146 -281.778017835
SCF_CYCLE: 2 -463.951585272 -282.640630962 2.82187E-02
SCF_CYCLE: 3 -464.049586229 -282.738631919 1.12648E-02
SCF_CYCLE: 4 -464.079700155 -282.768745845 8.95101E-03
SCF_CYCLE: 5 -464.098443005 -282.787488695 6.79396E-03
SCF_CYCLE: 6 -464.108208454 -282.797254144 5.37936E-03
SCF_CYCLE: 7 -464.123558036 -282.812603726
SCF_CYCLE: 8 -464.125156392 -282.814202081 2.45294E-03
SCF_CYCLE: 9 -464.125167327 -282.814213017 1.14132E-04
SCF_CYCLE: 10 -464.125171546 -282.814217235 7.41020E-05
SCF_CYCLE: 11 -464.125173237 -282.814218926 4.69041E-05
SCF_CYCLE: 12 -464.125174755 -282.814220445
SCF_CYCLE: 13 -464.125174427 -282.814220116 1.92040E-05
At termination total energy is -282.814220 Hartrees

Atom J K Vee JHF KHF
VeeHF
1 44.470645 -0.959420 43.511224 48.886603 -5.375379
43.511224

Atom Coulomb
1 53.047099
J_total 53.047099

Atom Kinetic(x) Kinetic(y) Kinetic(z) Total
1 12.930254 12.860021 12.902546 38.692820

Ttotal: 12.930254 12.860021 12.902546 38.692820

Atom Vne
1 -159.523693
Vne_total: -159.523693

Atomic properties for atom # 1
+++++
Number of Electrons, N = 6.1261059386
Pure Exchange, K ( 2K_ab) = -0.9594200871
HF Exchange, KHF ( 2K_ab+ Kaa) = -5.3753785852
Kinetic energy Numerical, T = 38.6928204030
Potential Energy Analytical, Vne = -156.5604706098
Potential Energy Numerical, Vne = -159.5236933970
Coulomb Energy Anal/Num, Vee = 53.0470991061
Pure Coulomb, J ( 4J_ab+ Jaa) = 44.4706445853
HF Coulomb, JHF ( 4J_ab+ 2Jaa) = 48.8866030835
Jaa = Kaa = 4.4159584981
Coulomb Numerically Over A = 19.3437436031

PROGRAM> end of inputs

Program terminated normally

Job: C2H5N02_C1_RHF_631Gd ended on :24-Aug-18 at 19:44:54
User: ibrahim
Cpu time: 00h00m39s66c on ibrahim-Lenovo-IdeaPad-P500
Elapsed time: 00h00m40s00c

*** RUNing the inputfile :: INPUT_ALL_0004.dat

```



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N\_molecules: 1  
Molecule is an asymmetric top.  
Point group: C1  
Cartesian coordinates for: TEST

| COORDINATES IN BOHR |    |             | COORDINATES IN ANGSTROMS |             |            |             |
|---------------------|----|-------------|--------------------------|-------------|------------|-------------|
| I                   | EL | AN          | X                        | Y           | Z          | X           |
|                     |    | Y           | Z                        |             |            |             |
| 1                   | O  | 8           | 2.35288801               | -0.36180632 | 1.33520098 | 4.44631363  |
|                     |    | -0.68371481 | 2.52316398               |             |            |             |
| 2                   | C  | 6           | 1.42338653               | 0.00000000  | 1.96785902 | 2.68981052  |
|                     |    | 0.00000000  | 3.71871433               |             |            |             |
| 3                   | O  | 8           | 1.56620070               | 0.37919230  | 3.23785258 | 2.95969016  |
|                     |    | 0.71656954  | 6.11865417               |             |            |             |
| 4                   | C  | 6           | 0.00000000               | 0.00000000  | 1.44084286 | 0.00000000  |
|                     |    | 0.00000000  | 2.72279820               |             |            |             |
| 5                   | H  | 1           | 0.65316574               | 0.62862176  | 3.52608235 | 1.23430428  |
|                     |    | 1.18792287  | 6.66332945               |             |            |             |
| 6                   | H  | 1           | -0.66879662              | 0.37530236  | 2.20646504 | -1.26384235 |
|                     |    | 0.70921862  | 4.16961433               |             |            |             |
| 7                   | H  | 1           | -0.28673497              | -1.01059766 | 1.17457750 | -0.54185053 |
|                     |    | -1.90975266 | 2.21962964               |             |            |             |
| 8                   | H  | 1           | -0.06099919              | 0.63521777  | 0.56490118 | -0.11527175 |
|                     |    | 1.20038753  | 1.06750844               |             |            |             |

Nuclear repulsion energy: 122.224021925

Distance Matrix for: TEST

|   | 1        | 2        | 3        | 4        | 5        | 6        |
|---|----------|----------|----------|----------|----------|----------|
| 7 | 8        |          |          |          |          |          |
| 1 | 0.000000 | 1.181157 | 2.188159 | 2.382886 | 2.944480 | 3.230017 |
|   | 2.722928 | 2.722916 |          |          |          |          |
| 2 | 1.181157 | 0.000000 | 1.333067 | 1.517819 | 1.848368 | 2.138928 |
|   | 2.138953 | 2.138970 |          |          |          |          |
| 3 | 2.188159 | 1.333067 | 0.000000 | 2.413714 | 0.989406 | 2.461501 |
|   | 3.101933 | 3.139745 |          |          |          |          |
| 4 | 2.382886 | 1.517819 | 2.413714 | 0.000000 | 2.273767 | 1.083660 |
|   | 1.083707 | 1.083742 |          |          |          |          |
| 5 | 2.944480 | 1.848368 | 0.989406 | 2.273767 | 0.000000 | 1.884979 |
|   | 3.016625 | 3.046091 |          |          |          |          |
| 6 | 3.230017 | 2.138928 | 2.461501 | 1.083660 | 1.884979 | 0.000000 |
|   | 1.769599 | 1.769663 |          |          |          |          |
| 7 | 2.722928 | 2.138953 | 3.101933 | 1.083707 | 3.016625 | 1.769599 |
|   | 0.000000 | 1.769568 |          |          |          |          |
| 8 | 2.722916 | 2.138970 | 3.139745 | 1.083742 | 3.046091 | 1.769663 |
|   | 1.769568 | 0.000000 |          |          |          |          |

Charge= 0, Number of electrons= 32

The basis set has now been re-ordered FDPS

The basis set has now been re-ordered FDPS

6-31G(d) Basis Set - Total number of basis functions: 68

Partitioning scheme set to: BECKE

```

The basis set has now been re-ordered FDPS
Projecting extended Huckel matrix (STO-3G) to 6-31G(d)
All integrals will be kept INCORE
NOTE: INTEGRALS .LE. 1.00E-07 (I2EACC) WERE NOT KEPT
Exponent cutoff used: 2.00E+01 PQCUT2 cutoff used: 1.00E-16

2017609 TWO-ELECTRON INTEGRALS CALCULATED IN IDFCCL (COMBINATIONS)
TOTAL NUMBER OF EACH OF THE 8 TYPES OF INTEGRALS SAVED:
IIKL: 65274 IJKJ: 65618 IJJL: 66535 IIKK: 2552
IJJJ: 1037 IIIL: 1043 IIII: 24 IJKL: 1815526
Number of integrals in INCORE buffers:
IIKK: 1332 IJJL: 34689 IJKJ: 34229
IIKL: 34433 IJKL: 653524

432582 TWO-ELECTRON INTEGRALS CALCULATED IN ISPCLC (COMBINATIONS)
TOTAL NUMBER OF EACH OF THE 8 TYPES OF INTEGRALS SAVED:
IIKL: 23628 IJKJ: 23882 IJJL: 23653 IIKK: 1807
IJJJ: 776 IIIL: 779 IIII: 44 IJKL: 358013
Number of integrals in INCORE buffers:
IIKK: 2278 IJJL: 47276 IJKJ: 46846
IIKL: 47049 IJKL: 784876
TOTAL OF 2450191 TWO-ELECTRON INTEGRALS CALCULATED (COMBINATIONS)

CLOSED SHELL SCF Nuclear Repulsion Energy is 122.224021925 Hartrees
Convergence on Density Matrix Required to Exit is 5.0000E-06

CYCLE ELECTRONIC ENERGY TOTAL ENERGY CONVERGENCE EXTRAPOLATION
SCF_CYCLE: 1 -349.147748716 -226.923726791
SCF_CYCLE: 2 -349.864153562 -227.640131637 3.23116E-02
SCF_CYCLE: 3 -349.947256630 -227.723234705 1.31150E-02
SCF_CYCLE: 4 -349.974926272 -227.750904347 1.04462E-02
SCF_CYCLE: 5 -349.992426777 -227.768404852 7.93076E-03
SCF_CYCLE: 6 -350.001568329 -227.777546404 6.23392E-03
SCF_CYCLE: 7 -350.018394666 -227.794372741 4-POINT
SCF_CYCLE: 8 -350.016674835 -227.792652910 2.80788E-03
SCF_CYCLE: 9 -350.016704229 -227.792682304 2.31884E-04
SCF_CYCLE: 10 -350.016716060 -227.792694135 1.55219E-04
SCF_CYCLE: 11 -350.016725831 -227.792703906 4-POINT
SCF_CYCLE: 12 -350.016724517 -227.792702592 6.16014E-05
At termination total energy is -227.792703 Hartrees

Atom J K Vee JHF KHF
VeeHF
1 60.217312 -1.863492 58.353820 66.554663 -8.200843
58.353820

Atom Coulomb
1 63.929855
J_total 63.929855

Atom Kinetic(x) Kinetic(y) Kinetic(z) Total
1 24.705415 24.642234 24.878383 74.226033
Ttotal: 24.705415 24.642234 24.878383 74.226033

Atom Vne
1 -231.797194
Vne_total: -231.797194

Atomic properties for atom # 1
++++

```

```

Number of Electrons, N      =      8.1424657625
Pure Exchange, K ( 2K_ab)  =      -1.8634917350
HF Exchange, KHF ( 2K_ab+ Kaa) =      -8.2008426955
Kinetic energy Numerical, T =      74.2260329430
Potential Energy Analytical, Vne = -232.1835909075
Potential Energy Numerical, Vne = -231.7971943095
Coulomb Energy Anal/Num, Vee =      63.9298549686
Pure Coulomb, J ( 4J_ab+ Jaa) =      60.2173116265
HF Coulomb, JHF ( 4J_ab+ 2Jaa) =      66.5546625870
Jaa = Kaa                  =      6.3373509605
Coulomb Numerically Over A =      37.9275985828

```

PROGRAM> end of inputs

Program terminated normally

```

Job: C2H4O2_C1_RHF_631Gd ended on :24-Aug-18 at 19:45:13
User: ibrahim
Cpu time: 00h00m19s69c on ibrahim-Lenovo-IdeaPad-P500
Elapsed time: 00h00m19s00c

```

\*\*\* RUNing the inputfile :: INPUT\_ALL\_0005.dat

Welcome to MUNgauss - July 9, 2018 Version 7

```

N_molecules: 1
Molecule is an asymmetric top.
Point group: C1
Cartesian coordinates for: TEST

```

| COORDINATES IN BOHR |             |             | COORDINATES IN ANGSTROMS |             |             |  |
|---------------------|-------------|-------------|--------------------------|-------------|-------------|--|
| I EL                | AN          | X           | Y                        | Z           | X           |  |
|                     | Y           | Z           |                          |             |             |  |
| 1 H                 | 1           | -0.52202178 | 0.87980599               | 1.80690743  | -0.98647812 |  |
|                     | 1.66259225  | 3.41455992  |                          |             |             |  |
| 2 C                 | 6           | 0.00000000  | 0.00000000               | 1.44084286  | 0.00000000  |  |
|                     | 0.00000000  | 2.72279820  |                          |             |             |  |
| 3 N                 | 7           | 0.00000000  | 0.00000000               | 0.00000000  | 0.00000000  |  |
|                     | 0.00000000  | 0.00000000  |                          |             |             |  |
| 4 C                 | 6           | 1.42338653  | 0.00000000               | 1.96785902  | 2.68981052  |  |
|                     | 0.00000000  | 3.71871433  |                          |             |             |  |
| 5 H                 | 1           | -0.48626030 | -0.86672789              | 1.89787416  | -0.91889872 |  |
|                     | -1.63787822 | 3.58646212  |                          |             |             |  |
| 6 H                 | 1           | 0.48872922  | -0.82369531              | -0.29610147 | 0.92356430  |  |
|                     | -1.55655844 | -0.55955064 |                          |             |             |  |
| 7 H                 | 1           | -0.95270540 | -0.09851616              | -0.29610147 | -1.80035215 |  |
|                     | -0.18616854 | -0.55955064 |                          |             |             |  |
| 8 H                 | 1           | 1.94391057  | -0.87751344              | 1.60269742  | 3.67345832  |  |
|                     | -1.65825995 | 3.02865896  |                          |             |             |  |
| 9 H                 | 1           | 1.93528625  | 0.89191159               | 1.62599031  | 3.65716073  |  |
|                     | 1.68546850  | 3.07267615  |                          |             |             |  |
| 10 H                | 1           | 1.40752416  | -0.01421698              | 3.05139149  | 2.65983499  |  |
|                     | -0.02686619 | 5.76629379  |                          |             |             |  |

Nuclear repulsion energy: 83.935233098

Distance Matrix for: TEST

|    | 1        | 2        | 3        | 4        | 5        | 6        |
|----|----------|----------|----------|----------|----------|----------|
| 7  | 8        | 9        | 10       |          |          |          |
| 1  | 0.000000 | 1.086540 | 2.076410 | 2.141163 | 1.749267 | 2.888976 |
|    | 2.359078 | 3.034913 | 2.463989 | 2.463973 |          |          |
| 2  | 1.086540 | 0.000000 | 1.440843 | 1.517819 | 1.093866 | 1.983508 |
|    | 1.983514 | 2.138928 | 2.138953 | 2.138970 |          |          |
| 3  | 2.076410 | 1.440843 | 0.000000 | 2.428682 | 2.142333 | 1.002500 |
|    | 1.002511 | 2.667856 | 2.680426 | 3.360404 |          |          |
| 4  | 2.141163 | 1.517819 | 2.428682 | 0.000000 | 2.098301 | 2.584100 |
|    | 3.283449 | 1.083660 | 1.083707 | 1.083742 |          |          |
| 5  | 1.749267 | 1.093866 | 2.142333 | 2.098301 | 0.000000 | 2.401247 |
|    | 2.370917 | 2.448056 | 3.005099 | 2.375668 |          |          |
| 6  | 2.888976 | 1.983508 | 1.002500 | 2.584100 | 2.401247 | 0.000000 |
|    | 1.613573 | 2.392882 | 2.954703 | 3.564428 |          |          |
| 7  | 2.359078 | 1.983514 | 1.002511 | 3.283449 | 2.370917 | 1.613573 |
|    | 0.000000 | 3.550022 | 3.607753 | 4.096767 |          |          |
| 8  | 3.034913 | 2.138928 | 2.667856 | 1.083660 | 2.448056 | 2.392882 |
|    | 3.550022 | 0.000000 | 1.769599 | 1.769663 |          |          |
| 9  | 2.463989 | 2.138953 | 2.680426 | 1.083707 | 3.005099 | 2.954703 |
|    | 3.607753 | 1.769599 | 0.000000 | 1.769568 |          |          |
| 10 | 2.463973 | 2.138970 | 3.360404 | 1.083742 | 2.375668 | 3.564428 |
|    | 4.096767 | 1.769663 | 1.769568 | 0.000000 |          |          |

Charge= 0, Number of electrons= 26

The basis set has now been re-ordered FDPs

The basis set has now been re-ordered FDPs

6-31G(d) Basis Set - Total number of basis functions: 59

Partitioning scheme set to: BECKE

The basis set has now been re-ordered FDPs

Projecting extended Huckel matrix (STO-3G) to 6-31G(d)

All integrals will be kept INCORE

NOTE: INTEGRALS .LE. 1.00E-07 (I2EACC) WERE NOT KEPT

Exponent cutoff used: 2.00E+01 PQCT2 cutoff used: 1.00E-16

887199 TWO-ELECTRON INTEGRALS CALCULATED IN IDFCLC (COMBINATIONS)

TOTAL NUMBER OF EACH OF THE 8 TYPES OF INTEGRALS SAVED:

|       |       |       |       |       |       |       |        |
|-------|-------|-------|-------|-------|-------|-------|--------|
| IIKL: | 30123 | IJKJ: | 31373 | IJJL: | 31967 | IIKK: | 1743   |
| IJJJ: | 501   | IIIL: | 501   | IIII: | 18    | IJKL: | 790973 |

Number of integrals in INCORE buffers:

|       |       |       |        |       |       |
|-------|-------|-------|--------|-------|-------|
| IIKK: | 891   | IJJL: | 16480  | IJKJ: | 16136 |
| IIKL: | 15693 | IJKL: | 280648 |       |       |

312893 TWO-ELECTRON INTEGRALS CALCULATED IN ISPCLC (COMBINATIONS)

TOTAL NUMBER OF EACH OF THE 8 TYPES OF INTEGRALS SAVED:

|       |       |       |       |       |       |       |        |
|-------|-------|-------|-------|-------|-------|-------|--------|
| IIKL: | 17502 | IJKJ: | 18404 | IJJL: | 17773 | IIKK: | 1588   |
| IJJJ: | 616   | IIIL: | 619   | IIII: | 41    | IJKL: | 256350 |

Number of integrals in INCORE buffers:

|       |       |       |        |       |       |
|-------|-------|-------|--------|-------|-------|
| IIKK: | 1711  | IJJL: | 25722  | IJKJ: | 25632 |
| IIKL: | 24823 | IJKL: | 372071 |       |       |

TOTAL OF 1200092 TWO-ELECTRON INTEGRALS CALCULATED (COMBINATIONS)

CLOSED SHELL SCF Nuclear Repulsion Energy is 83.935233098 Hartrees  
Convergence on Density Matrix Required to Exit is 5.0000E-06

```

CYCLE      ELECTRONIC ENERGY      TOTAL ENERGY      CONVERGENCE      EXTRAPOLATION
SCF_CYCLE: 1      -217.901046920      -133.965813822
SCF_CYCLE: 2      -218.158366283      -134.223133185      1.52595E-02
SCF_CYCLE: 3      -218.177683980      -134.242450883      4.76579E-03
SCF_CYCLE: 4      -218.180330274      -134.245097176      2.22264E-03
SCF_CYCLE: 5      -218.180811521      -134.245578424      9.73581E-04
SCF_CYCLE: 6      -218.180916317      -134.245683220      5.26773E-04
SCF_CYCLE: 7      -218.180941403      -134.245708305      2.43399E-04
SCF_CYCLE: 8      -218.180962171      -134.245729073
SCF_CYCLE: 9      -218.180950095      -134.245716997      2.13774E-04      4-POINT
At termination total energy is      -134.245717      Hartrees

Atom      J      K      Vee      JHF      KHF
VeeHF
1      4.777099      -0.173767      4.603332      4.935648      -0.332316
4.603332

Atom      Coulomb
1      3.666742
J_total      3.666742

Atom      Kinetic(x)      Kinetic(y)      Kinetic(z)      Total
1      0.199270      0.193756      0.197964      0.590990

Ttotal:      0.199270      0.193756      0.197964      0.590990

Atom      Vne
1      -7.870855
Vne_total:      -7.870855

Atomic properties for atom #      1
+++++
Number of Electrons, N      =      0.9393179250
Pure Exchange, K ( 2K_ab)      =      -0.1737670983
HF Exchange, KHF ( 2K_ab+ Kaa)      =      -0.3323162223
Kinetic energy Numerical, T      =      0.5909901007
Potential Energy Analytical, Vne      =      -8.6399581474
Potential Energy Numerical, Vne      =      -7.8708548747
Coulomb Energy Anal/Num, Vee      =      3.6667417499
Pure Coulomb, J ( 4J_ab+ Jaa)      =      4.7770989645
HF Coulomb, JHF ( 4J_ab+ 2Jaa)      =      4.9356480886
Jaa = Kaa      =      0.1585491240
Coulomb Numerically Over A      =      0.3757287077

PROGRAM> end of inputs

Program terminated normally

Job: C2H7N_C1_RHF_631Gd ended on :24-Aug-18 at 19:45:25
User: ibrahim
Cpu      time: 00h00m11s63c on ibrahim-Lenovo-IdeaPad-P500
Elapsed time: 00h00m12s00c

*** RUNing the inputfile :: INPUT_ALL_0006.dat

Welcome to MUNgauss - July 9, 2018 Version 7

```

N\_molecules: 1  
Molecule is an asymmetric top.  
Point group: C1  
Cartesian coordinates for: TEST

| COORDINATES IN BOHR |    |             | COORDINATES IN ANGSTROMS |             |             |             |
|---------------------|----|-------------|--------------------------|-------------|-------------|-------------|
| I                   | EL | AN          | X                        | Y           | Z           | X           |
|                     |    | Y           | Z                        |             |             |             |
| 1                   | H  | 1           | -0.48626030              | -0.86672789 | 1.89787416  | -0.91889872 |
|                     |    | -1.63787822 | 3.58646212               |             |             |             |
| 2                   | C  | 6           | 0.00000000               | 0.00000000  | 1.44084286  | 0.00000000  |
|                     |    | 0.00000000  | 2.72279820               |             |             |             |
| 3                   | N  | 7           | 0.00000000               | 0.00000000  | 0.00000000  | 0.00000000  |
|                     |    | 0.00000000  | 0.00000000               |             |             |             |
| 4                   | C  | 6           | 1.42338653               | 0.00000000  | 1.96785902  | 2.68981052  |
|                     |    | 0.00000000  | 3.71871433               |             |             |             |
| 5                   | H  | 1           | -0.52202178              | 0.87980599  | 1.80690743  | -0.98647812 |
|                     |    | 1.66259225  | 3.41455992               |             |             |             |
| 6                   | H  | 1           | 0.46862617               | 0.83529619  | -0.29610147 | 0.88557506  |
|                     |    | 1.57848092  | -0.55955064              |             |             |             |
| 7                   | H  | 1           | -0.95481287              | 0.07540156  | -0.29610147 | -1.80433469 |
|                     |    | 0.14248828  | -0.55955064              |             |             |             |
| 8                   | H  | 1           | 1.96357598               | 0.84116568  | 1.54958427  | 3.71062056  |
|                     |    | 1.58957264  | 2.92828967               |             |             |             |
| 9                   | H  | 1           | 1.91452219               | -0.92278526 | 1.68207074  | 3.61792234  |
|                     |    | -1.74381129 | 3.17865279               |             |             |             |
| 10                  | H  | 1           | 1.40861876               | 0.08144592  | 3.04843517  | 2.66190347  |
|                     |    | 0.15391046  | 5.76070717               |             |             |             |

Nuclear repulsion energy: 83.933362249

Distance Matrix for: TEST

|    | 1        | 2        | 3        | 4        | 5        | 6        |
|----|----------|----------|----------|----------|----------|----------|
| 7  | 8        | 9        | 10       |          |          |          |
| 1  | 0.000000 | 1.093866 | 2.142333 | 2.098301 | 1.749267 | 2.936362 |
|    | 2.433244 | 3.006643 | 2.411114 | 2.411097 |          |          |
| 2  | 1.093866 | 0.000000 | 1.440843 | 1.517819 | 1.086540 | 1.983508 |
|    | 1.983514 | 2.138928 | 2.138953 | 2.138970 |          |          |
| 3  | 2.142333 | 1.440843 | 0.000000 | 2.428682 | 2.076410 | 1.002500 |
|    | 1.002511 | 2.639015 | 2.710404 | 3.359136 |          |          |
| 4  | 2.098301 | 1.517819 | 2.428682 | 0.000000 | 2.141163 | 2.595150 |
|    | 3.284362 | 1.083660 | 1.083707 | 1.083742 |          |          |
| 5  | 1.749267 | 1.086540 | 2.076410 | 2.141163 | 0.000000 | 2.325083 |
|    | 2.292819 | 2.499181 | 3.033425 | 2.430256 |          |          |
| 6  | 2.936362 | 1.983508 | 1.002500 | 2.595150 | 2.325083 | 0.000000 |
|    | 1.613573 | 2.375177 | 3.015731 | 3.554969 |          |          |
| 7  | 2.433244 | 1.983514 | 1.002511 | 3.284362 | 2.292819 | 1.613573 |
|    | 0.000000 | 3.536940 | 3.625276 | 4.095335 |          |          |
| 8  | 3.006643 | 2.138928 | 2.639015 | 1.083660 | 2.499181 | 2.375177 |
|    | 3.536940 | 0.000000 | 1.769599 | 1.769663 |          |          |
| 9  | 2.411114 | 2.138953 | 2.710404 | 1.083707 | 3.033425 | 3.015731 |
|    | 3.625276 | 1.769599 | 0.000000 | 1.769568 |          |          |
| 10 | 2.411097 | 2.138970 | 3.359136 | 1.083742 | 2.430256 | 3.554969 |
|    | 4.095335 | 1.769663 | 1.769568 | 0.000000 |          |          |

Charge= 0, Number of electrons= 26

```

The basis set has now been re-ordered FDPS
The basis set has now been re-ordered FDPS
6-31G(d) Basis Set - Total number of basis functions:          59

Partitioning scheme set to: BECKE
The basis set has now been re-ordered FDPS
Projecting extended Huckel matrix (STO-3G) to 6-31G(d)
All integrals will be kept INCORE
NOTE: INTEGRALS .LE. 1.00E-07 (I2EACC) WERE NOT KEPT
Exponent cutoff used: 2.00E+01 PQ CUT2 cutoff used: 1.00E-16

      889780 TWO-ELECTRON INTEGRALS CALCULATED IN IDFCCL (COMBINATIONS)
TOTAL NUMBER OF EACH OF THE 8 TYPES OF INTEGRALS SAVED:
IIKL:      30174 IJKJ:      31418 IJJL:      32001 IIKK:      1740
IJJJ:      502 IIIL:      501 IIII:      18 IJKL:      793426
Number of integrals in INCORE buffers:
IIKK:      891 IJJL:      16491 IJKJ:      16154
IIKL:      15699 IJKL:      281062

      313488 TWO-ELECTRON INTEGRALS CALCULATED IN ISPCLC (COMBINATIONS)
TOTAL NUMBER OF EACH OF THE 8 TYPES OF INTEGRALS SAVED:
IIKL:      17511 IJKJ:      18396 IJJL:      17787 IIKK:      1588
IJJJ:      617 IIIL:      619 IIII:      41 IJKL:      256929
Number of integrals in INCORE buffers:
IIKK:      1711 IJJL:      25737 IJKJ:      25653
IIKL:      24829 IJKL:      372612
TOTAL OF      1203268 TWO-ELECTRON INTEGRALS CALCULATED (COMBINATIONS)

CLOSED SHELL SCF                      Nuclear Repulsion Energy is      83.933362249 Hartrees
Convergence on Density Matrix Required to Exit is 5.0000E-06

      CYCLE      ELECTRONIC ENERGY      TOTAL ENERGY      CONVERGENCE      EXTRAPOLATION
SCF_CYCLE:      1      -217.899573327      -133.966211078
SCF_CYCLE:      2      -218.155122907      -134.221760658      1.52152E-02
SCF_CYCLE:      3      -218.174192099      -134.240829850      4.70431E-03
SCF_CYCLE:      4      -218.176781933      -134.243419684      2.19982E-03
SCF_CYCLE:      5      -218.177250021      -134.243887771      9.52724E-04
SCF_CYCLE:      6      -218.177351734      -134.243989485      5.20729E-04
SCF_CYCLE:      7      -218.177376093      -134.244013844      2.39070E-04
SCF_CYCLE:      8      -218.177396346      -134.244034097
SCF_CYCLE:      9      -218.177384544      -134.244022295      2.12360E-04      4-POINT
At termination total energy is      -134.244022 Hartrees

Atom      J      K      Vee      JHF      KHF
VeeHF
1      4.731195      -0.172328      4.558866      4.888402      -0.329536
4.558866

Atom      Coulomb
1      3.635814
J_total      3.635814

Atom      Kinetic(x)      Kinetic(y)      Kinetic(z)      Total
1      0.199600      0.192762      0.192561      0.584923

Ttotal:      0.199600      0.192762      0.192561      0.584923

Atom      Vne
1      -7.805313
Vne_total:      -7.805313

```

```

Atomic properties for atom #      1
+++++
Number of Electrons, N           =    0.9362212077
Pure Exchange, K ( 2K_ab)       =   -0.1723284183
HF Exchange, KHF ( 2K_ab+ Kaa)  =   -0.3295360224
Kinetic energy Numerical, T      =    0.5849226569
Potential Energy Analytical, Vne =   -8.5931541860
Potential Energy Numerical, Vne  =   -7.8053134927
Coulomb Energy Anal/Num, Vee     =    3.6358137319
Pure Coulomb, J ( 4J_ab+ Jaa)   =    4.7311946586
HF Coulomb, JHF ( 4J_ab+ 2Jaa)  =    4.8884022626
Jaa = Kaa                        =    0.1572076040
Coulomb Numerically Over A      =    0.3719479049

PROGRAM> end of inputs

Program terminated normally

Job: C2H7N_C1_RHF_631Gd ended on :24-Aug-18 at 19:45:37
User: ibrahim
Cpu      time: 00h00m11s69c on ibrahim-Lenovo-IdeaPad-P500
Elapsed time: 00h00m12s00c

*** RUNing the inputfile :: INPUT_ALL_0007.dat

Welcome to MUNgauss - July 9, 2018 Version 7

N_molecules: 1
Molecule is an asymmetric top.
Point group: C1
Cartesian coordinates for: TEST
-----

```

| COORDINATES IN BOHR |    |             | COORDINATES IN ANGSTROMS |             |             |             |
|---------------------|----|-------------|--------------------------|-------------|-------------|-------------|
| I                   | EL | AN          | X                        | Y           | Z           | X           |
|                     |    | Y           | Z                        |             |             |             |
| 1                   | H  | 1           | 0.73050930               | -0.58944902 | -0.34663973 | 1.38046242  |
|                     |    | -1.11389713 | -0.65505411              |             |             |             |
| 2                   | N  | 7           | 0.00000000               | 0.00000000  | 0.00000000  | 0.00000000  |
|                     |    | 0.00000000  | 0.00000000               |             |             |             |
| 3                   | C  | 6           | 0.00000000               | 0.00000000  | 1.44084286  | 0.00000000  |
|                     |    | 0.00000000  | 2.72279820               |             |             |             |
| 4                   | H  | 1           | -0.87404490              | -0.32011104 | -0.36473314 | -1.65170536 |
|                     |    | -0.60492216 | -0.68924570              |             |             |             |
| 5                   | H  | 1           | -0.79508214              | 0.64155293  | 1.80219079  | -1.50248738 |
|                     |    | 1.21235925  | 3.40564677               |             |             |             |
| 6                   | H  | 1           | -0.15801825              | -1.00939572 | 1.80219079  | -0.29861119 |
|                     |    | -1.90748133 | 3.40564677               |             |             |             |
| 7                   | H  | 1           | 0.95326447               | 0.36771031  | 1.80219079  | 1.80140864  |
|                     |    | 0.69487173  | 3.40564677               |             |             |             |

```

-----
Nuclear repulsion energy:      42.600440492

```



```

Distance Matrix for: TEST
      1      2      3      4      5      6
7
1      0.000000      1.000626      2.018957      1.627103      2.908654      2.362903
2.362889
2      1.000626      0.000000      1.440843      0.999728      2.071627      2.071652
2.071670
3      2.018957      1.440843      0.000000      2.031386      1.083660      1.083707
1.083742
4      1.627103      0.999728      2.031386      0.000000      2.372044      2.383981
2.916799
5      2.908654      2.071627      1.083660      2.372044      0.000000      1.769599
1.769663
6      2.362903      2.071652      1.083707      2.383981      1.769599      0.000000
1.769568
7      2.362889      2.071670      1.083742      2.916799      1.769663      1.769568
0.000000

Charge=      0, Number of electrons=      18

The basis set has now been re-ordered FDPS
The basis set has now been re-ordered FDPS
6-31G(d) Basis Set - Total number of basis functions:      40

Partitioning scheme set to: BECKE
The basis set has now been re-ordered FDPS
Projecting extended Huckel matrix (STO-3G) to 6-31G(d)
All integrals will be kept INCORE
NOTE: INTEGRALS .LE. 1.00E-07 (I2EACC) WERE NOT KEPT
Exponent cutoff used: 2.00E+01 PQCUT2 cutoff used: 1.00E-16

183482 TWO-ELECTRON INTEGRALS CALCULATED IN IDFCCL (COMBINATIONS)
TOTAL NUMBER OF EACH OF THE 8 TYPES OF INTEGRALS SAVED:
IIKL:      8340 IJKJ:      8637 IJJL:      9078 IIKK:      802
IJJJ:      194 IIIL:      194 IIII:      12 IJKL:      156225
Number of integrals in INCORE buffers:
IIKK:      402 IJJL:      4546 IJKJ:      4323
IIKL:      4193 IJKL:      52435

71542 TWO-ELECTRON INTEGRALS CALCULATED IN ISPCLC (COMBINATIONS)
TOTAL NUMBER OF EACH OF THE 8 TYPES OF INTEGRALS SAVED:
IIKL:      5336 IJKJ:      5623 IJJL:      5351 IIKK:      748
IJJJ:      269 IIIL:      270 IIII:      28 IJKL:      53917
Number of integrals in INCORE buffers:
IIKK:      780 IJJL:      7236 IJKJ:      7151
IIKL:      6883 IJKL:      70608
TOTAL OF      255024 TWO-ELECTRON INTEGRALS CALCULATED (COMBINATIONS)

CLOSED SHELL SCF      Nuclear Repulsion Energy is      42.600440492 Hartrees
Convergence on Density Matrix Required to Exit is 5.0000E-06

      CYCLE      ELECTRONIC ENERGY      TOTAL ENERGY      CONVERGENCE      EXTRAPOLATION
SCF_CYCLE:      1      -137.590780320      -94.990339828
SCF_CYCLE:      2      -137.789680229      -95.189239737      1.90274E-02
SCF_CYCLE:      3      -137.805890701      -95.205450209      6.03033E-03
SCF_CYCLE:      4      -137.808182367      -95.207741875      2.82643E-03
SCF_CYCLE:      5      -137.808604191      -95.208163699      1.31109E-03
SCF_CYCLE:      6      -137.808696194      -95.208255702      7.00445E-04
SCF_CYCLE:      7      -137.808718120      -95.208277628      3.41503E-04
SCF_CYCLE:      8      -137.808738113      -95.208297621
SCF_CYCLE:      9      -137.808725573      -95.208285081      3.23420E-04      4-POINT

```

```

At termination total energy is      -95.208285  Hartrees

Atom      J      K      Vee      JHF      KHF
VeeHF
  1      3.283657    -0.158273    3.125384    3.464038    -0.338654
  3.125384

Atom      Coulomb
  1      3.002990
J_total    3.002990

Atom      Kinetic(x)    Kinetic(y)    Kinetic(z)    Total
  1      0.227447      0.233828      0.226529      0.687805

Ttotal:    0.227447      0.233828      0.226529      0.687805

Atom      Vne
  1      -6.583226
Vne_total:    -6.583226

  Atomic properties for atom #      1
  ++++++
Number of Electrons, N      =      0.9122002778
Pure Exchange, K ( 2K_ab)    =      -0.1582727854
HF Exchange, KHF ( 2K_ab+ Kaa) =      -0.3386542079
Kinetic energy Numerical, T    =      0.6878047756
Potential Energy Analytical, Vne =      -7.3027568430
Potential Energy Numerical, Vne =      -6.5832255905
Coulomb Energy Anal/Num, Vee    =      3.0029900015
Pure Coulomb, J ( 4J_ab+ Jaa)   =      3.2836565308
HF Coulomb, JHF ( 4J_ab+ 2Jaa)  =      3.4640379533
Jaa = Kaa                      =      0.1803814224
Coulomb Numerically Over A      =      0.3613782943

PROGRAM> end of inputs

Program terminated normally

Job: CH5N_C1_RHF_631Gd ended on :24-Aug-18 at 19:45:41
User: ibrahim
Cpu      time:  00h00m04s67c on ibrahim-Lenovo-IdeaPad-P500
Elapsed time:  00h00m04s00c

*** RUNing the inputfile :: INPUT_ALL_0008.dat

Welcome to MUNgauss - July 9, 2018 Version 7

N_molecules:  1
Molecule is an asymmetric top.
Point group: C1
Cartesian coordinates for: TEST

-----
                                COORDINATES IN ANGSTROMS
COORDINATES IN BOHR
I EL      AN      X      Y      Z      X

```

|     |   | Y           | Z           |             |             |
|-----|---|-------------|-------------|-------------|-------------|
| 1 H | 1 | -0.87404490 | -0.32011104 | -0.36473314 | -1.65170536 |
|     |   | -0.60492216 | -0.68924570 |             |             |
| 2 N | 7 | 0.00000000  | 0.00000000  | 0.00000000  | 0.00000000  |
|     |   | 0.00000000  | 0.00000000  |             |             |
| 3 C | 6 | 0.00000000  | 0.00000000  | 1.44084286  | 0.00000000  |
|     |   | 0.00000000  | 2.72279820  |             |             |
| 4 H | 1 | 0.73050930  | -0.58944902 | -0.34663973 | 1.38046242  |
|     |   | -1.11389713 | -0.65505411 |             |             |
| 5 H | 1 | 0.95932437  | 0.35134388  | 1.80219079  | 1.81286020  |
|     |   | 0.66394366  | 3.40564677  |             |             |
| 6 H | 1 | -0.17544770 | -1.00651259 | 1.80219079  | -0.33154808 |
|     |   | -1.90203300 | 3.40564677  |             |             |
| 7 H | 1 | -0.78407467 | 0.65509612  | 1.80219079  | -1.48168628 |
|     |   | 1.23795217  | 3.40564677  |             |             |

Nuclear repulsion energy: 42.600440385

Distance Matrix for: TEST

|   | 1        | 2        | 3        | 4        | 5        | 6        |
|---|----------|----------|----------|----------|----------|----------|
| 7 |          |          |          |          |          |          |
| 1 | 0.000000 | 0.999728 | 2.031386 | 1.627103 | 2.916788 | 2.377971 |
|   | 2.377958 |          |          |          |          |          |
| 2 | 0.999728 | 0.000000 | 1.440843 | 1.000626 | 2.071627 | 2.071652 |
|   | 2.071670 |          |          |          |          |          |
| 3 | 2.031386 | 1.440843 | 0.000000 | 2.018957 | 1.083660 | 1.083707 |
|   | 1.083742 |          |          |          |          |          |
| 4 | 1.627103 | 1.000626 | 2.018957 | 0.000000 | 2.356888 | 2.369002 |
|   | 2.908665 |          |          |          |          |          |
| 5 | 2.916788 | 2.071627 | 1.083660 | 2.356888 | 0.000000 | 1.769599 |
|   | 1.769663 |          |          |          |          |          |
| 6 | 2.377971 | 2.071652 | 1.083707 | 2.369002 | 1.769599 | 0.000000 |
|   | 1.769568 |          |          |          |          |          |
| 7 | 2.377958 | 2.071670 | 1.083742 | 2.908665 | 1.769663 | 1.769568 |
|   | 0.000000 |          |          |          |          |          |

Charge= 0, Number of electrons= 18

The basis set has now been re-ordered FDPS

The basis set has now been re-ordered FDPS

6-31G(d) Basis Set - Total number of basis functions: 40

Partitioning scheme set to: BECKE

The basis set has now been re-ordered FDPS

Projecting extended Huckel matrix (STO-3G) to 6-31G(d)

All integrals will be kept INCORE

NOTE: INTEGRALS .LE. 1.00E-07 (I2EACC) WERE NOT KEPT

Exponent cutoff used: 2.00E+01 PQCUT2 cutoff used: 1.00E-16

183462 TWO-ELECTRON INTEGRALS CALCULATED IN IDFCLC (COMBINATIONS)

TOTAL NUMBER OF EACH OF THE 8 TYPES OF INTEGRALS SAVED:

IIKL: 8340 IJKJ: 8637 IJJL: 9078 IIKK: 802

IJJJ: 194 IIIL: 194 IIII: 12 IJKL: 156205

Number of integrals in INCORE buffers:

IIKK: 402 IJJL: 4546 IJKJ: 4323

IIKL: 4193 IJKL: 52437

71530 TWO-ELECTRON INTEGRALS CALCULATED IN ISPCLC (COMBINATIONS)

TOTAL NUMBER OF EACH OF THE 8 TYPES OF INTEGRALS SAVED:

IIKL: 5335 IJKJ: 5624 IJJL: 5351 IIKK: 748

```

IJJJ:      269 IJIL:      270 IIII:      28 IJKL:      53905
Number of integrals in INCORE buffers:
IIKK:      780 IJJL:      7236 IJKJ:      7151
IIKL:      6883 IJKL:      70610
TOTAL OF      254992 TWO-ELECTRON INTEGRALS CALCULATED (COMBINATIONS)

CLOSED SHELL SCF          Nuclear Repulsion Energy is      42.600440385 Hartrees
Convergence on Density Matrix Required to Exit is  5.0000E-06

      CYCLE      ELECTRONIC ENERGY      TOTAL ENERGY      CONVERGENCE      EXTRAPOLATION
SCF_CYCLE:  1      -137.590779928      -94.990339544
SCF_CYCLE:  2      -137.789680198      -95.189239813      1.90274E-02
SCF_CYCLE:  3      -137.805890762      -95.205450378      6.03040E-03
SCF_CYCLE:  4      -137.808182454      -95.207742069      2.82647E-03
SCF_CYCLE:  5      -137.808604285      -95.208163901      1.31112E-03
SCF_CYCLE:  6      -137.808696290      -95.208255906      7.00458E-04
SCF_CYCLE:  7      -137.808718216      -95.208277832      3.41509E-04
SCF_CYCLE:  8      -137.808738210      -95.208297826
SCF_CYCLE:  9      -137.808725670      -95.208285285      3.23428E-04      4-POINT
At termination total energy is      -95.208285 Hartrees

Atom      J      K      Vee      JHF      KHF
VeeHF
  1      3.268000      -0.158437      3.109563      3.448276      -0.338712
3.109563

Atom      Coulomb
  1      2.998325
J_total      2.998325

Atom      Kinetic(x)      Kinetic(y)      Kinetic(z)      Total
  1      0.230469      0.231221      0.226525      0.688215
Ttotal:      0.230469      0.231221      0.226525      0.688215

Atom      Vne
  1      -6.574218
Vne_total:      -6.574218

Atomic properties for atom # 1
++++
Number of Electrons, N      =      0.9120293225
Pure Exchange, K ( 2K_ab)      =      -0.1584367635
HF Exchange, KHF ( 2K_ab+ Kaa)      =      -0.3387121932
Kinetic energy Numerical, T      =      0.6882153433
Potential Energy Analytical, Vne      =      -7.2924262401
Potential Energy Numerical, Vne      =      -6.5742177172
Coulomb Energy Anal/Num, Vee      =      2.9983253516
Pure Coulomb, J ( 4J_ab+ Jaa)      =      3.2680001524
HF Coulomb, JHF ( 4J_ab+ 2Jaa)      =      3.4482755821
Jaa = Kaa      =      0.1802754296
Coulomb Numerically Over A      =      0.3612338864

PROGRAM> end of inputs

Program terminated normally

Job: CH5N_C1_RHF_631Gd ended on :24-Aug-18 at 19:45:46

```

User: ibrahim  
 Cpu time: 00h00m04s64c on ibrahim-Lenovo-IdeaPad-P500  
 Elapsed time: 00h00m05s00c

\*\*\* RUNing the inputfile :: INPUT\_ALL\_0009.dat

Welcome to MUNgauss - July 9, 2018 Version 7

N\_molecules: 1  
 Molecule is an asymmetric top.  
 Point group: C1  
 Cartesian coordinates for: TEST

| COORDINATES IN BOHR |    |             | COORDINATES IN ANGSTROMS |             |            |             |
|---------------------|----|-------------|--------------------------|-------------|------------|-------------|
| I                   | EL | AN          | X                        | Y           | Z          | X           |
|                     |    | Y           | Z                        |             |            |             |
| 1                   | O  | 8           | 1.56620070               | 0.37919230  | 3.23785258 | 2.95969016  |
|                     |    | 0.71656954  | 6.11865417               |             |            |             |
| 2                   | C  | 6           | 1.42338653               | 0.00000000  | 1.96785902 | 2.68981052  |
|                     |    | 0.00000000  | 3.71871433               |             |            |             |
| 3                   | H  | 1           | 0.75176248               | 0.68026803  | 3.61808458 | 1.42062510  |
|                     |    | 1.28552018  | 6.83718846               |             |            |             |
| 4                   | O  | 8           | 2.35288801               | -0.36180632 | 1.33520098 | 4.44631363  |
|                     |    | -0.68371481 | 2.52316398               |             |            |             |
| 5                   | C  | 6           | 0.00000000               | 0.00000000  | 1.44084286 | 0.00000000  |
|                     |    | 0.00000000  | 2.72279820               |             |            |             |
| 6                   | H  | 1           | -0.00222494              | -0.32209824 | 0.40616133 | -0.00420453 |
|                     |    | -0.60867742 | 0.76753362               |             |            |             |
| 7                   | H  | 1           | -0.60406692              | -0.67858912 | 2.03164044 | -1.14152095 |
|                     |    | -1.28234749 | 3.83924375               |             |            |             |
| 8                   | H  | 1           | -0.41037651              | 1.00075379  | 1.50851356 | -0.77549915 |
|                     |    | 1.89115044  | 2.85067729               |             |            |             |

Nuclear repulsion energy: 122.214452707

Distance Matrix for: TEST

|   | 1        | 2        | 3        | 4        | 5        | 6        |
|---|----------|----------|----------|----------|----------|----------|
| 7 | 8        |          |          |          |          |          |
| 1 | 0.000000 | 1.333067 | 0.947910 | 2.188159 | 2.413714 | 3.312136 |
|   | 2.698872 | 2.698853 |          |          |          |          |
| 2 | 1.333067 | 0.000000 | 1.907115 | 1.181157 | 1.517819 | 2.138928 |
|   | 2.138953 | 2.138970 |          |          |          |          |
| 3 | 0.947910 | 1.907115 | 0.000000 | 2.976757 | 2.401727 | 3.448142 |
|   | 2.490295 | 2.429726 |          |          |          |          |
| 4 | 2.188159 | 1.181157 | 2.976757 | 0.000000 | 2.382886 | 2.532044 |
|   | 3.054335 | 3.085812 |          |          |          |          |
| 5 | 2.413714 | 1.517819 | 2.401727 | 2.382886 | 0.000000 | 1.083660 |
|   | 1.083707 | 1.083742 |          |          |          |          |
| 6 | 3.312136 | 2.138928 | 3.448142 | 2.532044 | 1.083660 | 0.000000 |
|   | 1.769599 | 1.769663 |          |          |          |          |
| 7 | 2.698872 | 2.138953 | 2.490295 | 3.054335 | 1.083707 | 1.769599 |
|   | 0.000000 | 1.769568 |          |          |          |          |
| 8 | 2.698853 | 2.138970 | 2.429726 | 3.085812 | 1.083742 | 1.769663 |
|   | 1.769568 | 0.000000 |          |          |          |          |

```

Charge=      0, Number of electrons=    32

The basis set has now been re-ordered FDPS
The basis set has now been re-ordered FDPS
6-31G(d) Basis Set - Total number of basis functions:      68

Partitioning scheme set to: BECKE
The basis set has now been re-ordered FDPS
Projecting extended Huckel matrix (STO-3G) to 6-31G(d)
All integrals will be kept INCORE
NOTE: INTEGRALS .LE. 1.00E-07 (I2EACC) WERE NOT KEPT
Exponent cutoff used: 2.00E+01 PQCT2 cutoff used: 1.00E-16

2016067 TWO-ELECTRON INTEGRALS CALCULATED IN IDFCLC (COMBINATIONS)
TOTAL NUMBER OF EACH OF THE 8 TYPES OF INTEGRALS SAVED:
IIKL:      65204 IJKJ:      65543 IJJL:      66540 IIKK:      2555
IJJJ:      1037 IIIL:      1043 IIII:      24 IJKL:      1814121
Number of integrals in INCORE buffers:
IIKK:      1332 IJJL:      34694 IJKJ:      34199
IIKL:      34436 IJKL:      653603

432221 TWO-ELECTRON INTEGRALS CALCULATED IN ISPCLC (COMBINATIONS)
TOTAL NUMBER OF EACH OF THE 8 TYPES OF INTEGRALS SAVED:
IIKL:      23612 IJKJ:      23852 IJJL:      23629 IIKK:      1807
IJJJ:      773 IIIL:      778 IIII:      44 IJKL:      357726
Number of integrals in INCORE buffers:
IIKK:      2278 IJJL:      47269 IJKJ:      46807
IIKL:      47049 IJKL:      784963
TOTAL OF      2448288 TWO-ELECTRON INTEGRALS CALCULATED (COMBINATIONS)

CLOSED SHELL SCF                      Nuclear Repulsion Energy is      122.214452707 Hartrees
Convergence on Density Matrix Required to Exit is 5.0000E-06

      CYCLE      ELECTRONIC ENERGY      TOTAL ENERGY      CONVERGENCE      EXTRAPOLATION
SCF_CYCLE:    1      -349.154741317      -226.940288610
SCF_CYCLE:    2      -349.862343544      -227.647890837      3.25505E-02
SCF_CYCLE:    3      -349.944259385      -227.729806678      1.31496E-02
SCF_CYCLE:    4      -349.971468451      -227.757015743      1.06281E-02
SCF_CYCLE:    5      -349.988791020      -227.774338313      8.06894E-03
SCF_CYCLE:    6      -349.997923860      -227.783471152      6.36537E-03
SCF_CYCLE:    7      -350.013106898      -227.798654191
SCF_CYCLE:    8      -350.013249493      -227.798796786      2.87013E-03      4-POINT
SCF_CYCLE:    9      -350.013263223      -227.798810515      1.55083E-04
SCF_CYCLE:   10      -350.013268538      -227.798815831      1.01305E-04
SCF_CYCLE:   11      -350.013272483      -227.798819775      4-POINT
SCF_CYCLE:   12      -350.013272100      -227.798819393      3.91899E-05
At termination total energy is      -227.798819 Hartrees

Atom      J      K      Vee      JHF      KHF
VeeHF
1      54.706447      -2.023613      52.682834      60.872489      -8.189655
52.682834

Atom      Coulomb
1      63.515892
J_total      63.515892

Atom      Kinetic(x)      Kinetic(y)      Kinetic(z)      Total
1      24.705261      24.959704      24.455530      74.120495

```

Ttotal: 24.705261 24.959704 24.455530 74.120495

Atom Vne  
1 -231.650205  
Vne\_total: -231.650205

Atomic properties for atom # 1  
+++++  
Number of Electrons, N = 8.0096836596  
Pure Exchange, K ( 2K\_ab) = -2.0236129019  
HF Exchange, KHF ( 2K\_ab+ Kaa) = -8.1896546098  
Kinetic energy Numerical, T = 74.1204946192  
Potential Energy Analytical, Vne = -231.9860318506  
Potential Energy Numerical, Vne = -231.6502053280  
Coulomb Energy Anal/Num, Vee = 63.5158922613  
Pure Coulomb, J ( 4J\_ab+ Jaa) = 54.7064470025  
HF Coulomb, JHF ( 4J\_ab+ 2Jaa) = 60.8724887104  
Jaa = Kaa = 6.1660417079  
Coulomb Numerically Over A = 37.6294775256

PROGRAM> end of inputs

Program terminated normally

Job: C2H4O2\_C1\_RHF\_631Gd ended on :24-Aug-18 at 19:46:05  
User: ibrahim  
Cpu time: 00h00m19s37c on ibrahim-Lenovo-IdeaPad-P500  
Elapsed time: 00h00m19s00c

\*\*\* RUNing the inputfile :: INPUT\_ALL\_0010.dat

Welcome to MUNgauss - July 9, 2018 Version 7

N\_molecules: 1  
Molecule is an asymmetric top.  
Point group: C1  
Cartesian coordinates for: TEST

| COORDINATES IN BOHR |             | COORDINATES IN ANGSTROMS |             |            |            |
|---------------------|-------------|--------------------------|-------------|------------|------------|
| I EL                | AN          | X                        | Y           | Z          | X          |
|                     | Y           | Z                        |             |            |            |
| 1 H                 | 1           | 0.75176248               | 0.68026803  | 3.61808458 | 1.42062510 |
|                     | 1.28552018  | 6.83718846               |             |            |            |
| 2 O                 | 8           | 1.56620070               | 0.37919230  | 3.23785258 | 2.95969016 |
|                     | 0.71656954  | 6.11865417               |             |            |            |
| 3 C                 | 6           | 1.42338653               | 0.00000000  | 1.96785902 | 2.68981052 |
|                     | 0.00000000  | 3.71871433               |             |            |            |
| 4 H                 | 1           | 2.37886126               | -0.33411421 | 1.58087862 | 4.49539595 |
|                     | -0.63138430 | 2.98742741               |             |            |            |
| 5 H                 | 1           | 0.70717887               | -0.81134180 | 1.91135348 | 1.33637428 |
|                     | -1.53321369 | 3.61193435               |             |            |            |
| 6 H                 | 1           | 1.06777838               | 0.83714620  | 1.37860065 | 2.01780856 |
|                     | 1.58197694  | 2.60517747               |             |            |            |

Nuclear repulsion energy: 41.946468146

Distance Matrix for: TEST

|   | 1        | 2        | 3        | 4        | 5        | 6        |
|---|----------|----------|----------|----------|----------|----------|
| 1 | 0.000000 | 0.947910 | 1.907115 | 2.797611 | 2.267117 | 2.267105 |
| 2 | 0.947910 | 0.000000 | 1.333067 | 1.978582 | 1.978608 | 1.978627 |
| 3 | 1.907115 | 1.333067 | 0.000000 | 1.083660 | 1.083707 | 1.083742 |
| 4 | 2.797611 | 1.978582 | 1.083660 | 0.000000 | 1.769599 | 1.769663 |
| 5 | 2.267117 | 1.978608 | 1.083707 | 1.769599 | 0.000000 | 1.769568 |
| 6 | 2.267105 | 1.978627 | 1.083742 | 1.769663 | 1.769568 | 0.000000 |

Charge= 0, Number of electrons= 18

The basis set has now been re-ordered FDPS

The basis set has now been re-ordered FDPS

6-31G(d) Basis Set - Total number of basis functions: 38

Partitioning scheme set to: BECKE

The basis set has now been re-ordered FDPS

Projecting extended Huckel matrix (STO-3G) to 6-31G(d)

All integrals will be kept INCORE

NOTE: INTEGRALS .LE. 1.00E-07 (I2EACC) WERE NOT KEPT

Exponent cutoff used: 2.00E+01 PQCT2 cutoff used: 1.00E-16

202149 TWO-ELECTRON INTEGRALS CALCULATED IN IDFCCL (COMBINATIONS)

TOTAL NUMBER OF EACH OF THE 8 TYPES OF INTEGRALS SAVED:

|       |       |       |       |       |       |       |        |
|-------|-------|-------|-------|-------|-------|-------|--------|
| IIKL: | 10353 | IJKJ: | 10370 | IJJL: | 10576 | IIKK: | 756    |
| IJJJ: | 264   | IIIL: | 264   | IIII: | 12    | IJKL: | 169554 |

Number of integrals in INCORE buffers:

|       |      |       |       |       |      |
|-------|------|-------|-------|-------|------|
| IIKK: | 378  | IJJL: | 5294  | IJKJ: | 5186 |
| IIKL: | 5186 | IJKL: | 56669 |       |      |

59744 TWO-ELECTRON INTEGRALS CALCULATED IN ISPCLC (COMBINATIONS)

TOTAL NUMBER OF EACH OF THE 8 TYPES OF INTEGRALS SAVED:

|       |      |       |      |       |      |       |       |
|-------|------|-------|------|-------|------|-------|-------|
| IIKL: | 4891 | IJKJ: | 4939 | IJJL: | 4865 | IIKK: | 645   |
| IJJJ: | 265  | IIIL: | 265  | IIII: | 26   | IJKL: | 43848 |

Number of integrals in INCORE buffers:

|       |      |       |       |       |      |
|-------|------|-------|-------|-------|------|
| IIKK: | 703  | IJJL: | 7740  | IJKJ: | 7662 |
| IIKL: | 7644 | IJKL: | 71391 |       |      |

TOTAL OF 261893 TWO-ELECTRON INTEGRALS CALCULATED (COMBINATIONS)

CLOSED SHELL SCF Nuclear Repulsion Energy is 41.946468146 Hartrees  
Convergence on Density Matrix Required to Exit is 5.0000E-06

|            | CYCLE | ELECTRONIC ENERGY | TOTAL ENERGY   | CONVERGENCE | EXTRAPOLATION |
|------------|-------|-------------------|----------------|-------------|---------------|
| SCF_CYCLE: | 1     | -156.599321711    | -114.652853564 |             |               |
| SCF_CYCLE: | 2     | -156.936416265    | -114.989948119 | 3.10786E-02 |               |
| SCF_CYCLE: | 3     | -156.968403705    | -115.021935559 | 9.05486E-03 |               |
| SCF_CYCLE: | 4     | -156.974430451    | -115.027962305 | 5.34933E-03 |               |
| SCF_CYCLE: | 5     | -156.976003710    | -115.029535564 | 2.57505E-03 |               |
| SCF_CYCLE: | 6     | -156.976488557    | -115.030020410 | 1.63418E-03 |               |
| SCF_CYCLE: | 7     | -156.976798318    | -115.030330172 |             | 4-POINT       |
| SCF_CYCLE: | 8     | -156.976733251    | -115.030265104 | 6.30541E-04 |               |
| SCF_CYCLE: | 9     | -156.976733281    | -115.030265135 | 1.01697E-05 |               |

At termination total energy is -115.030265 Hartrees

| Atom  | J | K | Vee | JHF | KHF |
|-------|---|---|-----|-----|-----|
| VeeHF |   |   |     |     |     |



```

      1      3.189251      -0.148162      3.041089      3.381640      -0.340551
      3.041089

Atom      Coulomb
      1      3.134249
J_total      3.134249

Atom      Kinetic(x)      Kinetic(y)      Kinetic(z)      Total
      1      0.275231      0.259927      0.257344      0.792503

Ttotal:      0.275231      0.259927      0.257344      0.792503

Atom      Vne
      1      -6.886800
Vne_total:      -6.886800

  Atomic properties for atom #      1
  ++++++
Number of Electrons, N      =      0.8692118318
Pure Exchange, K ( 2K_ab)      =      -0.1481619415
HF Exchange, KHF ( 2K_ab+ Kaa)      =      -0.3405505985
Kinetic energy Numerical, T      =      0.7925026501
Potential Energy Analytical, Vne      =      -7.7815164006
Potential Energy Numerical, Vne      =      -6.8868003991
Coulomb Energy Anal/Num, Vee      =      3.1342492136
Pure Coulomb, J ( 4J_ab+ Jaa)      =      3.1892511907
HF Coulomb, JHF ( 4J_ab+ 2Jaa)      =      3.3816398477
Jaa = Kaa      =      0.1923886570
Coulomb Numerically Over A      =      0.3393663631

PROGRAM> end of inputs

Program terminated normally

Job: CH40_C1_RHF_631Gd ended on :24-Aug-18 at 19:46:09
User: ibrahim
Cpu      time: 00h00m03s97c on ibrahim-Lenovo-IdeaPad-P500
Elapsed time: 00h00m04s00c

+-----+
|      Calculated molecule properties using the stored fragments properties
|
|      (already calculated, using fragment weights and geometry)
|
+-----+-----+-----+-----+-----+-----+-----+-----+-----+
|Atom#|  Electrons  | Potential Vne| Vee Ana/Num  | HF Coulomb J | HF Exchange K |
|      | Coulomb J   | Exchange K   | Kinetic T    |               |               |
+-----+-----+-----+-----+-----+-----+-----+-----+
|      | 1|      7.16505105| -167.95397363| 46.80195251| 44.37803191| -6.64743280|
|      | 38.96641988| -1.23582077| 54.00913414|
|      | 2|      6.10120522| -147.17544847| 48.23267128| 47.81924437| -5.27389078|
|      | 43.47856678| -0.93321319| 37.96886000|
|      | 3|      6.12610594| -156.56047061| 53.04709911| 48.88660308| -5.37537859|

```

```

44.47064459| -0.95942009| 38.69282040|
| 4| 8.14246576| -232.18359091| 63.92985497| 66.55466259| -8.20084270|
60.21731163| -1.86349173| 74.22603294|
| 5| 0.93931793| -8.63995815| 3.66674175| 4.93564809| -0.33231622|
4.77709896| -0.17376710| 0.59099010|
| 6| 0.93622121| -8.59315419| 3.63581373| 4.88840226| -0.32953602|
4.73119466| -0.17232842| 0.58492266|
| 7| 0.91220028| -7.30275684| 3.00299000| 3.46403795| -0.33865421|
3.28365653| -0.15827279| 0.68780478|
| 8| 0.91202932| -7.29242624| 2.99832535| 3.44827558| -0.33871219|
3.26800015| -0.15843676| 0.68821534|
| 9| 8.00968366| -231.98603185| 63.51589226| 60.87248871| -8.18965461|
54.70644700| -2.02361290| 74.12049462|
| 10| 0.86921183| -7.78151640| 3.13424921| 3.38163985| -0.34055060|
3.18925119| -0.14816194| 0.79250265|
+-----+-----+-----+-----+-----+-----+
| Sum= | 40.11349219| -975.46932729| 291.96559017| 288.62903440| -35.36696872|
261.08859137| -7.82652569| 282.36177763|
+-----+-----+-----+-----+-----+-----+

```

```

+-----+-----+-----+-----+-----+-----+
| Calculated molecule properties using the stored |
| electron density in each radial grid point |
| NOTE: Current partitioning weight |
+-----+-----+-----+-----+-----+-----+
| Atom#| Electrons | Vne | Vee |
+-----+-----+-----+-----+-----+-----+
| 1| 7.15777802| -184.04853311| 55.02393274|
| 2| 6.10204839| -147.47178143| 48.47777008|
| 3| 6.12586589| -156.61882673| 53.16678291|
| 4| 8.12711617| -244.30502587| 70.03631834|
| 5| 0.93063294| -11.19681140| 4.80074561|
| 6| 0.92887027| -11.15165216| 4.76876720|
| 7| 0.88913766| -11.38502637| 4.72612464|
| 8| 0.90912020| -10.29924806| 4.33377802|
| 9| 8.00779253| -241.04401767| 68.04177982|
| 10| 0.85461039| -11.49622610| 4.65863240|
+-----+-----+-----+-----+-----+-----+
| Sum= | 40.03297246| -1029.01714890| 318.03463175|
+-----+-----+-----+-----+-----+-----+

```

```

Nuclear repulsion (Vnn) = 181.51074356
ThE Total Energy = -282.47696467
ThE Virial = 2.00040794

```

```

+-----+-----+-----+-----+-----+-----+
| Calculated molecule properties using the stored |
| electron density in each radial grid point |
| NOTE: Database partitioning weight |
+-----+-----+-----+-----+-----+-----+
| Atom#| Electrons | Vne | Vee |
+-----+-----+-----+-----+-----+-----+
| 1| 7.16505105| -184.20665368| 55.14699849|
| 2| 6.10120522| -147.61809518| 48.54746317|
| 3| 6.12610594| -156.76366675| 53.24110912|
| 4| 8.14246576| -244.47968969| 70.20929978|
| 5| 0.93931793| -11.21750473| 4.85323402|
| 6| 0.93622121| -11.17199079| 4.81545229|
| 7| 0.91220028| -11.41231399| 4.86341981|
| 8| 0.91202932| -10.31552881| 4.35566492|
| 9| 8.00968366| -241.19764270| 68.12952172|

```

```

| 10|      0.86921183|    -11.51729460|      4.74661561|
+---+-----+-----+-----+
|Sum= |      40.11349219|    -1029.90038093|      318.90877893|
+---+-----+-----+-----+

      Nuclear repulsion (Vnn)    =      181.51074356
      The Total Energy           =     -282.48604952
      The Virial                 =       2.00044012

Atom#      AIM_FD      AIM_CW      HF      |Error FD|      |Error CW|      |%Error
FD|      |%Error CW|
1      7.16505105      7.15777802      7.14766342      0.01738763      0.01011460
0.24326312      0.14150921
2      6.10120522      6.10204839      6.11995184      -0.01874662      -0.01790345
-0.30631971      -0.29254238
3      6.12610594      6.12586589      6.12381199      0.00229395      0.00205390
0.03745947      0.03353960
4      8.14246576      8.12711617      8.13943840      0.00302737      -0.01232223
0.03719379      -0.15138918
5      0.93931793      0.93063294      0.92410932      0.01520860      0.00652362
1.64575819      0.70593633
6      0.93622121      0.92887027      0.92516871      0.01105249      0.00370156
1.19464621      0.40009548
7      0.91220028      0.88913766      0.87210041      0.04009987      0.01703725
4.59807925      1.95358780
8      0.91202932      0.90912020      0.90122908      0.01080024      0.00789112
1.19839033      0.87559554
9      8.00968366      8.00779253      8.00374504      0.00593862      0.00404749
0.07419805      0.05056994
10     0.86921183      0.85461039      0.84266991      0.02654192      0.01194048
3.14974090      1.41698156
Sum=    40.11349219      40.03297246      39.99988812      0.11360407      0.03308434
0.28401098      0.08271108

PROGRAM> end of inputs

Program terminated normally

Job: RUN_Gly_ ended on :24-Aug-18 at 19:47:15
User: ibrahim
Cpu      time:  00h01m09s88c on ibrahim-Lenovo-IdeaPad-P500
Elapsed time: 00h03m58s00c

```

## B.1.4 Plotting

Here is an example of input file for plotting some molecular and atomic properties along the CO bond,

```

MOLECULE Multiplicity = 1 Charge = 0
Title = "CO"
Z-Matrix
C
O      C      BL
END ! Z-Matrix

```

```

DEFINE ! 6-311++G(d,p)
BL      = 1.10444701
END !DEFINE
END ! MOLECULE

BASIS name=6-311++G(d,p) end

PARTitioning
Scheme=IAWAD ! other examples: ABSw, IAWAD, Fermi, Becke (default is Becke)
COrre=Average ! the core (Average, MAX1, MAX2, MIN) (default is Average)
! Steepness=0.5 ! For IAWad weight (default is 0.5)
! KValue=15.0 ! For Femi weight (default is 7.0)
end

GRID
MESH
  ORigin= ( 0.0  0.0 -3.5 ) ! orign x,y,z
  MESH = ( 0.02 0.02 0.02 ) ! step size
  NX=0 NY=0 NZ=400 ! number of steps
end ! mesh
end ! grid

PLOT ATOM=false Molecule=true end

AIMDFT ROT=( 2 1 ) end ! Rotate atoms: 2 to (0,0,0), 1 to +Z, None to YZ plane

output object=AIMDFT:MOLECULE%ROTATE end ! Rotate

!output object=GRID:WEIGHTS%MESH end ! plot the weight
!output object=GRID:EXCHANGE%MESH end ! plot the exchange
!output object=GRID:KINETIC%MESH end ! plot the kinetic
output object=GRID:RADIAL_DENSITY%MESH end ! plot the RDN
!output object=GRID:COULOMB%MESH end ! plot J
!output object=GRID:VNE%MESH end ! plot Vne
stop

```

Here is the output file,

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```

N_molecules: 1
Molecule is a symmetric top.
Point group: Cinf_v
Free format Z-Matrix for: CO
C
0      C      BL

```

```

VARIABLES:
BL = 1.10444701

```

Z MATRIX FOR: CO

| ----- |    |          |          |      |       |    |      |
|-------|----|----------|----------|------|-------|----|------|
| I     | AN | Z1<br>Z4 | BL       | Z2   | ALPHA | Z3 | BETA |
| ----- |    |          |          |      |       |    |      |
| 1     | 6  |          |          |      |       |    |      |
| 2     | 8  | 1        | 1.104447 | ( 1) |       |    |      |
| ----- |    |          |          |      |       |    |      |

Cartesian coordinates for: CO

| COORDINATES IN BOHR |    |            |            | COORDINATES IN ANGSTROMS |            |            |  |
|---------------------|----|------------|------------|--------------------------|------------|------------|--|
| I                   | EL | AN         | X          | Y                        | Z          | X          |  |
|                     |    | Y          | Z          |                          |            |            |  |
| 1                   | C  | 6          | 0.00000000 | 0.00000000               | 0.00000000 | 0.00000000 |  |
|                     |    | 0.00000000 | 0.00000000 |                          |            |            |  |
| 2                   | O  | 8          | 0.00000000 | 0.00000000               | 1.10444701 | 0.00000000 |  |
|                     |    | 0.00000000 | 2.08710222 |                          |            |            |  |

Nuclear repulsion energy: 22.998394420

Charge= 0, Number of electrons= 14

The basis set has now been re-ordered FDPS

The basis set has now been re-ordered FDPS

6-311++G(d,p) Basis Set - Total number of basis functions: 46

Partitioning scheme set to: IAWAD

Last atomic core set to: r\_Average

Total number of grid points: 401

RotUpdate is TRUE

The Cartesian coordinates were updated

The new coordinates are: (Angstrom unit)

| Atom | X          | Y          | Z          |
|------|------------|------------|------------|
| C    | 0.00000000 | 0.00000000 | 1.10444701 |
| O    | 0.00000000 | 0.00000000 | 0.00000000 |

The basis set has now been re-ordered FDPS

Projecting extended Huckel matrix (STO-3G) to 6-311++G(d,p)

All integrals will be kept INCORE

NOTE: INTEGRALS .LE. 1.00E-07 (I2EACC) WERE NOT KEPT

Exponent cutoff used: 2.00E+01 PQCT2 cutoff used: 1.00E-16

104707 TWO-ELECTRON INTEGRALS CALCULATED IN IDFCLC (COMBINATIONS)

TOTAL NUMBER OF EACH OF THE 8 TYPES OF INTEGRALS SAVED:

|       |      |       |      |       |      |       |       |
|-------|------|-------|------|-------|------|-------|-------|
| IIKL: | 4482 | IJKJ: | 5770 | IJJL: | 6500 | IIKK: | 948   |
| IJJJ: | 118  | IIIL: | 118  | IIII: | 12   | IJKL: | 86759 |

Number of integrals in INCORE buffers:

|       |      |       |       |       |      |
|-------|------|-------|-------|-------|------|
| IIKK: | 474  | IJJL: | 3252  | IJKJ: | 2885 |
| IIKL: | 2243 | IJKL: | 28925 |       |      |

49463 TWO-ELECTRON INTEGRALS CALCULATED IN ISPCLC (COMBINATIONS)

TOTAL NUMBER OF EACH OF THE 8 TYPES OF INTEGRALS SAVED:

|       |      |       |      |       |      |       |       |
|-------|------|-------|------|-------|------|-------|-------|
| IIKL: | 3651 | IJKJ: | 5050 | IJJL: | 3459 | IIKK: | 1117  |
| IJJJ: | 169  | IIIL: | 169  | IIII: | 34   | IJKL: | 35814 |

Number of integrals in INCORE buffers:

|       |      |       |       |       |      |
|-------|------|-------|-------|-------|------|
| IIKK: | 1035 | IJJL: | 4984  | IJKJ: | 5413 |
| IIKL: | 4071 | IJKL: | 40877 |       |      |

TOTAL OF 154170 TWO-ELECTRON INTEGRALS CALCULATED (COMBINATIONS)

CLOSED SHELL SCF Nuclear Repulsion Energy is 22.998394420 Hartrees

Convergence on Density Matrix Required to Exit is 5.0000E-06

| CYCLE | ELECTRONIC ENERGY | TOTAL ENERGY | CONVERGENCE | EXTRAPOLATION |
|-------|-------------------|--------------|-------------|---------------|
|-------|-------------------|--------------|-------------|---------------|

```

SCF_CYCLE: 1 -135.138449669 -112.140055248
SCF_CYCLE: 2 -135.652326582 -112.653932162 2.93131E-02
SCF_CYCLE: 3 -135.686265729 -112.687871309 1.62184E-02
SCF_CYCLE: 4 -135.711088854 -112.712694434 1.17709E-02
SCF_CYCLE: 5 -135.723210145 -112.724815724 1.11787E-02
SCF_CYCLE: 6 -135.754336026 -112.755941606 4-POINT
SCF_CYCLE: 7 -135.769718423 -112.771324003 5.07036E-03
SCF_CYCLE: 8 -135.769733450 -112.771339030 2.97065E-04
SCF_CYCLE: 9 -135.769736659 -112.771342239 4-POINT
SCF_CYCLE: 10 -135.769736274 -112.771341854 6.45217E-05
SCF_CYCLE: 11 -135.769741458 -112.771347038 1.96897E-04
SCF_CYCLE: 12 -135.769745261 -112.771350841 1.71384E-04
SCF_CYCLE: 13 -135.769752379 -112.771357959 4-POINT
SCF_CYCLE: 14 -135.769756054 -112.771361634 7.83639E-05
At termination total energy is -112.771362 Hartrees

PROGRAM> end of inputs

Program terminated normally

Job: CO_Cinf_v_RHF_6311ppGdp ended on :29-Aug-18 at 13:44:11
User: ibrahim
Cpu time: 00h00m00s10c on ibrahim-Lenovo-IdeaPad-P500
Elapsed time: 00h00m00s00c

```

Here is an example of plot file,

```

Results calculated at: RHF/6-311++G(d,p)
Molecular Radial Density
X and Y fixed to: 0.000000 0.000000
Z, Molecular Radial Density
-3.500000 0.005525
-3.480000 0.005790
-3.460000 0.006066
-3.440000 0.006354
-3.420000 0.006653
-3.400000 0.006964
-3.380000 0.007288
-3.360000 0.007624
-3.340000 0.007973
-3.320000 0.008335
-3.300000 0.008711

```

## B.1.5 Expectation Value of Last Core Shell

Here is example of input file to calculate the expectation value  $\langle r \rangle$  of Cl atom,

```

MOLECULE Multiplicity = 2 Charge = 0
Title = "Cl"
Z-Matrix
Cl
END ! Z-Matrix
END ! MOLECULE
BASIS name=6-311++G(d,p) end
R0HF Ncore=6 Nopen=3 ITeRation=1000 run end

```

```

NUmercial
property=Radial
RAdial GRID=SG1 end
end
output object=QM:DENSITY%NUMERICAL end
stop

```

Here is the output file,

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N\_molecules: 1  
Point group: Kh  
Cartesian coordinates for: Cl

```

-----
COORDINATES IN BOHR      COORDINATES IN ANGSTROMS
I EL      AN      X      Y      Z      X
      Y      Z
-----
1 Cl      17      0.00000000      0.00000000      0.00000000      0.00000000
      0.00000000      0.00000000
-----

```

Nuclear repulsion energy: 0.000000000

Charge= 0, Number of electrons= 17

NOpen= 1 NCore= 8

NOTE: Assuming no degeneracy amongst open-shell orbitals

The basis set has now been re-ordered FDPS

The basis set has now been re-ordered FDPS

6-311++G(d,p) Basis Set - Total number of basis functions: 31

MENU\_SCF> Number of closed shells set to 6

MENU\_SCF> Number of open shells set to 3

MENU\_SCF> Maximum SCF iterations set to 1000

The basis set has now been re-ordered FDPS

Projecting extended Huckel matrix (STO-3G) to 6-311++G(d,p)

RHF OPEN SHELL SPECIFICATION

GROUP : FFFFFFFFFAVVVVVVVVVVVVVVVVVVVVV

ALPHA OCC : 111111110000000000000000000000

BETA OCC : 111111110000000000000000000000

ORBITAL GROUP: F A V

NUMBER OF MO: 8 1 22

OLEVEL DEGENERACY DOUBLE OCC SINGLE ALPHA SINGLE BETA CONFIGURATIONS

SAMPLE CONFIGURATION  
0 1 3 2 1 0 3

AB AB A

+

-- -- --

COULOMB COUPLING COEFFICIENTS A(I,J)

| MO     | OCCA   | OCCB   | CLOSED | OPEN 1 | OPEN 2 | OPEN 3 |
|--------|--------|--------|--------|--------|--------|--------|
| CLOSED | 1.0000 | 1.0000 | 2.0000 | 1.6667 | 1.6667 | 1.6667 |
| OPEN 2 | 1.0000 | 0.6667 | 1.6667 | 1.3333 | 1.3333 | 1.3333 |

|        |        |        |        |        |        |        |
|--------|--------|--------|--------|--------|--------|--------|
| OPEN 3 | 1.0000 | 0.6667 | 1.6667 | 1.3333 | 1.3333 | 1.3333 |
| OPEN 4 | 1.0000 | 0.6667 | 1.6667 | 1.3333 | 1.3333 | 1.3333 |

| EXCHANGE COUPLING COEFFICIENTS |        |        | B(I,J) |        |        |        |
|--------------------------------|--------|--------|--------|--------|--------|--------|
| MO                             | OCCA   | OCCB   | CLOSED | OPEN 1 | OPEN 2 | OPEN 3 |
| CLOSED                         | 1.0000 | 1.0000 | 1.0000 | 0.8333 | 0.8333 | 0.8333 |
| OPEN 2                         | 1.0000 | 0.6667 | 0.8333 | 0.6667 | 0.6667 | 0.6667 |
| OPEN 3                         | 1.0000 | 0.6667 | 0.8333 | 0.6667 | 0.6667 | 0.6667 |
| OPEN 4                         | 1.0000 | 0.6667 | 0.8333 | 0.6667 | 0.6667 | 0.6667 |

All integrals will be kept INCORE  
 NOTE: INTEGRALS .LE. 1.00E-07 (I2EACC) WERE NOT KEPT  
 Exponent cutoff used: 2.00E+01 PQCT2 cutoff used: 1.00E-16

9888 TWO-ELECTRON INTEGRALS CALCULATED IN IDFCLC (RAW)  
 TOTAL NUMBER OF EACH OF THE 8 TYPES OF INTEGRALS SAVED:  
 IIKL: 296 IJKJ: 836 IJJL: 1052 IIKK: 327  
 IJJJ: 24 IIIL: 24 IIII: 6 IJKL: 7323  
 Number of integrals in INCORE buffers:  
 IIKK: 165 IJJL: 526 IJKJ: 418  
 IIKL: 148 IJKL: 2453

8887 TWO-ELECTRON INTEGRALS CALCULATED IN ISPCLC (RAW)  
 TOTAL NUMBER OF EACH OF THE 8 TYPES OF INTEGRALS SAVED:  
 IIKL: 1147 IJKJ: 1276 IJJL: 610 IIKK: 597  
 IJJJ: 66 IIIL: 66 IIII: 25 IJKL: 5100  
 Number of integrals in INCORE buffers:  
 IIKK: 465 IJJL: 831 IJKJ: 1056  
 IIKL: 723 IJKL: 4153  
 TOTAL OF 18775 TWO-ELECTRON INTEGRALS CALCULATED (RAW)

RHF OPEN SHELL SCF  
 CONVERGENCE ON DENSITY MATRIX REQUIRED TO EXIT IS 5.0000D-06, INITIAL SCALE FACTOR (SCALST) IS 0.60000  
 SCALE FACTOR CUT RATIO (FACTDN) IS 0.66667, SCALE FACTOR INCREASE RATIO (FACTUP) IS 1.20000  
 SCALE FACTOR LOWER LIMIT (SLIMDN) IS 0.00050, SCALE FACTOR UPPER LIMIT (SLIMUP) IS 0.80000  
 SWITCH TO INCREASE MODE AT D CONVERGENCE 2.0000D-03 (SCRIT, RHF=INCR MODE ONLY).  
 Nuclear repulsion energy is 0.000000000 Hartrees  
 CYCLE ENERGY SCF CONVERGENCE EXTRAPOLATION

| SCALE      | RESET          | SCF            | ELECTRONIC | TOTAL | COEFF      | DENSITY    |
|------------|----------------|----------------|------------|-------|------------|------------|
| 0          | -458.772202561 | -458.772202561 |            |       |            |            |
| 1          | -458.891001035 | -458.891001035 |            |       |            | 4.9546D-02 |
| 6.0000D-01 | YES            |                |            |       |            |            |
| 2          | -459.451471361 | -459.451471361 |            |       | 1.6572D-02 | 2.9781D-02 |
| 6.0000D-01 | YES            |                |            |       |            |            |
| 3          | -459.224788562 | -459.224788562 |            |       | 1.5073D-02 | 2.5790D-02 |
| 6.0000D-01 | YES            |                |            |       |            |            |
| 4          | -459.289075435 | -459.289075435 |            |       | 2.0801D-02 | 3.0468D-02 |
| 6.0000D-01 | YES            |                |            |       |            |            |
| 5          | -459.021625012 | -459.021625012 |            |       | 3.4345D-02 | 4.9075D-02 |
| 6.0000D-01 | YES            |                |            |       |            |            |
| 6          | -458.280660837 | -458.280660837 |            |       | 6.3981D-02 | 7.9415D-02 |
| 6.0000D-01 | YES            |                |            |       |            |            |
| 7          | -453.860960916 | -453.860960916 |            |       | 1.3775D-01 | 1.7536D-01 |
| 6.0000D-01 | YES            |                |            |       |            |            |
| 8          | -455.707007894 | -455.707007894 |            |       | 1.6493D-01 | 2.9103D-01 |
| 1.1852D-01 | YES            |                |            |       |            |            |
| 9          | -455.612617494 | -455.612617494 |            |       | 1.5923D-01 | 2.6469D-01 |
| 6.0000D-01 | YES            |                |            |       |            |            |



|    |                |                |            |            |
|----|----------------|----------------|------------|------------|
| 10 | -447.162448243 | -447.162448243 | 3.6744D-02 | 2.9407D-01 |
|    | 4.0000D-01 YES |                |            |            |
| 11 | -449.900249936 | -449.900249936 | 1.4882D-02 | 3.1609D-01 |
|    | 2.6667D-01 YES |                |            |            |
| 12 | -457.075125871 | -457.075125871 | 6.0375D-02 | 3.4565D-01 |
|    | 1.1852D-01 YES |                |            |            |
| 13 | -455.905703310 | -455.905703310 | 8.0477D-03 | 3.4317D-01 |
|    | 7.9012D-02 YES |                |            |            |
| 14 | -453.294097820 | -453.294097820 | 1.2178D-02 | 3.0694D-01 |
|    | 1.7778D-01 YES |                |            |            |
| 15 | -454.843032412 | -454.843032412 | 3.4627D-03 | 3.7677D-01 |
|    | 1.0405D-02 YES |                |            |            |
| 16 | -456.592350352 | -456.592350352 | 2.4616D-03 | 3.2049D-01 |
|    | 5.2675D-02 YES |                |            |            |
| 17 | -455.560469276 | -455.560469276 | 2.7767D-03 | 3.5625D-01 |
|    | 1.7778D-01 YES |                |            |            |
| 18 | -448.898879015 | -448.898879015 | 8.7994D-03 | 9.6854D-01 |
|    | 6.0000D-01 YES |                |            |            |
| 19 | -457.700926667 | -457.700926667 | 3.2563D-03 | 1.0743D+00 |
|    | 1.7778D-01 YES |                |            |            |
| 20 | -447.435208609 | -447.435208609 | 8.6254D-03 | 1.0155D+00 |
|    | 6.0000D-01 YES |                |            |            |
| 21 | -456.234213849 | -456.234213849 | 2.1204D-03 | 1.0590D+00 |
|    | 1.7778D-01 YES |                |            |            |
| 22 | -457.100355907 | -457.100355907 | 1.8084D-01 | 3.3080D-01 |
|    | 1.1852D-01 YES |                |            |            |
| 23 | -455.679752922 | -455.679752922 | 1.2540D-01 | 3.4269D-01 |
|    | 3.5117D-02 YES |                |            |            |
| 24 | -460.800574864 | -460.800574864 | 9.5108D-03 | 3.5824D-01 |
|    | 6.0000D-01 YES |                |            |            |
| 25 | -459.008498553 | -459.008498553 | 9.1972D-02 | 2.4129D-01 |
|    | 6.0000D-01 YES |                |            |            |
| 26 | -457.744912026 | -457.744912026 | 1.4814D-02 | 3.3384D-01 |
|    | 6.0000D-01 YES |                |            |            |
| 27 | -457.579926153 | -457.579926153 | 9.9838D-02 | 4.4324D-01 |
|    | 6.0000D-01 YES |                |            |            |
| 28 | -458.917448500 | -458.917448500 | 1.0648D-02 | 4.6793D-01 |
|    | 2.6667D-01 YES |                |            |            |
| 29 | -458.226714489 | -458.226714489 | 7.8724D-02 | 5.2723D-01 |
|    | 3.0829D-03 YES |                |            |            |
| 30 | -460.260373064 | -460.260373064 | 1.4062D-01 | 3.9332D-01 |
|    | 6.0000D-01 YES |                |            |            |
| 31 | -457.625850891 | -457.625850891 | 1.2905D-01 | 2.5640D-01 |
|    | 1.1852D-01 YES |                |            |            |
| 32 | -458.832022002 | -458.832022002 | 1.3738D-01 | 2.5314D-01 |
|    | 6.0000D-01 YES |                |            |            |
| 33 | -454.939760389 | -454.939760389 | 8.3623D-04 | 3.7504D-01 |
|    | 5.2675D-02 YES |                |            |            |
| 34 | -456.739664891 | -456.739664891 | 7.7015D-04 | 3.8178D-01 |
|    | 5.2675D-02 YES |                |            |            |
| 35 | -456.945230394 | -456.945230394 | 6.8413D-04 | 2.0000D-01 |
|    | 5.2675D-02 YES |                |            |            |
| 36 | -457.195639916 | -457.195639916 | 7.3119D-04 | 1.6140D-01 |
|    | 7.9012D-02 YES |                |            |            |
| 37 | -454.197688783 | -454.197688783 | 2.7958D-03 | 3.7077D-01 |
|    | 4.0000D-01 YES |                |            |            |
| 38 | -458.316579026 | -458.316579026 | 8.0862D-02 | 3.5366D-01 |
|    | 2.0553D-03 YES |                |            |            |
| 39 | -457.542854523 | -457.542854523 | 9.0986D-03 | 1.2178D-01 |
|    | 4.0000D-01 YES |                |            |            |
| 40 | -455.875478405 | -455.875478405 | 1.0238D-01 | 8.6068D-02 |
|    | 6.0000D-01 YES |                |            |            |
| 41 | -457.655165507 | -457.655165507 | 1.0550D-03 | 2.0983D-01 |
|    | 7.9012D-02 YES |                |            |            |

|    |                |                |            |            |
|----|----------------|----------------|------------|------------|
| 42 | -458.042087729 | -458.042087729 | 1.4133D-03 | 1.7545D-01 |
|    | 1.1852D-01 YES |                |            |            |
| 43 | -457.940168781 | -457.940168781 | 1.1831D-03 | 1.8627D-01 |
|    | 1.1852D-01 YES |                |            |            |
| 44 | -456.682022967 | -456.682022967 | 1.3207D-01 | 2.0372D-01 |
|    | 2.6667D-01 YES |                |            |            |
| 45 | -458.263330179 | -458.263330179 | 4.1016D-04 | 2.0688D-01 |
|    | 5.2675D-02 YES |                |            |            |
| 46 | -447.322880391 | -447.322880391 | 3.0212D-03 | 4.1350D-01 |
|    | 4.0000D-01 YES |                |            |            |
| 47 | -457.726647483 | -457.726647483 | 8.3823D-02 | 3.8888D-01 |
|    | 3.5117D-02 YES |                |            |            |
| 48 | -456.971708875 | -456.971708875 | 7.1904D-02 | 1.3138D-01 |
|    | 6.0000D-01 YES |                |            |            |
| 49 | -454.245572658 | -454.245572658 | 2.9059D-03 | 3.1669D-01 |
|    | 1.7778D-01 YES |                |            |            |
| 50 | -444.366751154 | -444.366751154 | 3.0660D-03 | 3.9233D-01 |
|    | 2.6667D-01 YES |                |            |            |
| 51 | -453.384895324 | -453.384895324 | 1.5424D-03 | 3.6964D-01 |
|    | 1.7778D-01 YES |                |            |            |
| 52 | -454.391573147 | -454.391573147 | 1.1290D-03 | 2.5721D-01 |
|    | 1.7778D-01 YES |                |            |            |
| 53 | -457.626406762 | -457.626406762 | 3.7611D-04 | 2.0999D-01 |
|    | 5.2675D-02 YES |                |            |            |
| 54 | -447.357529946 | -447.357529946 | 1.0202D-03 | 2.8888D-01 |
|    | 2.6667D-01 YES |                |            |            |
| 55 | -458.294503303 | -458.294503303 | 4.5901D-02 | 3.0412D-01 |
|    | 4.6244D-03 YES |                |            |            |
| 56 | -454.954413052 | -454.954413052 | 8.8269D-02 | 1.6685D-01 |
|    | 6.0000D-01 YES |                |            |            |
| 57 | -456.079224307 | -456.079224307 | 1.1615D-01 | 2.0551D-01 |
|    | 7.9012D-02 YES |                |            |            |
| 58 | -456.478964339 | -456.478964339 | 1.0304D-01 | 1.6610D-01 |
|    | 7.9012D-02 YES |                |            |            |
| 59 | -452.428015695 | -452.428015695 | 1.4993D-03 | 2.3197D-01 |
|    | 1.7778D-01 YES |                |            |            |
| 60 | -437.866322770 | -437.866322770 | 1.8896D-03 | 4.7869D-01 |
|    | 2.6667D-01 YES |                |            |            |
| 61 | -449.135513951 | -449.135513951 | 1.2285D-03 | 4.6380D-01 |
|    | 1.7778D-01 YES |                |            |            |
| 62 | -448.138431820 | -448.138431820 | 9.6326D-04 | 2.0784D-01 |
|    | 1.7778D-01 YES |                |            |            |
| 63 | -456.893586077 | -456.893586077 | 9.0528D-02 | 1.9815D-01 |
|    | 7.9012D-02 YES |                |            |            |
| 64 | -456.903750176 | -456.903750176 | 1.2922D-01 | 2.5547D-01 |
|    | 7.9012D-02 YES |                |            |            |
| 65 | -451.915269653 | -451.915269653 | 2.7655D-03 | 2.9528D-01 |
|    | 2.6667D-01 YES |                |            |            |
| 66 | -455.327229310 | -455.327229310 | 5.8918D-04 | 3.2280D-01 |
|    | 1.5607D-02 YES |                |            |            |
| 67 | -436.887744424 | -436.887744424 | 4.2152D-03 | 4.4820D-01 |
|    | 6.0000D-01 YES |                |            |            |
| 68 | -447.067971151 | -447.067971151 | 2.6786D-03 | 4.6887D-01 |
|    | 4.0000D-01 YES |                |            |            |
| 69 | -441.190981179 | -441.190981179 | 9.0666D-03 | 4.3297D-01 |
|    | 6.0000D-01 YES |                |            |            |
| 70 | -452.406584642 | -452.406584642 | 1.5222D-02 | 4.2577D-01 |
|    | 1.7778D-01 YES |                |            |            |
| 71 | -448.804980190 | -448.804980190 | 2.2324D-02 | 4.6997D-01 |
|    | 2.6667D-01 YES |                |            |            |
| 72 | -431.798180374 | -431.798180374 | 7.6692D-02 | 6.0945D-01 |
|    | 6.0000D-01 YES |                |            |            |
| 73 | -455.101912192 | -455.101912192 | 2.1836D-02 | 6.1540D-01 |
|    | 2.3411D-02 YES |                |            |            |

|     |                |                |            |            |          |
|-----|----------------|----------------|------------|------------|----------|
| 74  | -453.899689856 | -453.899689856 | 1.7685D-02 | 2.3582D-01 |          |
|     | 5.2675D-02 YES |                |            |            |          |
| 75  | -448.312914967 | -448.312914967 | 1.4525D-02 | 3.6670D-01 |          |
|     | 1.1852D-01 YES |                |            |            |          |
| 76  | -455.236117297 | -455.236117297 | 1.1749D-01 | 2.7521D-01 |          |
|     | 7.9012D-02 YES |                |            |            |          |
| 77  | -449.169209766 | -449.169209766 | 5.0789D-03 | 3.3640D-01 |          |
|     | 1.1852D-01 YES |                |            |            |          |
| 78  | -448.387891593 | -448.387891593 | 5.9289D-03 | 5.5145D-01 |          |
|     | 1.1852D-01 YES |                |            |            |          |
| 79  | -451.560113800 | -451.560113800 | 3.5098D-03 | 4.7033D-01 |          |
|     | 7.9012D-02 YES |                |            |            |          |
| 80  | -448.738088207 | -448.738088207 | 3.2885D-02 | 6.0441D-01 |          |
|     | 2.6667D-01 YES |                |            |            |          |
| 81  | -453.181303442 | -453.181303442 | 8.6422D-03 | 5.4121D-01 |          |
|     | 1.1852D-01 YES |                |            |            |          |
| 82  | -456.112617133 | -456.112617133 | 1.5943D-03 | 2.3072D-01 |          |
|     | 5.2675D-02 YES |                |            |            |          |
| 83  | -455.638201172 | -455.638201172 | 2.1297D-03 | 3.4554D-01 |          |
|     | 5.2675D-02 YES |                |            |            |          |
| 84  | -450.339918600 | -450.339918600 | 7.7392D-03 | 6.6935D-01 |          |
|     | 1.7778D-01 YES |                |            |            |          |
| 85  | -456.923733136 | -456.923733136 | 1.0000D-01 | 6.9107D-01 |          |
|     | 4.6244D-03 YES |                |            |            |          |
| 86  | -456.694962016 | -456.694962016 | 2.1827D-02 | 3.0850D-01 |          |
|     | 6.0000D-01 YES |                |            |            |          |
| 87  | -454.729010130 | -454.729010130 | 2.0747D-02 | 3.5780D-01 |          |
|     | 6.0000D-01 YES |                |            |            |          |
| 88  | -453.438188031 | -453.438188031 | 1.0312D-01 | 3.2874D-01 |          |
|     | 6.0000D-01 YES |                |            |            |          |
| 89  | -456.011614567 | -456.011614567 | 9.9600D-02 | 3.9415D-01 |          |
|     | 5.2675D-02 YES |                |            |            |          |
| 90  | -449.448233250 | -449.448233250 | 3.1992D-02 | 7.9750D-01 |          |
|     | 6.0000D-01 YES |                |            |            |          |
| 91  | -455.344774594 | -455.344774594 | 1.1749D-01 | 7.2010D-01 |          |
|     | 4.0000D-01 YES |                |            |            |          |
| 92  | -454.762043639 | -454.762043639 | 9.6388D-02 | 2.2241D-01 |          |
|     | 1.3702D-03 YES |                |            |            |          |
| 93  | -431.712627116 | -431.712627116 | 1.5712D-01 | 2.1443D+00 |          |
|     | 6.0000D-01 YES |                |            |            |          |
| 94  | -455.513311722 | -455.513311722 | 6.1955D-02 | 2.1559D+00 |          |
|     | 2.3411D-02 YES |                |            |            |          |
| 95  | -455.750871685 | -455.750871685 | 2.0241D-02 | 1.7350D-01 |          |
|     | 3.5117D-02 YES |                |            |            |          |
| 96  | -449.427045980 | -449.427045980 | 7.8276D-02 | 1.0687D+00 |          |
|     | 7.9012D-02 YES |                |            |            |          |
| 97  | -457.517141473 | -457.517141473 | 2.1674D-02 | 9.5998D-01 |          |
|     | 5.2675D-02 YES |                |            |            |          |
| 98  | -453.569703593 | -453.569703593 | 2.3082D-02 | 4.7470D-01 |          |
|     | 1.1852D-01 YES |                |            |            |          |
| 99  | -453.664651395 | -453.664651395 | 2.4803D-02 | 7.9378D-01 |          |
|     | 1.7778D-01 YES |                |            |            |          |
| 100 | -456.526812179 | -456.526812179 | 4.2875D-03 | 9.2407D-01 |          |
|     | 3.5117D-02 YES |                |            |            |          |
| 101 | -438.884921053 | -438.884921053 | 0.0000D+00 | 1.7528D+00 | NO MATCH |
|     | 4.0000D-01 YES |                |            |            |          |
| 102 | -453.656537816 | -453.656537816 | 4.1753D-01 | 1.8939D+00 |          |
|     | 3.5117D-02 YES |                |            |            |          |
| 103 | -456.547793192 | -456.547793192 | 8.6809D-03 | 7.0902D-01 |          |
|     | 1.1852D-01 YES |                |            |            |          |
| 104 | -458.113195660 | -458.113195660 | 2.0644D-03 | 2.6528D-01 |          |
|     | 3.5117D-02 YES |                |            |            |          |
| 105 | -456.035628537 | -456.035628537 | 2.9774D-03 | 4.5746D-01 |          |
|     | 5.2675D-02 YES |                |            |            |          |

|     |                |                |            |            |
|-----|----------------|----------------|------------|------------|
| 106 | -457.390485014 | -457.390485014 | 3.1543D-03 | 5.9934D-01 |
|     | 1.5607D-02 YES |                |            |            |
| 107 | -450.494088975 | -450.494088975 | 8.3199D-03 | 6.9104D-01 |
|     | 1.7778D-01 YES |                |            |            |
| 108 | -439.483169905 | -439.483169905 | 2.7253D-02 | 9.0450D-01 |
|     | 6.0000D-01 YES |                |            |            |
| 109 | -447.631577692 | -447.631577692 | 2.0383D-02 | 9.1803D-01 |
|     | 2.6667D-01 YES |                |            |            |
| 110 | -456.988950620 | -456.988950620 | 9.4336D-02 | 1.2380D+00 |
|     | 5.2675D-02 YES |                |            |            |
| 111 | -455.970837211 | -455.970837211 | 3.4148D-02 | 5.9403D-01 |
|     | 6.0000D-01 YES |                |            |            |
| 112 | -455.177960301 | -455.177960301 | 2.5303D-02 | 1.8506D-01 |
|     | 7.9012D-02 YES |                |            |            |
| 113 | -455.391677589 | -455.391677589 | 3.1686D-03 | 4.8206D-01 |
|     | 7.9012D-02 YES |                |            |            |
| 114 | -456.128288715 | -456.128288715 | 1.8729D-02 | 8.0846D-01 |
|     | 6.0000D-01 YES |                |            |            |
| 115 | -456.028505180 | -456.028505180 | 1.4637D-02 | 6.9377D-01 |
|     | 4.0000D-01 YES |                |            |            |
| 116 | -455.560838470 | -455.560838470 | 5.6778D-02 | 5.0960D-01 |
|     | 6.0000D-01 YES |                |            |            |
| 117 | -441.202514437 | -441.202514437 | 2.8229D-01 | 8.0787D-01 |
|     | 6.0000D-01 YES |                |            |            |
| 118 | -452.011202344 | -452.011202344 | 4.3893D-01 | 6.3418D-01 |
|     | 7.9012D-02 YES |                |            |            |
| 119 | -446.844288444 | -446.844288444 | 3.2707D-02 | 7.1145D-01 |
|     | 2.6667D-01 YES |                |            |            |
| 120 | -456.193020147 | -456.193020147 | 2.9450D-01 | 6.7176D-01 |
|     | 1.1852D-01 YES |                |            |            |
| 121 | -454.982137538 | -454.982137538 | 1.2830D-01 | 5.8592D-01 |
|     | 6.9366D-03 YES |                |            |            |
| 122 | -456.060060596 | -456.060060596 | 1.5909D-01 | 4.8046D-01 |
|     | 1.5607D-02 YES |                |            |            |
| 123 | -455.803818422 | -455.803818422 | 9.4987D-02 | 2.1697D-01 |
|     | 4.6244D-03 YES |                |            |            |
| 124 | -458.582057119 | -458.582057119 | 1.1612D-01 | 2.5448D-01 |
|     | 1.1852D-01 YES |                |            |            |
| 125 | -457.615073226 | -457.615073226 | 1.0170D-01 | 2.1476D-01 |
|     | 4.0000D-01 YES |                |            |            |
| 126 | -456.237165736 | -456.237165736 | 1.9626D-01 | 5.0461D-01 |
|     | 6.0000D-01 YES |                |            |            |
| 127 | -454.733095648 | -454.733095648 | 3.7172D-02 | 5.7665D-01 |
|     | 1.1852D-01 YES |                |            |            |
| 128 | -454.393732753 | -454.393732753 | 1.0321D-02 | 4.8821D-01 |
|     | 1.1852D-01 YES |                |            |            |
| 129 | -456.334497002 | -456.334497002 | 2.6595D-01 | 4.6979D-01 |
|     | 1.7778D-01 YES |                |            |            |
| 130 | -452.753890116 | -452.753890116 | 1.5173D-02 | 3.7527D-01 |
|     | 6.0000D-01 YES |                |            |            |
| 131 | -458.545643216 | -458.545643216 | 1.5767D-01 | 4.4122D-01 |
|     | 3.5117D-02 YES |                |            |            |
| 132 | -453.847443961 | -453.847443961 | 1.2508D-02 | 3.5267D-01 |
|     | 6.0000D-01 YES |                |            |            |
| 133 | -455.738598519 | -455.738598519 | 2.0058D-03 | 3.8019D-01 |
|     | 2.3411D-02 YES |                |            |            |
| 134 | -455.505378108 | -455.505378108 | 4.4980D-03 | 4.0308D-01 |
|     | 1.7778D-01 YES |                |            |            |
| 135 | -455.586577343 | -455.586577343 | 8.6627D-03 | 2.7458D-01 |
|     | 2.6667D-01 YES |                |            |            |
| 136 | -455.853594719 | -455.853594719 | 9.2160D-03 | 2.4010D-01 |
|     | 4.0000D-01 YES |                |            |            |
| 137 | -453.940010542 | -453.940010542 | 7.9523D-03 | 2.0088D-01 |
|     | 1.1852D-01 YES |                |            |            |

|     |                |                |            |            |
|-----|----------------|----------------|------------|------------|
| 138 | -457.687669885 | -457.687669885 | 1.0274D-01 | 2.1307D-01 |
|     | 7.9012D-02 YES |                |            |            |
| 139 | -457.118423226 | -457.118423226 | 1.0074D-02 | 1.5201D-01 |
|     | 6.0000D-01 YES |                |            |            |
| 140 | -457.172213836 | -457.172213836 | 8.1226D-02 | 1.3011D-01 |
|     | 6.0000D-01 YES |                |            |            |
| 141 | -455.583567341 | -455.583567341 | 4.8841D-02 | 3.5927D-01 |
|     | 4.0000D-01 YES |                |            |            |
| 142 | -455.975833331 | -455.975833331 | 1.7003D-01 | 4.8339D-01 |
|     | 4.0000D-01 YES |                |            |            |
| 143 | -453.121149627 | -453.121149627 | 6.8089D-02 | 8.5461D-01 |
|     | 1.7778D-01 YES |                |            |            |
| 144 | -453.372902920 | -453.372902920 | 1.9321D-02 | 8.2024D-01 |
|     | 4.0000D-01 YES |                |            |            |
| 145 | -454.561286980 | -454.561286980 | 1.7298D-01 | 5.2852D-01 |
|     | 1.1852D-01 YES |                |            |            |
| 146 | -453.794757371 | -453.794757371 | 1.1233D-02 | 4.5954D-01 |
|     | 2.6667D-01 YES |                |            |            |
| 147 | -447.353572473 | -447.353572473 | 7.1233D-03 | 1.5410D+00 |
|     | 4.0000D-01 YES |                |            |            |
| 148 | -446.308673831 | -446.308673831 | 1.2180D-02 | 1.2186D+00 |
|     | 6.0000D-01 YES |                |            |            |
| 149 | -451.900338927 | -451.900338927 | 2.1935D-01 | 5.8358D-01 |
|     | 1.7778D-01 YES |                |            |            |
| 150 | -450.901291059 | -450.901291059 | 1.2171D-03 | 1.0038D+00 |
|     | 1.1852D-01 YES |                |            |            |
| 151 | -457.905806337 | -457.905806337 | 6.4717D-04 | 8.2476D-01 |
|     | 5.2675D-02 YES |                |            |            |
| 152 | -442.083114884 | -442.083114884 | 3.5373D-03 | 1.0845D+00 |
|     | 4.0000D-01 YES |                |            |            |
| 153 | -458.048735208 | -458.048735208 | 2.7017D-01 | 1.1782D+00 |
|     | 2.3411D-02 YES |                |            |            |
| 154 | -457.391264532 | -457.391264532 | 1.0364D-01 | 3.4627D-01 |
|     | 5.2675D-02 YES |                |            |            |
| 155 | -457.460444985 | -457.460444985 | 9.6426D-02 | 2.2765D-01 |
|     | 7.9012D-02 YES |                |            |            |
| 156 | -456.761152731 | -456.761152731 | 3.5376D-03 | 1.7529D-01 |
|     | 4.0000D-01 YES |                |            |            |
| 157 | -457.841611865 | -457.841611865 | 5.4545D-03 | 1.4129D-01 |
|     | 6.0000D-01 YES |                |            |            |
| 158 | -457.113956243 | -457.113956243 | 5.4431D-03 | 2.5876D-01 |
|     | 6.0000D-01 YES |                |            |            |
| 159 | -456.071374353 | -456.071374353 | 4.5220D-03 | 3.0893D-01 |
|     | 2.6667D-01 YES |                |            |            |
| 160 | -457.456643367 | -457.456643367 | 4.7129D-03 | 3.3357D-01 |
|     | 1.7778D-01 YES |                |            |            |
| 161 | -456.724853201 | -456.724853201 | 6.7561D-03 | 4.0987D-01 |
|     | 6.0000D-01 YES |                |            |            |
| 162 | -457.639588798 | -457.639588798 | 1.0201D-03 | 2.4081D-01 |
|     | 3.5117D-02 YES |                |            |            |
| 163 | -457.467337082 | -457.467337082 | 8.9818D-03 | 2.2318D-01 |
|     | 6.0000D-01 YES |                |            |            |
| 164 | -458.527509174 | -458.527509174 | 6.0769D-03 | 2.0607D-01 |
|     | 2.6667D-01 YES |                |            |            |
| 165 | -457.902628945 | -457.902628945 | 4.8515D-03 | 3.0217D-01 |
|     | 6.0000D-01 YES |                |            |            |
| 166 | -455.887828830 | -455.887828830 | 7.3446D-02 | 2.9623D-01 |
|     | 6.0000D-01 YES |                |            |            |
| 167 | -460.265262456 | -460.265262456 | 8.4497D-02 | 2.3123D-01 |
|     | 2.6667D-01 YES |                |            |            |
| 168 | -459.663599144 | -459.663599144 | 5.4585D-02 | 1.0157D-01 |
|     | 6.0000D-01 YES |                |            |            |
| 169 | -458.883295440 | -458.883295440 | 7.5442D-02 | 1.1264D-01 |
|     | 6.0000D-01 YES |                |            |            |

|     |                |                |            |            |
|-----|----------------|----------------|------------|------------|
| 170 | -459.756666018 | -459.756666018 | 1.3061D-01 | 1.6793D-01 |
|     | 6.0000D-01 YES |                |            |            |
| 171 | -449.887182141 | -449.887182141 | 2.8125D-01 | 5.7231D-01 |
|     | 6.0000D-01 YES |                |            |            |
| 172 | -453.862703638 | -453.862703638 | 1.7487D-01 | 6.1297D-01 |
|     | 4.0000D-01 YES |                |            |            |
| 173 | -452.822856134 | -452.822856134 | 1.0897D-01 | 5.6887D-01 |
|     | 2.6667D-01 YES |                |            |            |
| 174 | -454.410414965 | -454.410414965 | 3.8329D-02 | 5.8423D-01 |
|     | 4.0000D-01 YES |                |            |            |
| 175 | -452.076775963 | -452.076775963 | 2.4586D-01 | 6.0878D-01 |
|     | 2.6667D-01 YES |                |            |            |
| 176 | -455.014357087 | -455.014357087 | 3.0426D-03 | 4.0386D-01 |
|     | 1.7778D-01 YES |                |            |            |
| 177 | -438.157465778 | -438.157465778 | 4.2669D-03 | 7.0351D-01 |
|     | 4.0000D-01 YES |                |            |            |
| 178 | -454.577415384 | -454.577415384 | 3.1389D-01 | 1.0121D+00 |
|     | 1.1852D-01 YES |                |            |            |
| 179 | -459.996999966 | -459.996999966 | 2.4606D-01 | 1.0423D+00 |
|     | 2.3411D-02 YES |                |            |            |
| 180 | -455.103725753 | -455.103725753 | 4.4992D-03 | 2.2080D-01 |
|     | 4.0000D-01 YES |                |            |            |
| 181 | -454.576065383 | -454.576065383 | 9.6354D-03 | 2.6841D-01 |
|     | 6.0000D-01 YES |                |            |            |
| 182 | -455.016049434 | -455.016049434 | 1.1865D-01 | 3.5135D-01 |
|     | 6.0000D-01 YES |                |            |            |
| 183 | -458.189079984 | -458.189079984 | 1.2007D-01 | 3.2480D-01 |
|     | 2.6667D-01 YES |                |            |            |
| 184 | -450.508559328 | -450.508559328 | 4.3101D-02 | 7.2652D-01 |
|     | 6.0000D-01 YES |                |            |            |
| 185 | -450.180101788 | -450.180101788 | 4.1646D-02 | 7.2691D-01 |
|     | 6.0000D-01 YES |                |            |            |
| 186 | -450.062407129 | -450.062407129 | 1.4628D-01 | 5.5950D-01 |
|     | 6.0000D-01 YES |                |            |            |
| 187 | -444.609479598 | -444.609479598 | 4.6065D-02 | 7.4726D-01 |
|     | 6.0000D-01 YES |                |            |            |
| 188 | -454.672721924 | -454.672721924 | 4.4608D-02 | 4.0973D-01 |
|     | 4.0000D-01 YES |                |            |            |
| 189 | -447.087937427 | -447.087937427 | 5.6410D-02 | 3.4835D-01 |
|     | 6.0000D-01 YES |                |            |            |
| 190 | -451.483647306 | -451.483647306 | 3.4277D-03 | 6.6457D-01 |
|     | 6.0000D-01 YES |                |            |            |
| 191 | -445.479046315 | -445.479046315 | 3.6888D-02 | 7.1563D-01 |
|     | 6.0000D-01 YES |                |            |            |
| 192 | -453.415938716 | -453.415938716 | 6.3666D-02 | 5.2770D-01 |
|     | 6.0000D-01 YES |                |            |            |
| 193 | -445.774095890 | -445.774095890 | 5.0452D-02 | 5.0065D-01 |
|     | 6.0000D-01 YES |                |            |            |
| 194 | -453.427510716 | -453.427510716 | 3.9955D-02 | 7.6799D-01 |
|     | 6.0000D-01 YES |                |            |            |
| 195 | -446.406717481 | -446.406717481 | 4.5605D-02 | 7.4540D-01 |
|     | 6.0000D-01 YES |                |            |            |
| 196 | -451.941588175 | -451.941588175 | 6.0722D-02 | 4.1957D-01 |
|     | 6.0000D-01 YES |                |            |            |
| 197 | -447.220859642 | -447.220859642 | 9.9924D-03 | 3.5630D-01 |
|     | 6.0000D-01 YES |                |            |            |
| 198 | -451.749806560 | -451.749806560 | 1.9007D-01 | 3.3944D-01 |
|     | 6.0000D-01 YES |                |            |            |
| 199 | -450.394486712 | -450.394486712 | 1.0113D-01 | 3.8526D-01 |
|     | 2.6667D-01 YES |                |            |            |
| 200 | -451.641583153 | -451.641583153 | 6.7024D-02 | 3.4714D-01 |
|     | 4.0000D-01 YES |                |            |            |
| 201 | -457.009545797 | -457.009545797 | 1.4699D-01 | 3.7683D-01 |
|     | 1.1852D-01 YES |                |            |            |

|     |                |                |            |            |         |
|-----|----------------|----------------|------------|------------|---------|
| 202 | -457.383351980 | -457.383351980 | 9.7237D-02 | 1.4901D-01 |         |
|     | 6.0000D-01 YES |                |            |            |         |
| 203 | -458.224109275 | -458.224109275 | 1.1273D-01 | 1.5200D-01 |         |
|     | 6.0000D-01 YES |                |            |            |         |
| 204 | -456.174700871 | -456.174700871 | 3.5312D-02 | 2.0588D-01 |         |
|     | 2.6667D-01 YES |                |            |            |         |
| 205 | -455.699523581 | -455.699523581 | 2.9008D-02 | 2.0776D-01 |         |
|     | 4.0000D-01 YES |                |            |            |         |
| 206 | -456.965784185 | -456.965784185 | 1.0039D-01 | 1.5532D-01 |         |
|     | 2.6667D-01 YES |                |            |            |         |
| 207 | -458.604008375 | -458.604008375 | 9.4526D-02 | 1.1695D-01 |         |
|     | 6.0000D-01 YES |                |            |            |         |
| 208 | -458.340131743 | -458.340131743 | 9.6752D-02 | 1.1716D-01 |         |
|     | 6.0000D-01 YES |                |            |            |         |
| 209 | -458.249458253 | -458.249458253 | 1.0680D-01 | 1.2698D-01 |         |
|     | 6.0000D-01 YES |                |            |            |         |
| 210 | -458.689944251 | -458.689944251 | 1.2053D-01 | 1.4777D-01 |         |
|     | 6.0000D-01 YES |                |            |            |         |
| 211 | -458.037168822 | -458.037168822 | 1.3802D-01 | 1.6271D-01 |         |
|     | 6.0000D-01 YES |                |            |            |         |
| 212 | -458.720931977 | -458.720931977 | 1.6471D-01 | 2.1054D-01 | 3-POINT |
|     | 6.0000D-01 YES |                |            |            |         |
| 213 | -458.680166912 | -458.680166912 |            | 8.6318D-03 |         |
|     | 6.0000D-01 YES |                |            |            |         |
| 214 | -458.674371048 | -458.674371048 | 1.0934D-02 | 1.5283D-02 |         |
|     | 6.0000D-01 YES |                |            |            |         |
| 215 | -458.823355541 | -458.823355541 | 1.3365D-02 | 1.7375D-02 |         |
|     | 6.0000D-01 YES |                |            |            |         |
| 216 | -458.942775578 | -458.942775578 | 2.0542D-02 | 2.7008D-02 |         |
|     | 6.0000D-01 YES |                |            |            |         |
| 217 | -459.146685169 | -459.146685169 | 2.4869D-02 | 3.1265D-02 |         |
|     | 6.0000D-01 YES |                |            |            |         |
| 218 | -459.303688800 | -459.303688800 | 3.2133D-02 | 3.9672D-02 |         |
|     | 6.0000D-01 YES |                |            |            |         |
| 219 | -459.431924182 | -459.431924182 | 3.7558D-02 | 4.5656D-02 |         |
|     | 6.0000D-01 YES |                |            |            |         |
| 220 | -459.419594052 | -459.419594052 | 4.2024D-02 | 4.8926D-02 |         |
|     | 6.0000D-01 YES |                |            |            |         |
| 221 | -459.496578143 | -459.496578143 | 4.6352D-02 | 5.4352D-02 |         |
|     | 6.0000D-01 YES |                |            |            |         |
| 222 | -459.359028266 | -459.359028266 | 4.7314D-02 | 5.3017D-02 |         |
|     | 6.0000D-01 YES |                |            |            |         |
| 223 | -459.524106000 | -459.524106000 | 4.8962D-02 | 5.5752D-02 |         |
|     | 6.0000D-01 YES |                |            |            |         |
| 224 | -459.307006991 | -459.307006991 | 4.6665D-02 | 5.0995D-02 |         |
|     | 6.0000D-01 YES |                |            |            |         |
| 225 | -459.563164716 | -459.563164716 | 4.5481D-02 | 5.0688D-02 |         |
|     | 6.0000D-01 YES |                |            |            |         |
| 226 | -459.297365026 | -459.297365026 | 4.0815D-02 | 4.3842D-02 | 4-POINT |
|     | 6.0000D-01 YES |                |            |            |         |
| 227 | -459.706147025 | -459.706147025 |            | 6.9408D-03 |         |
|     | 6.0000D-01 YES |                |            |            |         |
| 228 | -459.391281466 | -459.391281466 | 4.7995D-03 | 5.2560D-03 |         |
|     | 6.0000D-01 YES |                |            |            |         |
| 229 | -459.531637871 | -459.531637871 | 3.8063D-03 | 4.0181D-03 |         |
|     | 6.0000D-01 YES |                |            |            |         |
| 230 | -459.418970990 | -459.418970990 | 3.1986D-03 | 3.4004D-03 |         |
|     | 6.0000D-01 YES |                |            |            |         |
| 231 | -459.501625567 | -459.501625567 | 2.6673D-03 | 2.8064D-03 |         |
|     | 6.0000D-01 YES |                |            |            |         |
| 232 | -459.436218733 | -459.436218733 | 2.2513D-03 | 2.3777D-03 | 4-POINT |
|     | 6.0000D-01 YES |                |            |            |         |
| 233 | -459.466309910 | -459.466309910 |            | 6.9392D-04 |         |
|     | 6.0000D-01 YES |                |            |            |         |

```

234      -459.470459626      -459.470459626      4.3247D-04  4.4145D-04
        6.0000D-01      YES
235      -459.471109554      -459.471109554      3.9884D-04  4.0840D-04
        6.0000D-01      YES
236      -459.467996095      -459.467996095      2.7762D-04  2.8240D-04
        6.0000D-01      YES
237      -459.472715890      -459.472715890      2.6793D-04  2.7520D-04
        6.0000D-01      YES
238      -459.467373199      -459.467373199      1.9970D-04  2.0415D-04
        6.0000D-01      YES
239      -459.473045374      -459.473045374      1.9565D-04  2.0199D-04  4-POINT
        6.0000D-01      YES
240      -459.468461663      -459.468461663              5.8131D-05
        6.0000D-01      YES
241      -459.471708491      -459.471708491      4.3680D-05  4.5798D-05
        6.0000D-01      YES
242      -459.469106097      -459.469106097      3.4395D-05  3.5837D-05
        6.0000D-01      YES
243      -459.471207868      -459.471207868      3.0939D-05  3.2744D-05
        6.0000D-01      YES
244      -459.469460388      -459.469460388      2.4190D-05  2.5321D-05
        6.0000D-01      YES
245      -459.470929708      -459.470929708      2.4075D-05  2.5708D-05
        6.0000D-01      YES
246      -459.469671268      -459.469671268      1.8459D-05  1.9412D-05
        6.0000D-01      YES
247      -459.470758951      -459.470758951      1.9605D-05  2.0853D-05
        6.0000D-01      YES
248      -459.469806459      -459.469806459      1.4660D-05  1.5463D-05  4-POINT
        6.0000D-01      YES
249      -459.470162194      -459.470162194              6.2938D-06
        6.0000D-01      YES
250      -459.470301566      -459.470301566      3.8669D-06  4.0741D-06
        6.0000D-01      YES
251      -459.470211928      -459.470211928      3.4584D-06  3.6241D-06
        6.0000D-01      YES
252      -459.470268047      -459.470268047      2.2615D-06  2.3675D-06
        6.0000D-01      YES
253      -459.470246996      -459.470246996              0.0000D+00
        6.0000D-01      YES
At termination total energy is      -459.470247      Hartrees
Orbital convergence is 4.61331D-06
Energy components:
Kinetic =      459.478881804
Potential =      -1094.308816416
Kinetic + Potential =      -634.829934612
Coulomb repulsion =      202.868115385
Exchange =      -27.311163122
Coulomb+Exchange =      175.556952264
Nuclear =      0.000000000
Total electronic =      -459.272982348
Total energy =      -459.272982348

Virial =      1.999551885

Dipole moment (Debye):
*****
X: -3.727308E-05      Y: -8.172788E-05      Z: -2.095933E-04      TOTAL: 0.000228

Radial grid used for numerical integration: SG1
Weights used for numerical integration:      BECKE
The Bragg-Slater radius of H has been set to 0.35
*****
Radial by atom and MO: <r> (bohr)

```



```

-----
      Atom,MO:      1      2      3      4      5      6      7
      1      0.18261  0.88353  0.81147  0.81147  0.81147  3.11692  3.07513
      8      3.07513  3.07515
Total <r>:      15.84289
-----

Radial by atom and MO: <r^2> (bohr^2)
-----
      Atom,MO:      1      2      3      4      5      6      7
      1      0.02240  0.46262  0.40870  0.40870  0.40870  5.65437  6.80701
      8      6.80703  6.80708
Total <r^2>:      27.78660
-----

Total number of grid points:      3760

PROGRAM> end of inputs

Program terminated normally

Job: Cl_Kh_ROHF_6311ppGdp ended on :24-Aug-18 at 13:05:06
User: ibrahim
Cpu      time:  00h00m01s00c on ibrahim-Lenovo-IdeaPad-P500
Elapsed time:  00h00m01s00c

```

## B.2 Code Source

Here you can find the source code of AIMD,

```

      SUBROUTINE GET_AIMDFT_object (class, Objname, Modality)
!*****
!      Date last modified: July 22, 2015
!      Author: Ibrahim Awad I. Awad
!      Description: Objects belonging to the class AIMDFT
!*****
! MODULEs:
!      USE program_files
!      USE program_objects
!
!      implicit none
!
! Input scalar:
!      character*(*) :: class
!      character*(*) :: Objname
!      character*(*) :: Modality
!
! Local scalars:
!      integer :: Object_number
!
! Begin:

```



```

return
end SUBROUTINE GET_AIMDFT_object

SUBROUTINE BLD_AIMDFT_objects
!*****
!      Date last modified July 22, 2015
!      Author: Ibrahim Awad
!      Description:
!*****
! MODULES:
USE program_objects

implicit none

! Local scalar:
integer :: Iobject

!
! Begin:
OBJ_AIMDFT(1:Max_objects)%modality = 'other'
OBJ_AIMDFT(1:Max_objects)%class = 'AIMDFT'
OBJ_AIMDFT(1:Max_objects)%depend = .true.
NAIMDFTobjects = 0

!
! Class of (AIMDFT)
NAIMDFTobjects = NAIMDFTobjects + 1
OBJ_AIMDFT(NAIMDFTobjects)%name = 'FRAGCART'
OBJ_AIMDFT(NAIMDFTobjects)%modality = 'DBASE'
OBJ_AIMDFT(NAIMDFTobjects)%routine = 'Add_To_DB'

NAIMDFTobjects = NAIMDFTobjects + 1
OBJ_AIMDFT(NAIMDFTobjects)%name = 'FRAGCART'
OBJ_AIMDFT(NAIMDFTobjects)%modality = 'GRIDS'
OBJ_AIMDFT(NAIMDFTobjects)%routine = 'Store_result_direct'

NAIMDFTobjects = NAIMDFTobjects + 1
OBJ_AIMDFT(NAIMDFTobjects)%name = 'MOLECULE'
OBJ_AIMDFT(NAIMDFTobjects)%modality = 'BUILD'
OBJ_AIMDFT(NAIMDFTobjects)%routine = 'CalcMoleProperties'

NAIMDFTobjects = NAIMDFTobjects + 1
OBJ_AIMDFT(NAIMDFTobjects)%name = 'MOLECULE'
OBJ_AIMDFT(NAIMDFTobjects)%modality = 'ROTATE'
OBJ_AIMDFT(NAIMDFTobjects)%routine = 'get_databaseform'

NAIMDFTobjects = NAIMDFTobjects + 1
OBJ_AIMDFT(NAIMDFTobjects)%name = 'FRAGMENT'
OBJ_AIMDFT(NAIMDFTobjects)%modality = 'DISPLAY'
OBJ_AIMDFT(NAIMDFTobjects)%routine = 'FragmentsDisplay'

NAIMDFTobjects = NAIMDFTobjects + 1
OBJ_AIMDFT(NAIMDFTobjects)%name = 'FRAGMENT'
OBJ_AIMDFT(NAIMDFTobjects)%modality = 'BUILD'
OBJ_AIMDFT(NAIMDFTobjects)%routine = 'Build_FragFiles'

!
! NAIMDFTobjects = NAIMDFTobjects + 1
! OBJ_AIMDFT(NAIMDFTobjects)%name = 'FRAGCART'
! OBJ_AIMDFT(NAIMDFTobjects)%modality = 'MESH'
! OBJ_AIMDFT(NAIMDFTobjects)%routine = 'DENFRAG'

! Dummy Class
NAIMDFTobjects = NAIMDFTobjects + 1
OBJ_AIMDFT(NAIMDFTobjects)%name = '?'
OBJ_AIMDFT(NAIMDFTobjects)%modality = '?'
OBJ_AIMDFT(NAIMDFTobjects)%routine = '?'

```

```

!
do Iobject=1,NAIMDFTobjects
  OBJ_AIMDFT(Iobject)%exist=.false.
  OBJ_AIMDFT(Iobject)%Current=.false.
end do

return
end SUBROUTINE BLD_AIMDFT_objects

SUBROUTINE MENU_AIMDFT
!*****
! Date last modified: *
! Author: Ibrahim Awad *
! Description: *
!*****
! MODULES:
  USE program_parser
  USE program_constants
  USE MENU_gets
  USE AIMDFT_type

  implicit none

!
! Local scalars:
  integer :: length
  logical done
  ChangeTerminalAtoms=.false.
!
! Begin:
  call PRG_manager ('enter', 'MENU_AIMDFT', 'UTILITY')
! Defaults:
  done=.false.
! Menu:
  do while (.not.done)
    call new_token (' AIMDFT:')
!
    if(token(1:4).eq.'HELP')then
      write(UNIout,'(a)') &
      ' Command AIMDFT:', &
      ' Purpose: Request AIMDFT codes', &
      ' Syntax :', &
      ' Level_Number = <integer>, Override the default', &
      ' TErминаl = <integer>, Override the default', &
      ' end'
!
    ROTlist
      else if(token(1:3).eq.'ROT')then
        call GET_value(ROTlist, NROTlist)

! Level_Number
      else if(token(1:1).EQ.'L')then
        call GET_value (Level_Number)

! Terminal Atom
      else if(token(1:1).EQ.'T')then
        call GET_value (TerminalAtom)

! INDeXNUM (needed to RUN the input files for the fragments)
      else if(token(1:1).EQ.'I')then
        call GET_value (IndexNumber)

!DIrectMethod
      else if(token(1:1).EQ.'D')then
        DirectMethod=.true.

```

```

!Use the Becke weight from the Database
!   else if(token(1:5).EQ.'NODBW')then
!       DatabaseBWeight=.False.

! ChangeTerm
!   else if(token(1:2).EQ.'CH')then
!       ChangeTerminalAtoms=.true.

! OPT
!   else if(token(1:1).EQ.'O')then
!       OPT_status="OPT"

!   else
!       call MENU_end (done)
!   end if

! end do !(.not.done)

!
! end of routine MENU_AIMDFT
call PRG_manager ('exit', 'MENU_AIMDFT', 'UTILITY')
return
end

```

```

MODULE AIMDFT_type
!*****
! Date last modified: *
! Author: Ibrahim Awad *
! Description: *
!*****
!

USE program_constants

implicit none

! Parameters for fragment atom
integer :: Level_Number
integer, parameter :: SymMax = 64
integer :: TerminalAtom ! Add H as terminal
logical :: ChangeTerminalAtoms
logical :: DirectMethod
logical :: DatabaseBWeight
integer :: IndexNumber
integer :: MaxGridPnt
logical :: RotUpdate

integer, parameter :: MAX_ROTlist=4 ! Maximum number of ROT in ROTlist
integer :: NROTlist
integer, dimension(:) :: ROTlist(Max_ROTlist)

! the output unit (Angstrom)
double precision, parameter :: CartPrnFactor = Bohr_to_Angstrom

! Database types
character (128) :: DirLoc ! Database Location
character (128) :: DBFileExt ! Database file type (Default txt)
character (128) :: CsvSep !
character (128) :: IndexFileName !
character (128) :: CartFileName
character (128) :: PropFileName

character (128) :: homedir

```

```

character (len=:), allocatable :: IndexFilePath
character (len=:), allocatable :: CartFilePath
character (len=:), allocatable :: PropFilePath
character (len=:), allocatable :: GridPath

character (128) :: fmt_index
character (128) :: fmt_cart
character (128) :: fmt_prop
character (128) :: fmt_index_title
character (128) :: fmt_cart_title
character (128) :: fmt_prop_title

character(len=8) :: OPT_status

character(len=8) :: fmt_xyz
character(len=8) :: fmt_w
character(len=8) :: fmt_GridPts
character(len=8) :: fmt_rho
character(len=32) :: Pfmt_rho
character(len=32) :: Pfmt_rho_w
character(len=256) :: Pfmt_rho_w_v
character(len=64) :: Pfmt_rho_dd

type :: CartesianCoordinate
double precision :: X
double precision :: Y
double precision :: Z
end type CartesianCoordinate

type :: GridsDataBase
double precision :: X
double precision :: Y
double precision :: Z
double precision :: W ! Total Angular Weight
double precision :: BW ! Becke Weight
double precision :: D ! Electron Density
double precision :: Q ! W*BW*D
double precision :: VpotA ! I1E_V12dr2 (analytical term)
double precision :: VpotN ! I1E_V12dr2 (numerical term)
end type GridsDataBase

type :: FragAtomInfo
integer :: Atomic_Number
integer :: MUNIdx ! the index of atom in MUNgauss
double precision :: X
double precision :: Y
double precision :: Z
character(len=8) :: ELEMENT ! Atom Symbol (e.g H for Hydrogen)
integer :: type ! Atom type
integer :: number ! Total number of a given atom type
integer :: factor ! Valence factor
character(len=16) :: Ctype ! The atom type as a character string
integer :: level = 0
double precision :: bond_order
logical :: sorted = .false. ! Have UNiqE Type
end type FragAtomInfo

type :: FragInfo
! Number of fragment atoms
integer :: NAtoms
! Number of neighbour atoms
integer :: NumNeighbourAtoms
! (atoms number : level)
character (SymMax) , dimension(10000,10) :: symbol

```

```

! Rotation matrix
double precision, dimension (3,3)          :: RMat
! Translation matrix
double precision, dimension (3)            :: TMat
! connectivity matrix
integer, dimension(100,100)                :: Connect
end type FragsInfo

! Two dimension array (integer Values)
type :: TwoDiArrInt
integer, dimension(:,,:), allocatable      :: ijarrray
end type TwoDiArrInt

type :: TypeFragMat
double precision, dimension (3,3)          :: RMat
double precision, dimension (3)            :: TMat
double precision, dimension (3)            :: TMatCorr
end type TypeFragMat

type :: IndexFile
integer                                     :: IndexNum          ! column # 1
character(len=SymMax) :: Symbol              ! column # 2
integer                                     :: NAToms            ! column # 3
integer                                     :: GridsNum           ! column # 4 Grids number
character (len=32)    :: GridType             ! column # 5 Grid type
character (len=32)    :: GridTypeExtra        ! column # 6
character (len=32)    :: Method               ! column # 7 RHF, ....
character (len=32)    :: BasisSet             ! column # 8 Basis set
character (len=16)    :: DEN_Partitioning     ! column # 9 Becke, ...
character (len=16)    :: CoreSize             ! column # 10 Average,...
character (len=8)     :: OPT                  ! column # 11 NO_OPT, OPT
logical               :: Availability          ! Availability
end type IndexFile

type :: PropFile
integer                                     :: IndexNum          ! column # 1
double precision       :: ElectronNum        ! column # 2
double precision       :: K                   ! column # 3
double precision       :: KHF                 ! column # 4
double precision       :: T                   ! column # 5
double precision       :: Vne_A               ! column # 6
double precision       :: Vne_N               ! column # 7
double precision       :: Vee_AN              ! column # 8
double precision       :: J                   ! column # 9
double precision       :: JHF                 ! column # 10
double precision       :: Jaa                 ! column # 11
double precision       :: VeeA                ! column # 12
logical               :: Availability          ! Availability
end type PropFile

end MODULE AIMDFT_type

```

```

SUBROUTINE SET_defaults_AIMDFT
!*****
!   Date last modified:                                     *
!   Author: Ibrahim Awad                                    *
!   Description:                                             *
!*****
!
USE AIMDFT_type

implicit none

!   Parameters for fragment atom

```





```

SUBROUTINE GetMoleculeAtomsIndex(MolIdxInfo,&
                                SearchSym,&
                                SearchSymNum,&
                                IndexFoundNum)
!*****
!   Date last modified:
!   Author: Ibrahim Awad
!   Description: Return the required information from index file
!               for the given unique symbols, where different
!               conditions were applied.
!*****
!
USE module_grid_points
USE NI_defaults
USE type_Weights
USE QM_defaults

implicit none

integer, intent(out) :: IndexFoundNum
integer :: iSymbol
integer :: File_Unit
integer :: Reason
logical :: Lerror
character (SymMax):: CurrentSearchSymbol
character (20) :: Radnote
type(IndexFile) :: CurrentIndexInfo

integer, intent(in) :: SearchSymNum
character (SymMax), dimension (:), allocatable, intent(in) :: SearchSym
type (IndexFile), dimension (:), allocatable, intent(out) :: MolIdxInfo

allocate (MolIdxInfo(Natoms))

call GET_object ('GRID', 'RADIAL', RADIAL_grid)
! check the database and return index
IndexFoundNum=0 ! count number of unique symbols in the database for mol.
MolIdxInfo%Availability=.false.
do iSymbol=1, SearchSymNum
    call GET_unit (IndexFilePath, File_Unit, Lerror)
    open(UNIT=File_Unit,file=IndexFilePath, status='old',form='formatted')
    CurrentSearchSymbol=SearchSym(iSymbol)
    read(File_Unit,*)
    do ! read the index file
        read(File_Unit,fmt_index,IOSTAT=Reason)
                                &
                                CurrentIndexInfo%IndexNum,      &
                                CurrentIndexInfo%Symbol,         &
                                CurrentIndexInfo%NAtons,         &
                                CurrentIndexInfo%GridsNum,        &
                                CurrentIndexInfo%GridType,        &
                                CurrentIndexInfo%GridTypeExtra,   &
                                CurrentIndexInfo%Method,          &
                                CurrentIndexInfo%BasisSet,        &
                                CurrentIndexInfo%DEN_Partitioning,&
                                CurrentIndexInfo%CoreSize,        &
                                CurrentIndexInfo%OPT
        if (Reason.lt.0) exit
        if (trim(ADJUSTL(CurrentIndexInfo%Symbol)).ne.&
            trim(ADJUSTL(CurrentSearchSymbol))) cycle ! first condition
        if (trim(ADJUSTL(CurrentIndexInfo%GridType)).ne.&
            trim(ADJUSTL(RADIAL_grid))) cycle ! second condition
        if (trim(RADIAL_grid).eq.'GILL') then
            Radnote=""
        end if
    end do
    if (IndexFoundNum==0) then
        IndexFoundNum=iSymbol
    end if
end do

```

```

        write(Radnote, '(I2,A1,I2,A1,I2,A1,I3,A1)') NRPoints_Gill, &
        & '(', NApoints_Gill(1), ',', &
        NApoints_Gill(2), ',', &
        NApoints_Gill(3), ')')

        ! second condition for GILL
        if (trim(ADJUSTL(Radnote)).ne.&
            trim(ADJUSTL(CurrentIndexInfo%GridTypeExtra))) cycle
    end if
    if (trim(ADJUSTL(CurrentIndexInfo%BasisSet)).ne.&
        trim(ADJUSTL(Basis_set_name))) cycle ! third condition
    if (trim(ADJUSTL(CurrentIndexInfo%DEN_Partitioning)).ne.&
        trim(ADJUSTL(DEN_Partitioning))) cycle ! forth condition
    if (trim(ADJUSTL(CurrentIndexInfo%OPT)).ne.&
        trim(ADJUSTL(OPT_status))) cycle ! fifth condition

    IndexFoundNum=IndexFoundNum+1
    MolIdxInfo(iSymbol) = CurrentIndexInfo
    MolIdxInfo(iSymbol)%Availability = .true.
end do ! end of read index file
close(UNIT=File_Unit)
end do ! end loop of condition

write (*, '(A,I0)') "Number of indexes available in the database   = ", &
    IndexFoundNum
write (*, '(A,I0)') "Number of indexes NOT available in the database = ", &
    SearchSymNum-IndexFoundNum

return
end SUBROUTINE GetMoleculeAtomsIndex
SUBROUTINE GetMaxIndex (MaxIndex)
!*****
!   Date last modified:
!   Author: Ibrahim Awad
!   Description: Return the maximum index number from index file
!*****
!
    implicit none

    type(IndexFile) :: CurrentIndexInfo
    integer :: File_unit
    integer :: Reason
    logical :: Lerror

    integer, intent(out) :: MaxIndex
    CurrentIndexInfo%IndexNum=0
    MaxIndex=0
    call GET_unit (trim(IndexFilePath), File_unit, Lerror)
    open(UNIT=File_unit, file=trim(IndexFilePath), &
        status='old', form='formatted')
    read(File_Unit,*)
    Do
        read(File_Unit,fmt_index,IOSTAT=Reason) &
            CurrentIndexInfo%IndexNum, &
            CurrentIndexInfo%Symbol, &
            CurrentIndexInfo%NAtoms, &
            CurrentIndexInfo%GridsNum, &
            CurrentIndexInfo%GridType, &
            CurrentIndexInfo%GridTypeExtra, &
            CurrentIndexInfo%Method, &
            CurrentIndexInfo%BasisSet, &
            CurrentIndexInfo%DEN_Partitioning, &
            CurrentIndexInfo%CoreSize, &
            CurrentIndexInfo%OPT
    if (CurrentIndexInfo%IndexNum.gt.MaxIndex) MaxIndex=&

```

```

CurrentIndexInfo%IndexNum

        if(Reason.lt.0) exit
    end do
    close(unit=File_unit)
    write(*,'(A,I0)') "MaxIndex  =  ", MaxIndex
    return
end SUBROUTINE GetMaxIndex
SUBROUTINE GetFragmentDBCart (MolIdxInfo,UniqueSymNum,FragmentCart)
!*****
!   Date last modified:
!   Author: Ibrahim Awad
!   Description: Return the cartesian coordinates of the fragments
!               from the database cartesian file.
!*****
!

    implicit none

    integer, intent(in) :: UniqueSymNum
    type(IndexFile),dimension(:),allocatable, intent(in) :: MolIdxInfo
    type(FragAtomInfo),dimension(:,:),allocatable, intent(out) :: FragmentCart

    integer :: iatom, latom, indexi, newindex
    integer :: File_unit, Reason
    character(len=8) :: Element
    double precision :: XCart, YCart, ZCart
    logical :: Lerror

    allocate (FragmentCart(Natoms,Natoms))

    ! Return the coordinate of fragments in the database for the wanted index
    ! loop over the wanted index
    do indexi=1, UniqueSymNum
        ! check if the symbol is available within the database
        if (.not.MolIdxInfo(indexi)%Availability) cycle
        ! loop over the atoms of the fragment
        do iatom=1 , MolIdxInfo(indexi)%NAtoms
            call GET_unit (CartFilePath, File_unit, Lerror)
            open(UNIT=File_unit,file=CartFilePath, &
                status='old',form='formatted')
            read(File_unit,*)
            do ! start, reading the file
                read(File_unit,fmt_cart,IOSTAT=Reason) newindex, latom, Element
                read(File_unit,fmt_cart,IOSTAT=Reason) XCart, YCart, ZCart
                if (Reason.lt.0) exit
                if(newindex.eq.MolIdxInfo(indexi)%IndexNum.and.latom.eq.iatom)then
                    FragmentCart(indexi,iatom)%element=Element
                    FragmentCart(indexi,iatom)%x=XCart
                    FragmentCart(indexi,iatom)%y=YCart
                    FragmentCart(indexi,iatom)%z=ZCart
                end if
            end do ! reading the file
            close(unit=File_unit)
        end do ! iatom
    end do ! indexi
end SUBROUTINE GetFragmentDBCart
SUBROUTINE GetFragmentsProperties (MolIdxInfo,UniqueSymNum,FragProp)
!*****
!   Date last modified:
!   Author: Ibrahim Awad
!   Description: Return the cartesian coordinates of the fragments
!               from the database cartesian file.
!*****
!

```

```

!

implicit none

integer, intent(in) :: UniqueSymNum
type (IndexFile), dimension(:), allocatable, intent(in) :: MolIdxInfo
type (PropFile), dimension(:), allocatable, intent(out) :: FragProp

integer :: indexi, newindex
integer :: File_unit, Reason
double precision :: EleNum, EV_Anal, EV_Num, EX, EXJ, ET
double precision :: EC, Kaa, Jaa, Vee_A, ECK, EC_AnaNum
logical :: Lerror

allocate (FragProp(UniqueSymNum))

! Return the coordinate of fragments in the database for the wanted index
! loop over the wanted index
FragProp%Availability=.false.
do indexi=1, UniqueSymNum
    ! check if the symbol is available within the database
    if (.not. MolIdxInfo(indexi)%Availability) cycle
    FragProp(indexi)%Availability=.true.
    ! loop over the atoms of the fragment
    call GET_unit (PropFilePath, File_unit, Lerror)
    open(UNIT=File_unit, file=PropFilePath, status='old', form='formatted')
    read(File_unit,*)
    do ! start, reading the file
        read(File_unit,fmt_prop,IOSTAT=Reason) &
            & newindex, &
            & EleNum, &
            & EX, &
            & EXJ, &
            & ET, &
            & EV_Anal, &
            & EV_Num, &
            & EC_AnaNum, &
            & EC, &
            & ECK, &
            & Jaa, &
            & vee_A

        if (Reason.lt.0) exit
        if (newindex.eq. MolIdxInfo(indexi)%IndexNum) then
            FragProp(indexi)%IndexNum=newindex
            FragProp(indexi)%ElectonNum=EleNum
            FragProp(indexi)%K=EX
            FragProp(indexi)%KHF=EXJ
            FragProp(indexi)%T=ET
            FragProp(indexi)%Vne_A=EV_Anal
            FragProp(indexi)%Vne_N=EV_Num
            FragProp(indexi)%Vee_AN=EC_AnaNum
            FragProp(indexi)%J=EC
            FragProp(indexi)%JHF=ECK
            FragProp(indexi)%Jaa=Jaa
            FragProp(indexi)%VeeA=Vee_A
            exit
        end if
    end do ! reading the file
    close(unit=File_unit)
end do ! indexi
end SUBROUTINE GetFragmentsProperties
end MODULE GetDataBaseInfo

```

```

SUBROUTINE Add_To_DB
! *****
! Date last modified: *
! Author: Ibrahim Awad *
! Description: *
! *****
! MODULES

USE type_density
USE QM_defaults
USE N_integration
USE NI_defaults
USE Sorted_AIMDFT
USE symbol_AIMDFT
USE type_plotting
USE GetMolecularProperties

implicit none

character (SymMax), dimension (:), allocatable :: symbols
type (FragInfo), dimension (:), allocatable :: FragInfo
type (FragAtomInfo), dimension (:,:), allocatable :: DatabaseFragAtoms
type (FragAtomInfo), dimension (:,:), allocatable :: SortedFragAtoms
integer, dimension (:), allocatable :: atomAdj
double precision, dimension (:), allocatable :: Vpot1
double precision, dimension (:), allocatable :: V12dr2
double precision :: TraceAB
double precision :: Xpt,Ypt,Zpt

integer :: Iatom, newindex, NApts_atom_New, Jatom
integer :: kfound
integer :: IApoint
integer :: Znum

allocate (FragInfo(Natoms))
allocate (SortedFragAtoms(Natoms,Natoms))
allocate (DatabaseFragAtoms(Natoms,Natoms))
allocate (symbols(Natoms))
allocate (atomAdj(Natoms))

CALL GET_object ('GRID', 'RADIAL', RADIAL_grid)
CALL GET_object ('QM', 'ENERGY_COMPONENTS', Wavefunction)
CALL GetSortCartTerm(Natoms,SortedFragAtoms,FragInfo)
CALL GetDBFormFragments(SortedFragAtoms,DatabaseFragAtoms,FragInfo)
CALL GetFragSymbols(Level_Number,symbols)

newindex=IndexNumber ! add the new symbols

Iatom=1 ! Just add the intrest atom within the fragment.
NAIMprint=1
AIMprint(1)=1

! write(*,*)
CALL GET_object ('QM', 'ENERGY_VEE', 'NUMERICAL')
! CALL GET_object ('QM', 'ENERGY_EXCHANGE', 'NUMERICAL')

! CALL GET_object ('QM', 'ENERGY_COULOMB', 'MO')
! write(*,*)
CALL GET_object ('QM', 'ENERGY_COULOMB', 'NUMERICAL')
write(*,*)
CALL GET_object ('QM', 'ENERGY_KINETIC', 'NUMERICAL')
write(*,*)
CALL GET_object ('QM', 'ENERGY_VNE', 'NUMERICAL')
write(*,*)

```



```

integer :: File_unit

! Save grid points
allocate (temp1(NApts_atom))
allocate (temp2(NApts_atom))

temp1%x=grid_points%x;temp1%y=grid_points%y;temp1%z=grid_points%z

CALL DoCartRotation(temp1,temp2,NApts_atom,&
                    FragInfo(Iatom)%RMat,FragInfo(Iatom)%TMat)
CALL INPUT_File_GRID (newindex, GridFileNames) ! just frag atom
CALL GET_unit (GridFileNames, File_unit, Lerror)

open(UNIT=File_unit,file=GridFileNames, status='REPLACE',form='formatted')
write(File_unit,'(A20,7A24)') "X","Y","Z","Rho",&
                             "BW","AW","Q=w.W.Rho","Vpol"

NApts_atom_New=0
do IApoint=1,NApts_atom
! if(rho_Atom(IApoint)*grid_points(IApoint)%w*Bweights(IApoint).lt.1.0D-09)&
! cycle
    NApts_atom_New=NApts_atom_New+1
    write(File_unit,Pfmt_rho_w_v) temp2(IApoint)%X, &
    & temp2(IApoint)%Y, &
    & temp2(IApoint)%Z, &
    & rho_Atom(IApoint), &
    & Bweights(IApoint), &
    & grid_points(IApoint)%w, &
    & rho_Atom(IApoint)*grid_points(IApoint)%w*Bweights(IApoint), &
    & Vpot1(IApoint)
end do ! Ipoint
close(UNIT=File_unit)
deallocate(temp1)
deallocate(temp2)
end SUBROUTINE Store_grid_points
SUBROUTINE add_to_index_file()
! *****e*****
! Date last modified: *
! Author: Ibrahim Awad *
! Description: *
! *****e*****
! MODULES

USE type_Weights

implicit none

character (28) :: Radnote, coreSize
logical :: Lerror
integer :: File_unit

Radnote=""
coreSize=""
if (trim(RADIAL_grid).eq.'GILL') then
    write(Radnote,'(I2,A1,I2,A1,I2,A1,I3,A1)') NRPoints_Gill,&
    & '( ',NApoints_Gill(1),', ',NApoints_Gill(2),', ',NApoints_Gill(3),') '
endif
if (trim(DEN_Partitioning).eq.'IAWAD') then
    write(coreSize,'(A)') Last_Core
endif

write(*,'(/A)') "*****"
write(*,*) "The fragment with symbol ", trim(Symbols(Iatom))
write(*,*) "has been added to the database with index # ", newindex
write(*,'(A/)') "*****"

```

```

CALL GET_unit (trim(IndexFilePath), File_unit, Lerror)
open(UNIT=File_unit,file=trim(IndexFilePath), &
     status='old',form='formatted',position="append")
write(File_unit,fmt_index) newindex, &
     & Symbols(Iatom), &
     & NAtoms, &
     & NApts_atom_New, &
     & trim(RADIAL_grid), &
     & trim(Radnote), &
     & trim(Wavefunction), &
     & trim(Basis_set_name), &
     & trim(DEN_Partitioning), &
     & trim(coreSize), &
     & trim(OPT_status)
close(unit=File_unit)

end SUBROUTINE add_to_index_file
SUBROUTINE add_to_prop_file(PropFilePath)
!*****e*****
! Date last modified: *
! Author: Ibrahim Awad *
! Description: *
!*****e*****
! MODULES

implicit none

character (20) :: Radnote
logical :: Lerror
integer :: File_unit
double precision :: EV_Anal, EV_Num, EX, ET, EC, EXJ, ECK, EC_AnalNum
double precision :: EleNum, vee_A
character (*) :: PropFilePath

write(*,'(a,I5)') " Atomic properties for atom #", Iatom
write(*,'(a)') " ++++++"
CALL CalAtomicEle(grid_points, NApts_atom, rho_Atom, Bweights, EleNum) !
write(*,'(a,F15.10)') "Number of Electrons, N = ", EleNum

EX = K_Atomic(Iatom)
write(*,'(a,F15.10)') "Pure Exchange, K ( 2K_ab) = ", EX

EXJ = KHF_Atomic(Iatom)
write(*,'(a,F15.10)') "HF Exchange, KHF ( 2K_ab+ Kaa) = ", EXJ

ET = Atomic_Kinetic(Iatom)
write(*,'(a,F15.10)') "Kinetic energy Numerical, T = ", ET

! Calculate the atomic potential energy for interest fragment atom.
CALL get_Vne_Atom_Analytical(EV_Anal)
write(*,'(a,F15.10)') "Potential Energy Analytical, Vne = ", EV_Anal

EV_Num = Atomic_Vne(Iatom)
write(*,'(a,F15.10)') "Potential Energy Numerical, Vne = ", EV_Num

EC_AnalNum = Atomic_Coulomb(Iatom)
write(*,'(a,F15.10)') "Coulomb Energy Anal/Num, Vee = ", EC_AnalNum

EC = J_Atomic(Iatom)
write(*,'(a,F15.10)') "Pure Coulomb, J ( 4J_ab+ Jaa) = ", EC

ECK = JHF_Atomic(Iatom)
write(*,'(a,F15.10)') "HF Coulomb, JHF ( 4J_ab+ 2Jaa) = ", ECK

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```

write(*,'(a,F15.10)') "Jaa = Kaa" = ", ECK-EC

CALL CalcVeeSelfA(grid_points, NApts_atom, rho_Atom, Bweights, vee_A)
write(*,'(a,F15.10)') "Coulomb Numerically Over A" = ", vee_A
write(*,*)

CALL GET_unit (trim(PropFilePath), File_unit, Lerror)
open(UNIT=File_unit,file=trim(PropFilePath), &
      status='old',form='formatted',position="append")
write(File_unit,fmt_prop) newindex, &
      & EleNum, &
      & EX, &
      & EXJ, &
      & ET, &
      & EV_Anal, &
      & EV_Num, &
      & EC_AnalNum, &
      & EC, &
      & ECK, &
      & ECK-EC, &
      & vee_A

close(unit=File_unit)

end SUBROUTINE add_to_prop_file
SUBROUTINE add_to_cart_file()
!*****e*****
! Date last modified: *
! Author: Ibrahim Awad *
! Description: *
!*****e*****
! MODULES

implicit none
integer :: katom
integer :: File_unit
logical :: Lerror

CALL GET_unit (trim(CartFilePath), File_unit, Lerror)
open(UNIT=File_unit,file=trim(CartFilePath), &
      status='old',form='formatted',position="append")

do katom=1,NAtons
write(File_unit,fmt_cart) newindex, &
      & katom, &
      & DatabaseFragAtoms(Iatom,katom)%element, &
      & DatabaseFragAtoms(Iatom,katom)%x, &
      & DatabaseFragAtoms(Iatom,katom)%y, &
      & DatabaseFragAtoms(Iatom,katom)%z

end do
close(unit=File_unit)

end SUBROUTINE add_to_cart_file
end SUBROUTINE Add_To_DB

SUBROUTINE CalcMolePropertiesDB(DatabaseInform, FragProp, FRgrids)
!*****e*****
! Date last modified: *
! Author: Ibrahim Awad *
! Description: *
!*****e*****
! MODULES:

```

```

USE module_grid_points
USE mod_build
USE GetDataBaseInfo

implicit none

! Grid points by Jatom after RR
type (GridsDataBase), dimension(:,,:), allocatable, INTENT(IN) :: FRgrids
type (IndexFile), dimension (:), allocatable, INTENT(IN) :: DatabaseInform
type (PropFile), dimension (:), allocatable, INTENT(IN) :: FragProp
type (type_grid_points), dimension(:,,:), allocatable :: grid_point_DB

double precision, dimension(:,,:), allocatable :: Bweights
double precision, dimension(:,,:), allocatable :: Pweights
double precision, dimension(:,,:), allocatable :: Vpot_r2

double precision, dimension (:), allocatable :: EleNum
double precision, dimension (:), allocatable :: pot_energy
double precision, dimension (:), allocatable :: colA_energy
double precision, dimension (:), allocatable :: colN_energy

double precision :: EleNum_tot
double precision :: pot_energy_tot
double precision :: colA_energy_tot
double precision :: colN_energy_tot
double precision :: EleNum_tot_prop
double precision :: pot_energy_A_tot_prop
double precision :: VeeAN_energy_tot_prop
double precision :: HFcoul_energy_tot_prop
double precision :: HFexchange_tot_prop
double precision :: coul_energy_tot_prop
double precision :: exchange_tot_prop
double precision :: kinetic_tot_prop
double precision :: Vnn_tot
double precision :: col_FullNum
double precision :: Virial

integer :: avlN, Jatom, points_num

CHARACTER (64) :: message

allocate (EleNum(Natoms))
allocate (pot_energy(Natoms))
allocate (colA_energy(Natoms))
allocate (colN_energy(Natoms))

call GET_object ('GRID', 'RADIAL', RADIAL_grid)

! compute the weight for all grid points
allocate (grid_point_DB(Natoms,MaxGridPnt))
allocate (Pweights(NAtoms,MaxGridPnt))
allocate (Bweights(NAtoms,MaxGridPnt))
allocate (Vpot_r2(NAtoms,MaxGridPnt))

grid_point_DB(1:Natoms,1:MaxGridPnt)%x=FRgrids(1:Natoms,1:MaxGridPnt)%x
grid_point_DB(1:Natoms,1:MaxGridPnt)%y=FRgrids(1:Natoms,1:MaxGridPnt)%y
grid_point_DB(1:Natoms,1:MaxGridPnt)%z=FRgrids(1:Natoms,1:MaxGridPnt)%z

do Jatom=1, Natoms
  if (.not.DatabaseInform(Jatom)%Availability) cycle
  points_num=DatabaseInform(Jatom)%GridsNum
  CALL GET_weights (grid_point_DB(Jatom,1:points_num), points_num, &
    Jatom, Bweights(Jatom,1:points_num))

```



```

end SUBROUTINE prt_tot_energy
SUBROUTINE calc_density_prop(cw)
! *****
! Date last modified: *
! Author: Ibrahim Awad *
! Description: Calculate Number of electrons *
! *****
! MODULES:
implicit none

CHARACTER (64), INTENT(IN) :: cw

call calc_elec()
call calc_Vne()
call calc_Vee_FullNum()
call calc_Vee_anal()

write(*, '(A)') '+-----+'
write(*, '(A)') '| Calculated molecule properties using the stored |'
write(*, '(A)') '| electron density in each radial grid point |'
write(*, '(A8,A48,A2)') '| NOTE: ', cw, '|'
write(*, '(A)') '+-----+'
write(*, '(A)') '| Atom# | Electrons | Vne | Vee |'
write(*, '(A)') '+-----+'

do Jatom=1, NAtoms
if (.not. FragProp(Jatom)%Availability) cycle
write(*, '(A1,I5,3(A1,F16.8),A1)') '|', Jatom, &
& '|', EleNum(Jatom), &
& '|', -pot_energy(Jatom), &
& '|', colN_energy(Jatom), &
& '|'

end do
write(*, '(A)') '+-----+'
write(*, '(A6,3(A1,F16.8),A1)') '| Sum= ', &
& '|', EleNum_tot, &
& '|', pot_energy_tot, &
& '|', colN_energy_tot, '| '

write(*, '(A)') '+-----+'

end SUBROUTINE calc_density_prop
SUBROUTINE calc_elec()
! *****
! Date last modified: *
! Author: Ibrahim Awad *
! Description: Calculate Number of electrons *
! *****
! MODULES:
implicit none

double precision :: point_charge
integer :: Ipoint, Jatom

EleNum=0.0d0
EleNum_tot=0.0d0
do Jatom=1, NAtoms
if (.not. DatabaseInform(Jatom)%Availability) cycle
do Ipoint=1, DatabaseInform(Jatom)%GridsNum
point_charge = FRgrids(Jatom,Ipoint)%D*&
FRgrids(Jatom,Ipoint)%W*&
PWeights(Jatom,Ipoint)
EleNum(Jatom) = EleNum(Jatom)+point_charge
end do

```

```

        EleNum(Jatom) = FourPi*EleNum(Jatom)
        EleNum_tot = EleNum_tot + EleNum(Jatom)
    end do
    return
end SUBROUTINE calc_elec
SUBROUTINE calc_Vne()
!*****
!   Date last modified:
!   Author: Ibrahim Awad
!   Description: Calculate V_ne, Potential
!*****
! MODULES:
    implicit none

    double precision :: point_charge, NWeight, temp
    integer :: Ipoint, Jatom, Iatom, JZ_num

    pot_energy_tot=ZERO
    pot_energy = ZERO
    do Iatom=1, Natoms
        JZ_num = CARTESIAN(Iatom)%Atomic_Number
        if (JZ_num .le. 0) cycle
        do Jatom = 1, Natoms
            if (.not.DatabaseInform(Jatom)%Availability) cycle
            do Ipoint = 1, DatabaseInform(Jatom)%GridsNum
                point_charge = FRgrids(Jatom,Ipoint)%D*&
                             FRgrids(Jatom,Ipoint)%W*&
                             PWeights(Jatom,Ipoint)
                temp = JZ_num*point_charge/ &
                     dsqrt((FRgrids(Jatom,Ipoint)%x-CARTESIAN(Iatom)%X)**2 + &
                           (FRgrids(Jatom,Ipoint)%y-CARTESIAN(Iatom)%Y)**2 + &
                           (FRgrids(Jatom,Ipoint)%z-CARTESIAN(Iatom)%Z)**2)
                pot_energy(Iatom) = pot_energy(Iatom) + temp
            end do ! Ipoint
        end do ! Jatom
        pot_energy(Iatom)= FourPi*pot_energy(Iatom)
        pot_energy_tot = pot_energy_tot - pot_energy(Iatom)
    end do
    return
end SUBROUTINE calc_Vne
SUBROUTINE calc_Vnn()
!*****
!   Date last modified:
!   Author: Ibrahim Awad
!   Description: Calculate V_nn, Potential
!*****
! MODULES:
    implicit none

    double precision :: temp
    integer :: Jatom, Iatom, JZ_num, IZ_num
    double precision :: pot_energy_point

    Vnn_tot=ZERO
    do Iatom=1, Natoms
        pot_energy_point = ZERO
        IZ_num = CARTESIAN(Iatom)%Atomic_Number
        if (IZ_num .le. 0) cycle
        do Jatom = Iatom+1, Natoms
            JZ_num = CARTESIAN(Jatom)%Atomic_Number
            if (JZ_num .le. 0) cycle
            temp = JZ_num*IZ_num/ &
                 dsqrt((CARTESIAN(Jatom)%x-CARTESIAN(Iatom)%X)**2 + &
                       (CARTESIAN(Jatom)%y-CARTESIAN(Iatom)%Y)**2 + &

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```

                (CARTESIAN(Jatom)%z-CARTESIAN(Iatom)%Z)**2)
        pot_energy_point = pot_energy_point + temp
    end do ! Jatom
    Vnn_tot = Vnn_tot + pot_energy_point
end do
write(*,'(A32,F16.8)') "Nuclear repulsion (Vnn)   =", Vnn_tot
return
end SUBROUTINE calc_Vnn
SUBROUTINE calc_Vee_anal()
! *****
!   Date last modified:
!   Author: Ibrahim Awad
!   Description:   Calculate V_ee (half analytical, from the fragments),
!                 Coulomb repulsion
! *****
! MODULES:
    implicit none

    double precision :: point_charge, NWeight
    integer :: Ipoint, Jatom, JZ_num

    colA_energy = ZERO
    colA_energy_tot=ZERO
    do Jatom=1, Natoms
        JZ_num = CARTESIAN(Jatom)%Atomic_Number
        if (JZ_num .le. 0) cycle
        do Ipoint = 1,DatabaseInform(Jatom)%GridsNum
            if (.not.DatabaseInform(Jatom)%Availability) cycle
                point_charge = FRgrids(Jatom,Ipoint)%VpotA*&
                             FRgrids(Jatom,Ipoint)%D*&
                             FRgrids(Jatom,Ipoint)%W*&
                             PWeights(Jatom,Ipoint)/TWO
                colA_energy(Jatom) = colA_energy(Jatom) + point_charge
            end do
            colA_energy(Jatom) = FourPi*colA_energy(Jatom)
            colA_energy_tot = colA_energy_tot + colA_energy(Jatom)
        end do
    end do
    return
end SUBROUTINE calc_Vee_anal
SUBROUTINE calc_Vee_FullNum()
! *****
!   Date last modified:
!   Author: Ibrahim Awad
!   Description:   Calculate V_ee (Full Numerical), Coulomb repulsion
! *****
! MODULES:
    implicit none

    double precision :: point_charge, NWeight
    integer :: Ipoint, Jatom, JZ_num
    double precision :: NWeightI, NWeightJ, temp, Pcharge
    integer :: Jpoint, Iatom

    colN_energy=ZERO
    colN_energy_tot=ZERO
    do Iatom=1, Natoms
        JZ_num = CARTESIAN(Iatom)%Atomic_Number
        if (JZ_num .le. 0) cycle
        do Ipoint = 1,DatabaseInform(Iatom)%GridsNum
            if (.not.DatabaseInform(Iatom)%Availability) cycle
                do Jatom=1, Natoms
                    JZ_num = CARTESIAN(Jatom)%Atomic_Number
                    if (JZ_num .le. 0) cycle
                    do Jpoint = 1,DatabaseInform(Jatom)%GridsNum

```

```

        if (.not.DatabaseInform(Jatom)%Availability) cycle
        if(Ipoint.eq.Jpoint .and. Iatom.eq.Jatom) cycle
        NWeightJ=PWeights(Jatom,Jpoint)
        NWeightI=PWeights(Iatom,Ipoint)
        Pcharge=FRgrids(Iatom,Ipoint)%D*FRgrids(Jatom,Jpoint)%D &
            & *FRgrids(Iatom,Ipoint)%W*FRgrids(Jatom,Jpoint)%W &
            & *NWeightJ*NWeightI
        temp = Pcharge/ &
            DSQRT((FRgrids(Jatom,Jpoint)%x-FRgrids(Iatom,Ipoint)%x)**2+ &
                (FRgrids(Jatom,Jpoint)%y-FRgrids(Iatom,Ipoint)%y)**2+ &
                (FRgrids(Jatom,Jpoint)%z-FRgrids(Iatom,Ipoint)%z)**2)
        colN_energy(Iatom)=colN_energy(Iatom)+temp
    end do ! Jpoint = Ipoint + 1,DatabaseInform(Jatom)%GridsNum
end do ! Jatom=1, Natoms
end do ! Ipoint = 1,DatabaseInform(Iatom)%GridsNum
colN_energy(Iatom)=0.5d0*FourPi*FourPi*colN_energy(Iatom)
colN_energy_tot=colN_energy_tot+colN_energy(Iatom)
end do ! Iatom=1, Natoms
return
end SUBROUTINE calc_Vee_FullNum
SUBROUTINE calc_Vpotr2_pt(Iatom,Ipoint,vpot_pt)
! *****
! Date last modified: *
! Author: Ibrahim Awad *
! Description: Calculate V_ee (Full Numerical), Coulomb repulsion *
! *****
! MODULES:
    implicit none

    integer :: Jatom,Jpoint,JZ_num,Iatom,Ipoint
    double precision :: NWeightJ, PchargeJ, densityXYZ,GET_density_at_xyz
    double precision :: vpot_pt, SQRTtr

    vpot_pt=ZERO
    do Jatom=1, Natoms
        JZ_num = CARTESIAN(Jatom)%Atomic_Number
        if (JZ_num .le. 0) cycle
        if (.not.DatabaseInform(Jatom)%Availability) cycle
        do Jpoint = 1,DatabaseInform(Jatom)%GridsNum
            if(Ipoint.eq.Jpoint .and. Iatom.eq.Jatom) cycle
            NWeightJ=PWeights(Jatom,Jpoint)
            PchargeJ=FRgrids(Jatom,Jpoint)%W*&
                NWeightJ*FRgrids(Jatom,Jpoint)%D !densityXYZ!
            SQRTtr=DSQRT((FRgrids(Jatom,Jpoint)%x-FRgrids(Iatom,Ipoint)%x)**2+&
                (FRgrids(Jatom,Jpoint)%y-FRgrids(Iatom,Ipoint)%y)**2+ &
                (FRgrids(Jatom,Jpoint)%z-FRgrids(Iatom,Ipoint)%z)**2)
            vpot_pt = vpot_pt + PchargeJ/SQRTtr
        end do ! Ipoint = 1,DatabaseInform(Iatom)%GridsNum
    end do ! Iatom=1, Natoms
    vpot_pt=vpot_pt*FourPi
    return
end SUBROUTINE calc_Vpotr2_pt
SUBROUTINE calc_Vee_FullNum2()
! *****
! Date last modified: *
! Author: Ibrahim Awad *
! Description: Calculate V_ee (Full Numerical), Coulomb repulsion *
! *****
! MODULES:
    implicit none

    double precision :: point_chargeI, NWeight
    integer :: Ipoint, Jatom, JZ_num
    double precision :: NWeightI, vpot_pt, temp

```

```

double precision :: PchargeI, densityXYZ, GET_density_at_xyz
integer :: Jpoint, Iatom

colN_energy=ZERO
colN_energy_tot=ZERO
do Iatom=1, Natoms
  JZ_num = CARTESIAN(Iatom)%Atomic_Number
  if (JZ_num .le. 0) cycle
  do Ipoint = 1,DatabaseInform(Iatom)%GridsNum
    if (.not.DatabaseInform(Iatom)%Availability) cycle
    NWeightI=PWeights(Iatom,Ipoint)
    PchargeI=FRgrids(Iatom,Ipoint)%W*NWeightI*FRgrids(Iatom,Ipoint)%D
    call calc_Vpotr2_pt(Iatom,Ipoint,vpot_pt)
    temp = PchargeI*vpot_pt!FRgrids(Iatom,Ipoint)%VpotA!
    colN_energy(Iatom)=colN_energy(Iatom)+temp
  end do ! Ipoint = 1,DatabaseInform(Iatom)%GridsNum
  colN_energy(Iatom)=0.50d0*FourPi*colN_energy(Iatom)
  colN_energy_tot=colN_energy_tot+colN_energy(Iatom)
end do ! Iatom=1, Natoms
return
end SUBROUTINE calc_Vee_FullNum2
SUBROUTINE calc_atomic_prop()
! *****
! Date last modified: *
! Author: Ibrahim Awad *
! Description: Calculate Number of electrons *
! *****
! MODULES:
  implicit none

  integer :: Jatom

  EleNum_tot_prop=ZERO
  pot_energy_A_tot_prop=ZERO
  pot_energy_N_tot_prop=ZERO
  VeeAN_energy_tot_prop=ZERO
  HFexchange_tot_prop=ZERO
  HFcoul_energy_tot_prop=ZERO
  exchange_tot_prop=ZERO
  coul_energy_tot_prop=ZERO
  kinetic_tot_prop=ZERO

  write(*, '(A)') '+-----&
  &-----+
  write(*, '(A)') '| Calculated molecule properties using the stored frag&
  &ments properties |'
  write(*, '(A)') '| (already calculated, using fragment weigths and geom&
  &etry) |'
  write(*, '(A)') '+-----+-----+-----+-----+-----+-----&
  &-----+-----+-----+-----+-----+-----+-----+
  write(*, '(A)') '|Atom#| Electrons | Potential Vne| Vee Ana/Num | HF &
  &Coulomb J |HF Exchange K | Coulomb J | Exchange K | Kinetic T |'
  write(*, '(A)') '+-----+-----+-----+-----+-----+-----&
  &-----+-----+-----+-----+-----+-----+
  do Jatom=1, Natoms
    if (.not.FragProp(Jatom)%Availability) cycle
    write(*, '(A1,I5,8(A1,F14.8),A1)') '|',Jatom, &
      &'|',FragProp(Jatom)%ElectronNum, &
      &'|',FragProp(Jatom)%Vne_A, &
      &'|',FragProp(Jatom)%Vne_N, & ! No need same as Vne_A
      &'|',FragProp(Jatom)%Vee_AN, &
      &'|',FragProp(Jatom)%JHF, &
      &'|',FragProp(Jatom)%KHF, &
      &'|',FragProp(Jatom)%J, &

```





```

TAIMDensity2=ZERO
TotError1=ZERO
TotError2=ZERO
AtomDensity=ZERO
AtomAIMDFTDensity1=ZERO
AtomAIMDFTDensity2=ZERO
ErrorDensity_w1=ZERO
ErrorDensity_w2=ZERO
write(*,'(A6,A12,6A14)') 'Atom#','AIM_FD','AIM_CW','HF','|Error FD|', &
& ' |Error CW|', '%Error FD|', '%Error CW|'

do Jatom=1, NAtoms
  if (.not.DatabaseInform(Jatom)%Availability) cycle
  do Ipoint=1, DatabaseInform(Jatom)%GridsNum

    xpoint=FRgrids(Jatom,Ipoint)%x-CARTESIAN(Jatom)%x
    ypoint=FRgrids(Jatom,Ipoint)%y-CARTESIAN(Jatom)%y
    zpoint=FRgrids(Jatom,Ipoint)%z-CARTESIAN(Jatom)%z

    densityXYZ=GET_density_at_xyz(FRgrids(Jatom,Ipoint)%x,&
                                  FRgrids(Jatom,Ipoint)%y,&
                                  FRgrids(Jatom,Ipoint)%z)
    density_w=densityXYZ*FRgrids(Jatom,Ipoint)%W*BWeights(Jatom,Ipoint)
    AIMDFTsity_w1=FRgrids(Jatom,Ipoint)%D*&
                  FRgrids(Jatom,Ipoint)%W*&
                  FRgrids(Jatom,Ipoint)%BW
    AIMDFTsity_w2=FRgrids(Jatom,Ipoint)%D*&
                  FRgrids(Jatom,Ipoint)%W*&
                  BWeights(Jatom,Ipoint)
    ABSerror1=AIMDFTsity_w1-density_w
    ABSerror2=AIMDFTsity_w2-density_w

    AtomDensity(Jatom)=AtomDensity(Jatom)+density_w
    AtomAIMDFTDensity1(Jatom)=AtomAIMDFTDensity1(Jatom)+AIMDFTsity_w1
    AtomAIMDFTDensity2(Jatom)=AtomAIMDFTDensity2(Jatom)+AIMDFTsity_w2
  end do

  AtomDensity(Jatom)=FourPi*AtomDensity(Jatom)
  AtomAIMDFTDensity1(Jatom)=FourPi*AtomAIMDFTDensity1(Jatom)
  AtomAIMDFTDensity2(Jatom)=FourPi*AtomAIMDFTDensity2(Jatom)

  TDensity=TDensity+AtomDensity(Jatom)
  TAIMDensity1=TAIMDensity1+AtomAIMDFTDensity1(Jatom)
  TAIMDensity2=TAIMDensity2+AtomAIMDFTDensity2(Jatom)

  ErrorDensity_w1(Jatom)=AtomAIMDFTDensity1(Jatom)-AtomDensity(Jatom)
  ErrorDensity_w2(Jatom)=AtomAIMDFTDensity2(Jatom)-AtomDensity(Jatom)

  TotError1=TotError1+ErrorDensity_w1(Jatom)
  TotError2=TotError2+ErrorDensity_w2(Jatom)

  if (AtomDensity(Jatom).ne.ZERO) then
    write(*,'(I4,7F14.8)') Jatom, &
& AtomAIMDFTDensity1(Jatom), &
& AtomAIMDFTDensity2(Jatom), &
& AtomDensity(Jatom), &
& ErrorDensity_w1(Jatom), &
& ErrorDensity_w2(Jatom), &
& ErrorDensity_w1(Jatom)/AtomDensity(Jatom)*100.0d0, &
& ErrorDensity_w2(Jatom)/AtomDensity(Jatom)*100.0d0
  end if
end do
if (TDensity.ne.ZERO) then
  write(*,'(A4,7F14.8)') 'Sum=', TAIMDensity1,TAIMDensity2, &
& TDensity,TotError1,TotError2, &

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& TotError1/TDensity*100.0d0, &
& TotError2/TDensity*100.0d0

end if
return
end SUBROUTINE calc_error_density
SUBROUTINE Stores_error_files()
!*****
!   Date last modified:
!   Author: Ibrahim Awad
!   Description: ! Just to check the denisty at each radial grid point....
!*****
! MODULES:
implicit none

double precision :: densityXYZ, GET_density_at_xyz
integer :: File_unit
integer :: Ipoint
integer :: Jatom
logical :: Lerror
character (60) :: PlotFileNames
double precision :: xpoint, ypoint, zpoint
double precision :: ABSerror1, ABSerror2
double precision :: density_w, AIMDFTsity_w1, AIMDFTsity_w2

do Jatom=1, NAtoms
if (.not.DatabaseInform(Jatom)%Availability) cycle
!<<<<<<<<<<IMPORTANT::::: should be change
call INPUT_File_FRAG ('GRIDError_', '.dat', Jatom, PlotFileNames)
call GET_unit (PlotFileNames, File_unit, Lerror)
open(UNIT=File_unit, file=PlotFileNames, &
status='REPLACE', form='formatted')
write(File_unit, '(3A12,A18,6A17)') 'X', 'Y', 'Z', 'Bweight', 'RadWeight', &
& 'AIM_Density_FW', 'AIM_Density_CW', &
& 'Density_w', 'Diff1', 'Diff2'

do Ipoint=1, DatabaseInform(Jatom)%GridsNum

xpoint=FRgrids(Jatom, Ipoint)%x-CARTESIAN(Jatom)%x
ypoint=FRgrids(Jatom, Ipoint)%y-CARTESIAN(Jatom)%y
zpoint=FRgrids(Jatom, Ipoint)%z-CARTESIAN(Jatom)%z

densityXYZ=GET_density_at_xyz (FRgrids(Jatom, Ipoint)%x, &
FRgrids(Jatom, Ipoint)%y, &
FRgrids(Jatom, Ipoint)%z)

density_w=densityXYZ*&
FRgrids(Jatom, Ipoint)%W*&
BWeights(Jatom, Ipoint)
AIMDFTsity_w1=FRgrids(Jatom, Ipoint)%D*&
FRgrids(Jatom, Ipoint)%W*&
FRgrids(Jatom, Ipoint)%BW
AIMDFTsity_w2=FRgrids(Jatom, Ipoint)%D*&
FRgrids(Jatom, Ipoint)%W*&
BWeights(Jatom, Ipoint)
ABSerror1=AIMDFTsity_w1-density_w
ABSerror2=AIMDFTsity_w2-density_w

write(File_unit, Pfmt_rho_dd) xpoint, &
& ypoint, &
& zpoint, &
& FRgrids(Jatom, Ipoint)%BW, &
& FRgrids(Jatom, Ipoint)%W, &
& AIMDFTsity_w1, &
& AIMDFTsity_w2, &
& density_w, &
& ABSerror1, &

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```

& ABSerror2

    end do
    close(UNIT=File_unit)
end do
return
end SUBROUTINE Stores_error_files
end SUBROUTINE CalcMolePropertiesDB

SUBROUTINE Build_Molecule_DB(DatabaseInform,&
                             FragProp,&
                             FRgrids,&
                             CorrFragAtom)
!*****e*****
!   Date last modified:
!   Author: Ibrahim Awad
!   Description:
!*****e*****
! MODULES:

USE symbol_AIMDFT
USE sorted_AIMDFT
USE matrix_print

implicit none

! Grid points by Jatom after RR
type(GridsDataBase),dimension (:,:),allocatable,intent(out)::FRgrids
type(IndexFile),dimension (:),allocatable,intent(out)::DatabaseInform
type(PropFile),dimension (:),allocatable,intent(out)::FragProp
! Adjusted the rotated database coordinate to fit the current fragments
! coordinates and the intreseted atom in the center.
type(FragAtomInfo),dimension (:,:),allocatable,intent(out)::CorrFragAtom
type(FragAtomInfo),dimension (:,:),allocatable :: &
& CurrFragAtoms,      &! current fragment coordinates of the molecule
& DBCurrFragAtoms,    &! database form of current fragment coordinates
& DBUniqueFragAtoms, &! The database coordinates for the unique fragments
& DBFragAtoms,        &! The database coordinates for all atoms fragments
& RotDBFragAtoms      &! Rotated database coordinate to fit the
                        &! current fragemnts coordinates.
type(FragAtomInfo),dimension (:),allocatable :: term1, term2, term3
type(TypeFragMat),dimension (:),allocatable :: FragAtomsMatrix
! Information for current fragments of molecule.
type(FragsInfo),dimension (:),allocatable :: CurrFragInfo
! Contain index file information.
type(IndexFile),dimension (:),allocatable :: MolIdxInfo
! The atomic properties for the fragments from the database
type(PropFile),dimension (:),allocatable :: DBFragProp

double precision,dimension (:,:),allocatable :: RR
double precision,dimension (:),allocatable :: TT
double precision,dimension (:),allocatable :: TTCORR

character (SymMax),dimension (:),allocatable :: symbols
character (SymMax),dimension (:),allocatable :: UniSym
integer,dimension (:),allocatable :: UniSymMUNIdx

integer :: UniSymMum
integer :: IndexFoundNum
integer :: ifound
integer :: indexi
integer :: Ipoint
integer :: File_unit
integer :: Reason
integer :: Jatom

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```

integer :: iatom
integer :: symI

character (SymMax):: SymbolINDX
character (SymMax):: searchsym
character (25) :: GridType
character(len=:),allocatable :: GridFileNames
character (60) :: PlotFileNames
character (60) :: PlotFileNames2

logical :: Lerror2
logical :: Lerror

double precision,dimension (1:3) :: NewPoint
double precision,dimension (1:3) :: point

double precision :: DEN, BeckW, AngW, Qval, Vpot
character(len=8) :: ele

if(.not. allocated(FRgrids)) allocate (FRgrids(Natoms,MaxGridPnt))
if(.not. allocated(DatabaseInform)) allocate (DatabaseInform(Natoms))
if(.not. allocated(CorrFragAtom)) allocate (CorrFragAtom(Natoms,Natoms))
if(.not. allocated(FragProp)) allocate (FragProp(Natoms))
allocate (FragAtomsMatrix(Natoms))
allocate (CurrFragAtoms(Natoms,Natoms))
allocate (DBCurrFragAtoms(Natoms,Natoms))
allocate (DBUniqueFragAtoms(Natoms,Natoms))
allocate (DBFragAtoms(Natoms,Natoms))
allocate (RotDBFragAtoms(Natoms,Natoms))
allocate (CurrFragInfo(Natoms))
allocate (MolIdxInfo(Natoms))
allocate (DBFragProp(Natoms))
allocate (RR(1:3,1:3))
allocate (TT(1:3))
allocate (TTCORR(1:3))
allocate (symbols(Natoms))
allocate (UniSym(Natoms))
allocate (UniSymMUNIdx(Natoms))

! Partitioning the molecule to its fragemnt and complete the valance
CALL GetSortCartTerm(Level_Number,CurrFragAtoms,CurrFragInfo)
! CALL GenerateFragments(Level_Number,CurrFragAtoms,CurrFragInfo)
! find the symbols of the fragments
CALL GetFragSymbols(Level_Number,symbols)
! find the unique symbols for the molecule and their number
CALL GetUniSym (Level_Number, UniSym, UniSymMUNIdx, UniSymMum)
! return the MolIdxInfo (all information from the index file)
! and the availability, .....
! from the database at specific conditions
CALL GetMoleculeAtomsIndex (MolIdxInfo,UniSym,UniSymMum,IndexFoundNum)
! Return the coordinate of fragments from
! the database for the wanted index
CALL GetFragmentDBCart (MolIdxInfo,UniSymMum,DBUniqueFragAtoms)
! Return the atomic properties of fragments
! from the database for the wanted index
CALL GetFragmentsProperties (MolIdxInfo,UniSymMum,DBFragProp)
! Project the database information to the molecule atoms
DBFragAtoms%x=0.d0
DBFragAtoms%y=0.d0
DBFragAtoms%z=0.d0
DBFragAtoms%element="NOT"
DatabaseInform(1:Natoms)%Availability=.false.

```

```

FragProp(1:NAtoms)%Availability=.false.
DatabaseInform(1:NAtoms)%NAtoms=CurrFragInfo(1:NAtoms)%NAtoms
do Jatom=1, NAtoms ! loop over all atoms
  do indexi=1, UniSymMum
    ! loop over the available database
    if (.not.MolIdxInfo(indexi)%Availability) cycle
    if (symbols(Jatom).ne.MolIdxInfo(indexi)%Symbol) cycle

    ! first: project the index information
    DatabaseInform(Jatom)=MolIdxInfo(indexi)
    ! There is available index for the Jatom
    DatabaseInform(Jatom)%Availability=.true.

    ! Second: Project the cartizeian coordinates for the fragmnets
    DBFragAtoms(Jatom,1:MolIdxInfo(indexi)%NAtoms)= &
    & DBUniqueFragAtoms(indexi,1:MolIdxInfo(indexi)%NAtoms)

    ! Third: Project the atomic properties for the fragments
    FragProp(Jatom)=DBFragProp(indexi)
    ! There is available atomic properties for the Jatom
    FragProp(Jatom)%Availability=.true.
  end do
end do
DBFragAtoms%type=0
DBFragAtoms%number=0
DBFragAtoms%factor=0
DBFragAtoms%Ctype="NOT"

! Calculate RR and TT for each atom
do Jatom=1, NAtoms
  if (.not.DatabaseInform(Jatom)%Availability) cycle
  allocate (term1(DatabaseInform(Jatom)%NAtoms))
  allocate (term2(DatabaseInform(Jatom)%NAtoms))
  allocate (term3(DatabaseInform(Jatom)%NAtoms))
  ! Needed to be in one dimension to get RR and TT
  term1(1:DatabaseInform(Jatom)%NAtoms)=&
  CurrFragAtoms(Jatom,1:DatabaseInform(Jatom)%NAtoms)
  term2(1:DatabaseInform(Jatom)%NAtoms)=&
  DBFragAtoms(Jatom,1:DatabaseInform(Jatom)%NAtoms)
  call GetCartRotMatrixW(term1,term2,DatabaseInform(Jatom)%NAtoms,RR,TT)
  call DoCartRotation(term2,term3,DatabaseInform(Jatom)%NAtoms,RR,TT)
  FragAtomsMatrix(Jatom)%RMT=RR
  FragAtomsMatrix(Jatom)%TMT=TT
  RotDBFragAtoms(Jatom,1:DatabaseInform(Jatom)%NAtoms)=&
  term3(1:DatabaseInform(Jatom)%NAtoms)
  RotDBFragAtoms(Jatom,1:DatabaseInform(Jatom)%NAtoms)%element=&
  & DBFragAtoms(Jatom,1:DatabaseInform(Jatom)%NAtoms)%element
  deallocate (term1)
  deallocate (term2)
  deallocate (term3)
end do

! calculate TTCORR (let first atom at 0,0,0 )
do Jatom=1, NAtoms
  if (.not.DatabaseInform(Jatom)%Availability) cycle
  do ifound=1, DatabaseInform(Jatom)%NAtoms
    FragAtomsMatrix(Jatom)%TMTACORR(1) = -RotDBFragAtoms(Jatom,1)%x+&
    CurrFragAtoms(Jatom,1)%x
    FragAtomsMatrix(Jatom)%TMTACORR(2) = -RotDBFragAtoms(Jatom,1)%y+&
    CurrFragAtoms(Jatom,1)%y
    FragAtomsMatrix(Jatom)%TMTACORR(3) = -RotDBFragAtoms(Jatom,1)%z+&
    CurrFragAtoms(Jatom,1)%z
  end do
end do

```



```

!!!!!!!!!!!!!!!!!!!! end OF PRINTING
! JUST PRINTING (YOU CAN DELETE THE BELOW LINES)
!
write(*,*) "*****"
write(*,*) "    NEW FRAG CART (FROM DATABASE) "
write(*,*) "*****"
do JAtom=1, NAtoms
    write(UNIout,*) "Atom# ", JAtom
    if (.not.DatabaseInform(Jatom)%Availability) then
        write(UNIout,*) "the carts for atom#", JAtom, &
            "are not available in the database"

        cycle
    end if
    write(UNIout, '(A6,3A16)') "Atom", "X", "Y", "Z"
    do ifound=1, DatabaseInform(Jatom)%NAtoms
        write(UNIout, '(A6,2X,3F16.8)') &
            & DBFragAtoms(Jatom,ifound)%element, &
            & DBFragAtoms(Jatom,ifound)%x*CartPrnFactor, &
            & DBFragAtoms(Jatom,ifound)%y*CartPrnFactor, &
            & DBFragAtoms(Jatom,ifound)%z*CartPrnFactor

        end do
    end do
!!!!!!!!!!!!!!!!!!!! end OF PRINTING
! JUST PRINTING (YOU CAN DELETE THE BELOW LINES)
!
write(*,*) "*****"
write(*,*) " Where the fragment should be (database form) "
write(*,*) "*****"
call GetDBFormFragments(CurrFragAtoms,DBCurrFragAtoms,CurrFragInfo)
do JAtom=1, NAtoms
    write(UNIout,*) "Atom# ", JAtom
    if (.not.DatabaseInform(Jatom)%Availability) then
        write(UNIout,*) "the carts for atom#", JAtom, &
            "are not available in the database"

        cycle
    end if
    write(UNIout, '(A6,3A16)') "Atom", "X", "Y", "Z"
    do ifound=1, DatabaseInform(Jatom)%NAtoms
        write(UNIout, '(A6,2X,3F16.8)') &
            & DBCurrFragAtoms(Jatom,ifound)%element, &
            & DBCurrFragAtoms(Jatom,ifound)%x*CartPrnFactor, &
            & DBCurrFragAtoms(Jatom,ifound)%y*CartPrnFactor, &
            & DBCurrFragAtoms(Jatom,ifound)%z*CartPrnFactor

        end do
    end do
!!!!!!!!!!!!!!!!!!!! end OF PRINTING
! JUST PRINTING (YOU CAN DELETE THE BELOW LINES)
!
write(*,*) "*****"
write(*,*) " The error: (FROM DATABASE)- (Original) "
write(*,*) "*****"
call GetDBFormFragments(CurrFragAtoms,DBCurrFragAtoms,CurrFragInfo)
do JAtom=1, NAtoms
    write(UNIout,*) "Atom# ", JAtom
    if (.not.DatabaseInform(Jatom)%Availability) then
        write(UNIout,*) "the carts for atom#", JAtom, &
            "are not available in the database"

        cycle
    end if
    write(UNIout, '(A6,3A16)') "Atom", "X", "Y", "Z"
    do ifound=1, DatabaseInform(Jatom)%NAtoms
        write(UNIout, '(A6,2X,3F16.8)') &
            & DBCurrFragAtoms(Jatom,ifound)%element, &
            & (DBFragAtoms(Jatom,ifound)%x-&

```



```

& DBCurrFragAtoms(Jatom,ifound)%x)*CartPrnFactor, &
& (DBFragAtoms(Jatom,ifound)%y-&
& DBCurrFragAtoms(Jatom,ifound)%y)*CartPrnFactor, &
& (DBFragAtoms(Jatom,ifound)%z-&
& DBCurrFragAtoms(Jatom,ifound)%z)*CartPrnFactor
end do
end do
!!!!!!!!!!!!!!!!!!!! end OF PRINTING
! JUST PRINTING (YOU CAN DELETE THE BELOW LINES)
!
write(*,*) "*****"
write(*,*) " CART WANTED (CART OF MOLECULE) "
write(*,*) "*****"
do JAtom=1, NAtoms
  if (.not.DatabaseInform(Jatom)%Availability) cycle
  write(UNIout,*) "Atom# ", JAtom
  write(UNIout,'(A6,3A16)') "Atom","X","Y","Z"
  do ifound=1, DatabaseInform(Jatom)%NAtoms
    write(UNIout,'(A6,2X,3F16.8)') &
    & CurrFragAtoms(Jatom,ifound)%element, &
    & CurrFragAtoms(Jatom,ifound)%x*CartPrnFactor, &
    & CurrFragAtoms(Jatom,ifound)%y*CartPrnFactor, &
    & CurrFragAtoms(Jatom,ifound)%z*CartPrnFactor
  end do
end do
!!!!!!!!!!!!!!!!!!!! end OF PRINTING
! JUST PRINTING (YOU CAN DELETE THE BELOW LINES)
!
write(*,*) "*****"
write(*,*) " The DatabaseInform Matrix for each atom "
write(*,*) "*****"
do JAtom=1, NAtoms
  if (.not.DatabaseInform(Jatom)%Availability) cycle
  write(UNIout,*) "Atom# ", JAtom
  call PRT_matrix(FragAtomsMatrix(Jatom)%RMAT,3,3)
end do
!!!!!!!!!!!!!!!!!!!! end OF PRINTING
! JUST PRINTING (YOU CAN DELETE THE BELOW LINES)
!
write(*,*) "*****"
write(*,*) " CART AfteR ROT "
write(*,*) "*****"
do JAtom=1, NAtoms
  if (.not.DatabaseInform(Jatom)%Availability) cycle
  write(UNIout,*) "Atom# ", JAtom
  write(UNIout,'(A6,3A16)') "Atom","X","Y","Z"
  do ifound=1, DatabaseInform(Jatom)%NAtoms
    write(UNIout,'(A6,2X,3F16.8)') &
    & RotDBFragAtoms(Jatom,ifound)%element, &
    & RotDBFragAtoms(Jatom,ifound)%x*CartPrnFactor, &
    & RotDBFragAtoms(Jatom,ifound)%y*CartPrnFactor, &
    & RotDBFragAtoms(Jatom,ifound)%z*CartPrnFactor
  end do
end do
!!!!!!!!!!!!!!!!!!!! end OF PRINTING
! JUST PRINTING (YOU CAN DELETE THE BELOW LINES)
!
write(*,*) "*****"
write(*,*) " CART Diff error "
write(*,*) "*****"
do JAtom=1, NAtoms
  if (.not.DatabaseInform(Jatom)%Availability) cycle
  write(UNIout,*) "Atom# ", JAtom
  write(UNIout,'(A6,3A16)') "Atom","X","Y","Z"

```

```

do ifound=1, DatabaseInform(Jatom)%NAtoms
  write(UNIout,'(A6,2X,3F16.8)') &
    & RotDBFragAtoms(Jatom,ifound)%element, &
    & (RotDBFragAtoms(Jatom,ifound)%x-&
    & CurrFragAtoms(Jatom,ifound)%x)*CartPrnFactor, &
    & (RotDBFragAtoms(Jatom,ifound)%y-&
    & CurrFragAtoms(Jatom,ifound)%y)*CartPrnFactor, &
    & (RotDBFragAtoms(Jatom,ifound)%z-&
    & CurrFragAtoms(Jatom,ifound)%z)*CartPrnFactor
end do
end do
!!!!!!!!!!!!!!!!!!!! end OF PRINTING
! JUST PRINTING (YOU CAN DELETE THE BELOW LINES)
!
write(*,*) "*****"
write(*,*) "          CART after TTCORR          "
write(*,*) "*****"
CorrFragAtom=RotDBFragAtoms
do JAtom=1, NAtoms
  if (.not.DatabaseInform(Jatom)%Availability) cycle
  write(UNIout,*) "Atom# ", JAtom
  write(UNIout,'(A6,3A16)') "Atom","X","Y","Z"

  do ifound=1, DatabaseInform(Jatom)%NAtoms
    CorrFragAtom(Jatom,ifound)%element=&
      & RotDBFragAtoms(Jatom,ifound)%element
    CorrFragAtom(Jatom,ifound)%x=RotDBFragAtoms(Jatom,ifound)%x + &
      & FragAtomsMatrix(Jatom)%TMATCORR(1)
    CorrFragAtom(Jatom,ifound)%y=RotDBFragAtoms(Jatom,ifound)%y + &
      & FragAtomsMatrix(Jatom)%TMATCORR(2)
    CorrFragAtom(Jatom,ifound)%z=RotDBFragAtoms(Jatom,ifound)%z + &
      & FragAtomsMatrix(Jatom)%TMATCORR(3)

    write(UNIout,'(A6,2X,3F16.8)') &
      & CorrFragAtom(Jatom,ifound)%element, &
      & CorrFragAtom(Jatom,ifound)%x*CartPrnFactor, &
      & CorrFragAtom(Jatom,ifound)%y*CartPrnFactor, &
      & CorrFragAtom(Jatom,ifound)%z*CartPrnFactor

  end do
end do
!!!!!!!!!!!!!!!!!!!! end OF PRINTING
! JUST PRINTING (YOU CAN DELETE THE BELOW LINES)
!
write(*,*) "*****"
write(*,*) "          CART Diff error after TTCORR          "
write(*,*) "*****"
do JAtom=1, NAtoms
  if (.not.DatabaseInform(Jatom)%Availability) cycle
  write(UNIout,*) "Atom# ", JAtom
  write(UNIout,'(A6,3A16)') "Atom","X","Y","Z"
  do ifound=1, DatabaseInform(Jatom)%NAtoms
    write(UNIout,'(A6,2X,3F16.8)') &
      & RotDBFragAtoms(Jatom,ifound)%element, &
      & (CorrFragAtom(Jatom,ifound)%x-&
      & CurrFragAtoms(Jatom,ifound)%x)*CartPrnFactor, &
      & (CorrFragAtom(Jatom,ifound)%y-&
      & CurrFragAtoms(Jatom,ifound)%y)*CartPrnFactor, &
      & (CorrFragAtom(Jatom,ifound)%z-&
      & CurrFragAtoms(Jatom,ifound)%z)*CartPrnFactor
  end do
end do
!!!!!!!!!!!!!!!!!!!! end OF PRINTING

! deallocate (FRgrids)

```

```

        deallocate (FragAtomsMatrix)
!       deallocate (CorrFragAtom)
        deallocate (CurrFragAtoms)
        deallocate (DBCurrFragAtoms)
        deallocate (DBUniqueFragAtoms)
        deallocate (DBFragAtoms)
        deallocate (RotDBFragAtoms)
        deallocate (CurrFragInfo)
        deallocate (MolIdxInfo)
        deallocate (DBFragProp)
        deallocate (RR)
        deallocate (TT)
        deallocate (TTCORR)
        deallocate (symbols)
        deallocate (UniSym)
        deallocate (UniSymMUNIdx)
        return
    end SUBROUTINE Build_Molecule_DB

SUBROUTINE Build_Molecule_Direct(DatabaseInform,FragProp,FRgrids)
!*****e*****
!       Date last modified: *
!       Author: Ibrahim Awad *
!       Description: *
!*****e*****
! MODULES:

    USE AIMDFT_type
    USE type_molecule

    implicit none

    ! Grid points by Iatom
    type(GridsDataBase), dimension(:,:), allocatable, intent(out) :: FRgrids
    type(IndexFile), dimension (:), allocatable, INTENT(out) :: DatabaseInform
    type(PropFile), dimension(:), allocatable, INTENT(out) :: FragProp

    integer :: File_unit
    logical :: Lerror

    ! < should be change
    CALL GET_unit ('Molecule_Properties.txt', File_unit, Lerror)
    open(UNIT=File_unit,file=trim('Molecule_Properties.txt'), &
        status='REPLACE',form='formatted')
    write(File_unit,fmt_prop_title) &
        & "#", &
        & "Electrons", &
        & "K", &
        & "K_HF", &
        & "T", &
        & "Vne_Ana", &
        & "Vne_Num", &
        & "Vee (Ana/Num)", &
        & "J", &
        & "J_HF", &
        & "Jaa=Kaa", &
        & "Vee_DFT_AtomA"

    close(unit=File_unit)

    CALL GenFilesDirect()
    CALL GetGridsInform(DatabaseInform,FRgrids)
    CALL GetDirectFragmentsProperties(FragProp)

```



```

write(fileunit,'(A20,A10,A4/)' ) 'PARTitioning Scheme=',&
                                trim(DEN_Partitioning),' end'

if (trim(RADIAL_grid).eq.'GILL') then
write(radline,'(A20,I3,A10,I2,I3,I4,A10)' ) &
    'RADial Gill RPoints=',&
    NRPoints_Gill ,&
    ' APoints=(',&
    NApoints_Gill(1),&
    NApoints_Gill(2),&
    NApoints_Gill(3),' ) end end'
else
write(radline,'(A22,A7,A8)' ) 'NUMercial RADial GRID=',&
                                trim(RADIAL_grid),' end end'
endif

write(fileunit,'(A/)' ) radline

write(fileunit,'(A/)' ) 'output object=AIMDFT:FRAGCART%GRIDS end'
write(fileunit,'(A)' ) 'stop'
close(UNIT=fileunit)
write(*,'(///A,A/,A/)' ) "*** RUNing the inputfile :: ", inputfile
write(cmdd,'(a7,a20)' )"mgauss<",inputfile
CALL SYSTEM(cmdd)
! write(cmdd,'(a3,a20)' )"rm ",inputfile
! CALL SYSTEM(cmdd)
end do
write(*,'(A///)' )
deallocate (FragInfo)
deallocate (DBFragmentAtom)
return
end SUBROUTINE GenFilesDirect
SUBROUTINE GetGridsInform(DatabaseInform,FRgrids)
!*****e*****
! Date last modified: *
! Author: Ibrahim Awad *
! Description: *
!*****e*****
! MODULES:

USE AIMDFT_Files
USE type_molecule
USE AIMDFT_type

implicit none

integer :: Ipoint
integer :: Iatom
integer :: File_unit
integer :: Reason

character (60) :: PlotFileNames
character (60) :: cmdd
logical :: Lerror
double precision :: DEN, BeckW, AngW, Qval, Vpot
double precision, dimension (1:3) :: point

type(GridsDataBase), dimension(:,,:), allocatable, intent(out) :: FRgrids
type(IndexFile), dimension (:), allocatable, INTENT(out) :: DatabaseInform

allocate (FRgrids(Natoms,MaxGridPnt))
allocate(DatabaseInform(NAtoms))

DatabaseInform%Availability=.false.

```

```

do Iatom=1, NAtoms
  if (CARTESIAN(Iatom)%Atomic_Number.le.0) cycle
  call INPUT_File_FRAG ('GRID_DIRECT_', '.dat', Iatom, PlotFileNames)
  call GET_unit (PlotFileNames, File_unit, Lerror)
  open(UNIT=File_unit, file=PlotFileNames, status='OLD', form='formatted')
  Ipoint=0
  read(File_unit, *)
  do
    read(File_unit, Pfmt_rho_w_v, IOSTAT=Reason) point(1), &
      point(2), point(3), DEN, BeckW, AngW, Qval, Vpot
    if (Reason.lt.0) exit
    Ipoint=Ipoint+1
    FRgrids(Iatom, Ipoint)%x=Point(1)
    FRgrids(Iatom, Ipoint)%y=Point(2)
    FRgrids(Iatom, Ipoint)%z=Point(3)
    FRgrids(Iatom, Ipoint)%D=DEN
    FRgrids(Iatom, Ipoint)%BW=BeckW
    FRgrids(Iatom, Ipoint)%W=AngW
    FRgrids(Iatom, Ipoint)%Q=Qval
    FRgrids(Iatom, Ipoint)%VpotA=Vpot
  end do
  close(UNIT=File_unit)
!   write(cmdd, '(a3, a36)') "rm ", PlotFileNames
!   CALL SYSTEM(cmdd)

  DatabaseInform(Iatom)%Availability=.true.
  DatabaseInform(Iatom)%IndexNum=Iatom
  DatabaseInform(Iatom)%GridsNum=Ipoint

end do
end SUBROUTINE GetGridsInform
SUBROUTINE GetDirectFragmentsProperties(FragProp)
! *****
!   Date last modified: *
!   Author: Ibrahim Awad *
!   Description: Return the cartesian coordinates of the fragments *
!               from the database cartesian file. *
! *****
!

implicit none

type(PropFile), dimension(:), allocatable, intent(out) :: FragProp

integer :: newindex
integer :: File_unit, Reason
logical :: Lerror
double precision :: EleNum, EV_Anal, EV_Num
double precision :: EX, EXJ, ET, EC, Kaa, Jaa, Vee_A, ECK, EC_Anal
character (60) :: cmdd

allocate (FragProp(NAtoms))

! loop over the wanted index
FragProp%Availability=.false.
call GET_unit ('Molecule_Properties.txt', File_unit, Lerror)
open(UNIT=File_unit, file='Molecule_Properties.txt', &
  status='old', form='formatted')
read(File_unit, *)
do ! start, reading the file
  read(File_unit, fmt_prop, IOSTAT=Reason) &
    & newindex, &
    & EleNum, &

```

```

& EX, &
& EXJ, &
& ET, &
& EV_Anal, &
& EV_Num, &
& EC_AnalNum, &
& EC, &
& ECK, &
& Jaa, &
& vee_A

if (Reason.lt.0) exit
FragProp(newindex)%IndexNum=newindex
FragProp(newindex)%ElectronNum=EleNum
FragProp(newindex)%K=EX
FragProp(newindex)%KHF=EXJ
FragProp(newindex)%T=ET
FragProp(newindex)%Vne_A=EV_Anal
FragProp(newindex)%Vne_N=EV_Num
FragProp(newindex)%Vee_AN=EC_AnalNum
FragProp(newindex)%J=EC
FragProp(newindex)%JHF=ECK
FragProp(newindex)%Jaa=Jaa
FragProp(newindex)%VeeA=Vee_A
FragProp(newindex)%Availability=.true.
end do ! reading the file

close(unit=File_unit)
write(cmdd,'(a3,a24)') "rm ", "Molecule_Properties.txt"
CALL SYSTEM(cmdd)

end SUBROUTINE GetDirectFragmentsProperties
end SUBROUTINE Build_Molecule_Direct

```

```

SUBROUTINE Store_result_direct()
! *****
! Date last modified: *
! Author: Ibrahim Awad *
! Description: *
! *****
! MODULES

USE type_density
USE QM_defaults
USE QM_objects
USE N_integration
USE NI_defaults
USE type_molecule
USE AIMDFT_type
USE AIMDFT_files
USE type_plotting
USE GetMolecularProperties

implicit none

character (60):: PlotFileNames

double precision, dimension(:), allocatable :: Vpot1
double precision, dimension(:), allocatable :: V12dr2
double precision :: TraceAB
double precision :: Xpt,Ypt,Zpt

logical :: Lerror
integer :: IApoint

```

```

integer :: Znum, Iatom
integer :: kfound
integer :: File_unit
integer :: newindex

CALL GET_object ('GRID', 'RADIAL', RADIAL_grid)
! CALL GET_object ('QM', 'ENERGY_COMPONENTS', Wavefunction)

newindex=IndexNumber

Iatom=1 ! Just add the interest atom within the fragment.
NAIMprint=1
AIMprint(1)=1

! write(*,*)
CALL GET_object ('QM', 'ENERGY_VEE', 'NUMERICAL')
! CALL GET_object ('QM', 'ENERGY_EXCHANGE', 'NUMERICAL')
! CALL GET_object ('QM', 'ENERGY_COULOMB', 'MO')
write(*,*)
CALL GET_object ('QM', 'ENERGY_COULOMB', 'NUMERICAL')
write(*,*)
CALL GET_object ('QM', 'ENERGY_KINETIC', 'NUMERICAL')
write(*,*)
CALL GET_object ('QM', 'ENERGY_VNE', 'NUMERICAL')
write(*,*)

Znum=CARTESIAN(Iatom)%Atomic_Number
kfound=GRID_loc(Znum)
NApts_atom=NApoints_atom(kfound)

allocate(grid_points(1:NApts_atom))
! Save grid points
do IApoint=1,NApts_atom
    grid_points(IApoint)%X=Egridpts(Iatom,IApoint)%X+CARTESIAN(Iatom)%X
    grid_points(IApoint)%Y=Egridpts(Iatom,IApoint)%Y+CARTESIAN(Iatom)%Y
    grid_points(IApoint)%Z=Egridpts(Iatom,IApoint)%Z+CARTESIAN(Iatom)%Z
    grid_points(IApoint)%w=Egridpts(Iatom,IApoint)%w
end do ! IApoint

! compute the weight for all grid points

allocate(Bweights(NApts_atom))
CALL GET_weights (grid_points, NApts_atom, Iatom, Bweights)

! compute the charge for all grid points
allocate (rho_Atom(NApts_atom))
CALL GET_density (grid_points, NApts_atom, rho_Atom)

! compute the V12dr2 for all grid points!
! < IT IS Not nessessary to calculate it.
allocate(Vpot1(1:NApts_atom))
allocate(V12dr2(1:MATlen))
do IApoint=1,NApts_atom
    Xpt=grid_points(IApoint)%X
    Ypt=grid_points(IApoint)%Y
    Zpt=grid_points(IApoint)%Z
    CALL I1E_V12dr2 (V12dr2, MATlen,Xpt,Ypt,Zpt)
    Vpot1(IApoint)=-TraceAB (PM0, V12dr2, NBasis)
end do ! IApoint

CALL Store_grid_points_direct
CALL add_to_prop_file('Molecule_Properties.txt')

```







```

integer, intent(in) :: level
character (SymMax), dimension (:), allocatable, intent(out) :: UniSym
integer, dimension (:), allocatable, intent(out) :: UniSymMUNIdx
character (SymMax), dimension (:), allocatable :: symbols

integer, intent(out) :: UniSymMum
integer :: Jatom

allocate (UniSym(NAtoms))
allocate (symbols(NAtoms))
allocate (UniSymMUNIdx(NAtoms))

CALL GetFragSymbols(level,symbols)

UniSym = ""
UniSymMum = 0      ! count the unique symbols within the molecule
do Jatom=1, NAtoms
    if (Cartesian(Jatom)%Atomic_Number.eq.0) cycle ! exclude dummy atoms
    if (ANY(UniSym.eq.symbols(Jatom))) cycle ! Founded before
    UniSymMum=UniSymMum+1
    UniSym(UniSymMum)=symbols(Jatom)
    UniSymMUNIdx(UniSymMum)=Jatom ! Just the first match
end do
deallocate(symbols)
return
end SUBROUTINE GetUniSym
SUBROUTINE GetFragSymbols(level,symbols)
!*****
!      Date last modified:
!      Author: Ibrahim Awad
!      Description: Generate unique symbols for the molecule fragments
!                  [e.g. 7( 6( 6( 8 8 6) 1( 6) 1( 6)) 1() 1()) ]
!*****
!

implicit none

integer, intent(in) :: level
character (SymMax), dimension (:), intent(out), allocatable :: symbols

type (FragAtomInfo), dimension(:,,:), allocatable :: FragAtoms
type (FragInfo), dimension(:), allocatable :: FragInfo ! (main atom)

character (SymMax), dimension(:), allocatable :: tempOrdValue
integer, dimension(10) :: tempOrd
integer :: NtempOrd, TempCopy, curr,next
character (SymMax) :: TempCopyChar
character (10) :: atomicN
character (len=1), parameter :: SepSymbol=","

integer :: ilevel
integer :: ifound
integer :: Matom
integer :: Jatom
logical :: swap

allocate (symbols(NAtoms))
allocate (FragAtoms(NAtoms,NAtoms))
allocate (FragInfo(NAtoms))
allocate (tempOrdValue(NAtoms))

CALL GetSortCart(level,FragAtoms,FragInfo)

```

```

! JUST ::::defined symbol for level 1
do Jatom=1, NAtoms
if (Cartesian(Jatom)%Atomic_Number.eq.0) cycle ! exclude dummay atoms
do Matom=1, NAtoms ! loop for subatoms
! Defind the MainAtom
write(atomicN,'(I0)') Cartesian(Jatom)%Atomic_Number
FragInfo(Matom)%symbol(Jatom,1)=trim(atomicN)//"("
NtempOrd=0
tempOrd=0
do ifound=1, FragInfo(Jatom)%NAtoms ! loop over subatoms
if (FragAtoms(Jatom,ifound)%level.ne.1) cycle !Just level 1
! exclude subatom=Matom
if (FragAtoms(Jatom,ifound)%MUNIdx.eq.Matom) cycle
! exclude Mainatom=Mainatom
if (FragAtoms(Jatom,ifound)%MUNIdx.eq.Jatom) cycle
NtempOrd=NtempOrd+1
tempOrd(NtempOrd)=FragAtoms(Jatom,ifound)%Atomic_Number
end do
swap=.true.
do while(swap)
swap=.false.
do curr=1,NtempOrd-1
next=curr+1
if (tempOrd(curr).lt.tempOrd(next)) then
TempCopy=tempOrd(curr)
tempOrd(curr)=tempOrd(next)
tempOrd(next)=TempCopy
swap=.true.
endif !(valueB.lt.valueA)
end do !curr
end do !while(swap)
do ifound=1, NtempOrd
if (ifound.ne.NtempOrd) then
! the new subatom
write(atomicN,'(I0,A1)') tempOrd(ifound),Sepsymbol
else
write(atomicN,'(I0)') tempOrd(ifound) ! the new subatom
end if
! add the new subatom
FragInfo(Matom)%symbol(Jatom,1)=&
trim(FragInfo(Matom)%symbol(Jatom,1))//trim(atomicN)//"
end do
! just level 1, the neighbor
FragInfo(Matom)%symbol(Jatom,1)=&
trim(FragInfo(Matom)%symbol(Jatom,1))//")"
if (Matom.eq.Jatom) symbols(Jatom)=&
trim(FragInfo(Matom)%symbol(Jatom,1))
end do !Matom
end do ! Jatom

! Now from the symbols of the 1st level,
! we will build the symbols for ALL levels
do ilevel=2, level
do Jatom=1, NAtoms
do Matom=1, NAtoms ! loop over dumatoms
! Defind the Mainatom
write(atomicN,'(I0)') Cartesian(Jatom)%Atomic_Number
FragInfo(Matom)%symbol(Jatom,ilevel)=trim(atomicN)//"("
NtempOrd=0
tempOrd=0
tempOrdValue=""
do ifound=1, FragInfo(Jatom)%NAtoms ! loop over subatoms
if (FragAtoms(Jatom,ifound)%level.ne.1) cycle !Just level 1

```



```

implicit none

type (FragAtomInfo), dimension(:, :), intent(inout), allocatable :: FragAtoms
type (FragInfo), dimension(:), intent(inout), allocatable :: FragInfo
integer, intent(in) :: adlevel

type (FragAtomInfo), dimension(:), allocatable :: termatoms
type (FragAtomInfo), dimension(:), allocatable :: DBterm
type (FragAtomInfo), dimension(:), allocatable :: Newtermatoms
type (FragAtomInfo), dimension(:), allocatable :: NewDBterm

double precision, dimension (:, :), allocatable :: RR, RRI
double precision, dimension (:), allocatable :: TT
double precision, dimension (1:3) :: NewPoint, point

integer :: loop3, oldloop3, iloop, NewLoop
integer :: Jatom, iMUNatom, ifound, ifound2, ifound3
double precision :: bondtype
logical :: termenalTF

allocate (termatoms(Natoms))
allocate (DBterm(Natoms))
allocate (Newtermatoms(Natoms))
allocate (NewDBterm(Natoms))
allocate (RR(3,3))
allocate (RRI(3,3))
allocate (TT(3))

CALL GET_object ('MOL', 'GRAPH', 'CONNECT')
CALL GET_object ('QM', 'BOND_ORDER', 'MEYER')

do Jatom=1, Natoms
! loop over number of adjacent atoms for Jatom
do ifound=1, FragInfo(Jatom)%NAtoms
iMUNatom=FragAtoms(Jatom,ifound)%MUNIdx
if(ChangeTerminalAtoms) then
termenalTF=.true.
else
! In the case if the atom is terminal then leave it same
termenalTF=(FragInfo(iMUNatom)%NumNeighbourAtoms.ne.1)
end if
if ((FragAtoms(Jatom,ifound)%level.ne.(adlevel))) cycle
if (.not.termenalTF) cycle

! found one atom (2 >>> to be in z-axes)
termatoms(2)=FragAtoms(Jatom,ifound)
do ifound2=1, FragInfo(Jatom)%NAtoms
! Look for atom with (level-1) of the interest atom
if (FragAtoms(Jatom,ifound2)%level.ne.(adlevel-1)) cycle
! find the parent atom
if (Connect(FragAtoms(Jatom,ifound2)%MUNIdx,&
termatoms(2)%MUNIdx).ne.1) cycle
termatoms(1)=FragAtoms(Jatom,ifound2) ! found the origin atom
end do
loop3=2 ! number of terminal atoms
do ifound3=1, FragInfo(Jatom)%NAtoms
if (Connect(termatoms(1)%MUNIdx,&
FragAtoms(Jatom,ifound3)%MUNIdx).ne.1) cycle
if (FragAtoms(Jatom,ifound3)%MUNIdx.eq.termatoms(2)%MUNIdx) cycle
loop3=loop3+1
termatoms(loop3)=FragAtoms(Jatom,ifound3) ! the terminal atoms
end do
! find the bond type of the interest atom with the origin

```

```

bondtype= bond_order(termatoms(2)%MUNIdx,termatoms(1)%MUNIdx)
! write(*,*) termatoms(2)%MUNIdx,termatoms(1)%MUNIdx,bondtype
CALL GetDBCart(termatoms,DBterm,loop3,RR,TT)
CALL INV33(RR, RRI)

oldloop3=loop3
if(bondtype.le.0.5)then ! Partial single bond (TS?)
  CALL getterm1 (DBterm,loop3,NewDBterm,NewLoop)
else if(bondtype.le.1.15)then ! Single bond
  CALL getterm2 (DBterm,loop3,NewDBterm,NewLoop)
else if(bondtype.le.1.6)then ! Aromatic double bond
  CALL getterm3 (DBterm,loop3,NewDBterm,NewLoop)
else if(bondtype.le.2.2)then ! Double bond
  CALL getterm4 (DBterm,loop3,NewDBterm,NewLoop)
else if(bondtype.le.2.7)then ! Aromatic triple bond
  CALL getterm5 (DBterm,loop3,NewDBterm,NewLoop)
else if(bondtype.le.3.2)then ! Triple bond
  CALL getterm6 (DBterm,loop3,NewDBterm,NewLoop)
end if

do iloop=loop3+1, NewLoop
  point=(/NewDBterm(iloop)%x,NewDBterm(iloop)%y,NewDBterm(iloop)%z/)
  NewPoint=Matmul(RRI,(point-TT))
  Newtermatoms(iloop)%x=NewPoint(1)
  Newtermatoms(iloop)%y=NewPoint(2)
  Newtermatoms(iloop)%z=NewPoint(3)
end do

FragAtoms(Jatom,ifound)%Atomic_Number=NewDBterm(2)%Atomic_Number
FragAtoms(Jatom,ifound)%element=NewDBterm(2)%element

do iloop=loop3+1, NewLoop
  FragInfo(Jatom)%NAtoms=FragInfo(Jatom)%NAtoms+1
  FragAtoms(Jatom,FragInfo(Jatom)%NAtoms)=Newtermatoms(iloop)
  FragAtoms(Jatom,FragInfo(Jatom)%NAtoms)%Atomic_Number=&
    NewDBterm(iloop)%Atomic_Number
  FragAtoms(Jatom,FragInfo(Jatom)%NAtoms)%element=&
    NewDBterm(iloop)%element
  FragAtoms(Jatom,FragInfo(Jatom)%NAtoms)%type=999
  FragAtoms(Jatom,FragInfo(Jatom)%NAtoms)%ctype=&
    NewDBterm(iloop)%ctype
  FragAtoms(Jatom,FragInfo(Jatom)%NAtoms)%factor=-1
  FragAtoms(Jatom,FragInfo(Jatom)%NAtoms)%level=adlevel+1
  FragAtoms(Jatom,FragInfo(Jatom)%NAtoms)%MUNIdx=-1
end do
end do ! ifound=1, SATOMS(Jatom)
end do ! JAtom=1, NAtoms

CALL CalcFragConnect(FragAtoms,FragInfo)
CALL UpdateFragTypes(FragAtoms,FragInfo)
deallocate(termatoms,DBterm,Newtermatoms,NewDBterm,RR,TT,RRI)
return
end SUBROUTINE AddTerminalAtoms
SUBROUTINE getterm1 (DBterm,loop3,NewDBterm,NewLoop)
! *****
! Date last modified: *
! Author: Ibrahim Awad *
! Description: *
! *****
!
  implicit none

  type (FragAtomInfo), dimension(:), allocatable, intent(in) :: DBterm
  integer, intent(in) :: loop3 ! +1 for new atoms

```

```

type (FragAtomInfo), dimension(:), allocatable, intent(out) :: NewDBterm
integer, intent(out) :: NewLoop ! +1 for new atoms
type (FragAtomInfo), dimension(:), allocatable :: newatoms

double precision :: CH_bond, Anglea
integer :: NNewatoms, iloop

allocate (NewDBterm(Natoms))

NewDBterm=DBterm
NNewatoms=0

! select case(NewDBterm(2)%Atomic_Number)
!   case(1) ! H atom
!       !!!!!!!!!!!!!!!!!!!
!   case(2) ! He atom
!       !!!!!!!!!!!!!!!!!!!
!   case(3)
!       !!!!!!!!!!!!!!!!!!!
!   case(6) ! C atom
!       !!!!!!!!!!!!!!!!!!!
!   case(8) ! O atom
!       !!!!!!!!!!!!!!!!!!!
!   CASE DEFAULT
!       NewDBterm(2)%Atomic_Number=TerminalAtom
!       NewDBterm(2)%element=element_symbols(TerminalAtom)
! end select

NewLoop=loop3
do iloop=1, NNewatoms
    NewLoop=NewLoop+1
    NewDBterm(NewLoop)%x=newatoms(iloop)%x+DBterm(2)%x
    NewDBterm(NewLoop)%y=newatoms(iloop)%y+DBterm(2)%y
    NewDBterm(NewLoop)%z=newatoms(iloop)%z+DBterm(2)%z
    NewDBterm(NewLoop)%Atomic_Number=newatoms(iloop)%Atomic_Number
    NewDBterm(NewLoop)%element=newatoms(iloop)%element
    NewDBterm(NewLoop)%ctype=newatoms(iloop)%ctype
end do

if (allocated(newatoms)) deallocate(newatoms) ! if claUSE is necessary
end SUBROUTINE getterm1

SUBROUTINE getterm2 (DBterm,loop3,NewDBterm,NewLoop)
!*****
!   Date last modified: *
!   Author: Ibrahim Awad *
!   Description: *
!*****
!
implicit none

type (FragAtomInfo), dimension(:), allocatable, intent(in) :: DBterm
integer, intent(in) :: loop3 ! +1 for new atoms
type (FragAtomInfo), dimension(:), allocatable, intent(out) :: NewDBterm
integer, intent(out) :: NewLoop ! +1 for new atoms
type (FragAtomInfo), dimension(:), allocatable :: newatoms

double precision :: HO_bond, Anglea, CH_bond, NH_bond
integer :: NNewatoms, iloop

allocate (NewDBterm(Natoms))

NewDBterm=DBterm
NNewatoms=0

```



```

select case(NewDBterm(2)%Atomic_Number)
case(6)    ! C atom
  NNewatoms=3
  allocate (newatoms(NNewatoms))
  ! NEW ATOMS FOR THE TERMENAL
  ! Single bond (-CH3)
  CH_bond=2.04781965 ! Bohr
  !Anglea=31.65 *0.0174533 ! Need Degree to RAD
  !H1
  newatoms(1)%x=0.0d0
  newatoms(1)%y=-1.93061734
  newatoms(1)%z=0.68284856
  newatoms(1)%Atomic_Number=TerminalAtom
  newatoms(1)%element=element_symbols(TerminalAtom)
  newatoms(1)%ctype="-CH3"
  !H2
  newatoms(2)%x=1.67199927
  newatoms(2)%y=0.96543892
  newatoms(2)%z=0.68284856
  newatoms(2)%Atomic_Number=TerminalAtom
  newatoms(2)%element=element_symbols(TerminalAtom)
  newatoms(2)%ctype="-CH3"
  !H3
  newatoms(3)%x=-1.67199914
  newatoms(3)%y= 0.96557695
  newatoms(3)%z= 0.68284856
  newatoms(3)%Atomic_Number=TerminalAtom
  newatoms(3)%element=element_symbols(TerminalAtom)
  newatoms(3)%ctype="-CH3"

case(7)    ! N atom
  NNewatoms=2
  allocate (newatoms(NNewatoms))
  ! NEW ATOMS FOR THE TERMENAL
  ! Single bond (-NH2)
  NH_bond=1.89445030 ! Bohr
  !Anglea=31.65 *0.0174533 ! Need Degree to RAD
  !H1
  newatoms(1)%x=0.0d0
  newatoms(1)%y=-1.80992957
  newatoms(1)%z=0.55955062
  newatoms(1)%Atomic_Number=TerminalAtom
  newatoms(1)%element=element_symbols(TerminalAtom)
  newatoms(1)%ctype="-NH2"
  !H2
  newatoms(2)%x=-1.64331915
  newatoms(2)%y=0.75857027
  newatoms(2)%z=0.55955062
  newatoms(2)%Atomic_Number=TerminalAtom
  newatoms(2)%element=element_symbols(TerminalAtom)
  newatoms(2)%ctype="-NH2"

case(8)    ! O atom

  ! NEW ATOMS FOR THE TERMENAL
  ! Hydrogen bond (H-O)
  NNewatoms=1
  allocate (newatoms(NNewatoms))

  HO_bond=1.86970623 ! Bohr
  Anglea=14.5d0 *0.0174533 ! Need Degree to RAD
  !H1 for O atom

```

```

newatoms(1)%x=0.0d0
newatoms(1)%y=-HO_bond*Cos(Anglea)
newatoms(1)%z=HO_bond*Sin(Anglea)
newatoms(1)%Atomic_Number=TerminalAtom
newatoms(1)%element=element_symbols(TerminalAtom)
newatoms(1)%ctype="-OH"
!   case(8) ! O atom
!           !!!!!!!!!!!!!!!
CASE DEFAULT
  NewDBterm(2)%Atomic_Number=TerminalAtom
  NewDBterm(2)%element=element_symbols(TerminalAtom)
end select

NewLoop=loop3
do iloop=1, NNewatoms
  NewLoop=NewLoop+1
  NewDBterm(NewLoop)%x=newatoms(iloop)%x+DBterm(2)%x
  NewDBterm(NewLoop)%y=newatoms(iloop)%y+DBterm(2)%y
  NewDBterm(NewLoop)%z=newatoms(iloop)%z+DBterm(2)%z
  NewDBterm(NewLoop)%Atomic_Number=newatoms(iloop)%Atomic_Number
  NewDBterm(NewLoop)%element=newatoms(iloop)%element
  NewDBterm(NewLoop)%ctype=newatoms(iloop)%ctype
end do

if (allocated(newatoms)) deallocate(newatoms) ! if claUSE is necessary

end SUBROUTINE getterm2

SUBROUTINE getterm3 (DBterm,loop3,NewDBterm,NewLoop)
!*****
!   Date last modified:
!   Author: Ibrahim Awad
!   Description:
!*****
!
implicit none

type (FragAtomInfo), dimension(:), allocatable, intent(in) :: DBterm
integer, intent(in) :: loop3 ! +1 for new atoms
type (FragAtomInfo), dimension(:), allocatable, intent(out) :: NewDBterm
integer, intent(out) :: NewLoop ! +1 for new atoms
type (FragAtomInfo), dimension(:), allocatable :: newatoms

double precision :: CH_bond, Anglea
integer :: NNewatoms, iloop

allocate (NewDBterm(Natoms))

NewDBterm=DBterm
NNewatoms=0

!   select case(NewDBterm(2)%Atomic_Number)
!   case(1) ! H atom
!           !!!!!!!!!!!!!!!
!   case(2) ! He atom
!           !!!!!!!!!!!!!!!
!   case(3)
!           !!!!!!!!!!!!!!!
!   case(6) ! C atom
!           !!!!!!!!!!!!!!!
!   case(8) ! O atom
!           !!!!!!!!!!!!!!!
!   CASE DEFAULT

```

```

!       NewDBterm(2)%Atomic_Number=TerminalAtom
!       NewDBterm(2)%element=element_symbols(TerminalAtom)
!     end select

NewLoop=loop3
do iloop=1, NNewatoms
  NewLoop=NewLoop+1
  NewDBterm(NewLoop)%x=newatoms(iloop)%x+DBterm(2)%x
  NewDBterm(NewLoop)%y=newatoms(iloop)%y+DBterm(2)%y
  NewDBterm(NewLoop)%z=newatoms(iloop)%z+DBterm(2)%z
  NewDBterm(NewLoop)%Atomic_Number=newatoms(iloop)%Atomic_Number
  NewDBterm(NewLoop)%element=newatoms(iloop)%element
  NewDBterm(NewLoop)%ctype=newatoms(iloop)%ctype
end do

if (allocated(newatoms)) deallocate(newatoms) ! if claUSE is necessary
end SUBROUTINE getterm3
SUBROUTINE getterm4 (DBterm,loop3,NewDBterm,NewLoop)
!*****
!   Date last modified:
!   Author: Ibrahim Awad
!   Description:
!*****
!

implicit none

type (FragAtomInfo), dimension(:), allocatable, intent(in) :: DBterm
integer, intent(in) :: loop3 ! +1 for new atoms
type (FragAtomInfo), dimension(:), allocatable, intent(out) :: NewDBterm
integer, intent(out) :: NewLoop ! +1 for new atoms
type (FragAtomInfo), dimension(:), allocatable :: newatoms

double precision :: CH_bond, Anglea
integer :: NNewatoms, iloop

allocate (NewDBterm(Natoms))

NewDBterm=DBterm
NNewatoms=0

select case(NewDBterm(2)%Atomic_Number)
!   case(1) ! H atom
!       !!!!!!!!!!!!!!!
!   case(2) ! He atom
!       !!!!!!!!!!!!!!!
!   case(3)
!       !!!!!!!!!!!!!!!
!   case(6) ! C atom
NNewatoms=2
allocate (newatoms(NNewatoms))
! NEW ATOMS FOR THE TERMENAL
! Double bond (=CH2)
CH_bond=2.03456411 ! Bohr
Anglea=31.65 *0.0174533 ! Need Degree to RAD
!H1
newatoms(1)%x=0.0d0
newatoms(1)%y=CH_bond*Cos(Anglea)
newatoms(1)%z=CH_bond*Sin(Anglea)
newatoms(1)%Atomic_Number=TerminalAtom
newatoms(1)%element=element_symbols(TerminalAtom)
newatoms(1)%ctype="=CH2"
!H2
newatoms(2)%x=0.0d0
newatoms(2)%y=-CH_bond*Cos(Anglea)

```

```

newatoms(2)%z= CH_bond*Sin(Anglea)
newatoms(2)%Atomic_Number=TerminalAtom
newatoms(2)%element=element_symbols(TerminalAtom)
newatoms(2)%ctype="=CH2"

!   case(8) ! 0 atom
!   !!!!!!!!!!!!!!!
!   CASE DEFAULT
!       NewDBterm(2)%Atomic_Number=TerminalAtom
!       NewDBterm(2)%element=element_symbols(TerminalAtom)
end select

NewLoop=loop3
do iloop=1, NNewatoms
    NewLoop=NewLoop+1
    NewDBterm(NewLoop)%x=newatoms(iloop)%x+DBterm(2)%x
    NewDBterm(NewLoop)%y=newatoms(iloop)%y+DBterm(2)%y
    NewDBterm(NewLoop)%z=newatoms(iloop)%z+DBterm(2)%z
    NewDBterm(NewLoop)%Atomic_Number=newatoms(iloop)%Atomic_Number
    NewDBterm(NewLoop)%element=newatoms(iloop)%element
    NewDBterm(NewLoop)%ctype=newatoms(iloop)%ctype
end do

if (allocated(newatoms)) deallocate(newatoms) ! if claUSE is necessary
end SUBROUTINE getterm4

SUBROUTINE getterm5 (DBterm,loop3,NewDBterm,NewLoop)
!*****
!   Date last modified: *
!   Author: Ibrahim Awad *
!   Description: *
!*****
!
implicit none

type (FragAtomInfo), dimension(:), allocatable, intent(in) :: DBterm
integer, intent(in) :: loop3 ! +1 for new atoms
type (FragAtomInfo), dimension(:), allocatable, intent(out) :: NewDBterm
integer, intent(out) :: NewLoop ! +1 for new atoms
type (FragAtomInfo), dimension(:), allocatable :: newatoms

double precision :: CH_bond, Anglea
integer :: NNewatoms, iloop

allocate (NewDBterm(Natoms))

NewDBterm=DBterm
NNewatoms=0

!   select case(NewDBterm(2)%Atomic_Number)
!   case(1) ! H atom
!       !!!!!!!!!!!!!!!
!   case(2) ! He atom
!       !!!!!!!!!!!!!!!
!   case(3)
!       !!!!!!!!!!!!!!!
!   case(6) ! C atom
!       !!!!!!!!!!!!!!!
!   case(8) ! 0 atom
!       !!!!!!!!!!!!!!!
!   CASE DEFAULT
!       NewDBterm(2)%Atomic_Number=TerminalAtom

```

```

!       NewDBterm(2)%element=element_symbols(TerminalAtom)
!     end select

NewLoop=loop3
do iloop=1, NNewatoms
  NewLoop=NewLoop+1
  NewDBterm(NewLoop)%x=newatoms(iloop)%x+DBterm(2)%x
  NewDBterm(NewLoop)%y=newatoms(iloop)%y+DBterm(2)%y
  NewDBterm(NewLoop)%z=newatoms(iloop)%z+DBterm(2)%z
  NewDBterm(NewLoop)%Atomic_Number=newatoms(iloop)%Atomic_Number
  NewDBterm(NewLoop)%element=newatoms(iloop)%element
  NewDBterm(NewLoop)%ctype=newatoms(iloop)%ctype
end do

if (allocated(newatoms)) deallocate(newatoms) ! if claUSE is necessary
end SUBROUTINE getterm5

SUBROUTINE getterm6 (DBterm,loop3,NewDBterm,NewLoop)
!*****
!   Date last modified:
!   Author: Ibrahim Awad
!   Description:
!*****
!
implicit none

type (FragAtomInfo), dimension(:), allocatable, intent(in) :: DBterm
integer, intent(in) :: loop3 ! +1 for new atoms
type (FragAtomInfo), dimension(:), allocatable, intent(out) :: NewDBterm
integer, intent(out) :: NewLoop ! +1 for new atoms
type (FragAtomInfo), dimension(:), allocatable :: newatoms

double precision :: CH_bond, Anglea
integer :: NNewatoms, iloop

allocate (NewDBterm(Natoms))

NewDBterm=DBterm
NNewatoms=0

!   select case(NewDBterm(2)%Atomic_Number)
!     case(1) ! H atom
!       !!!!!!!!!!!!!!!
!     case(2) ! He atom
!       !!!!!!!!!!!!!!!
!     case(3)
!       !!!!!!!!!!!!!!!
!     case(6) ! C atom
!       !!!!!!!!!!!!!!!
!     case(8) ! O atom
!       !!!!!!!!!!!!!!!
!     CASE DEFAULT
!       NewDBterm(2)%Atomic_Number=TerminalAtom
!       NewDBterm(2)%element=element_symbols(TerminalAtom)
!     end select

NewLoop=loop3
do iloop=1, NNewatoms
  NewLoop=NewLoop+1
  NewDBterm(NewLoop)%x=newatoms(iloop)%x+DBterm(2)%x
  NewDBterm(NewLoop)%y=newatoms(iloop)%y+DBterm(2)%y
  NewDBterm(NewLoop)%z=newatoms(iloop)%z+DBterm(2)%z
  NewDBterm(NewLoop)%Atomic_Number=newatoms(iloop)%Atomic_Number

```



```

        MinZ=MINval(Zlist)
        do Ilist=1,Nlist
            ZI=Zlist(Ilist)
            if(ZI.le.MinZ)then
                Iterm=Iterm+1
                Itype=Itype+(ZI-2)*Iterm
                Zlist(Ilist)=999
            end if
        end do ! Ilist
    end do ! while
    FragAtoms(Jatom,Ivertex)%type=Itype
end do ! Ivertex
deallocate (Zlist)
end do ! Jatom
return
end SUBROUTINE UpdateFragTypes
SUBROUTINE CalcFragConnect(FragAtoms,FragInfo)
!*****
!   Date last modified:
!   Author: Ibrahim Awad
!   Description: Calculate the connectivity matrix for the fragments
!*****
!

    USE type_elements

    implicit none

    type(FragAtomInfo),dimension(:,:), intent(inout), allocatable :: FragAtoms
    type(FragInfo),dimension(:), intent(inout), allocatable :: FragInfo
    integer :: Jatom, IatomN

    ! In future you can USE different type of radii
    call Frag_CONN (BS_RADII, NELEMENTS, BS_scalef, FragAtoms, FragInfo)
    return
end SUBROUTINE CalcFragConnect
SUBROUTINE Frag_CONN (RADII, & ! Bragg-Slater atomic radii
    ELMLEN, & ! Length of data array RADII
    FUDGE, & ! Fudge factor for Connectivity determination
    FragAtoms, &
    FragInfo)
!*****
!   Date last modified:
!   Author: Ibrahim Awad
!   Description: Calculate the connectivity matrix for the fragments
!               using Bragg-Slater atomic radii
!*****
!

    implicit none

    type (FragAtomInfo), dimension(:,:), intent(inout), allocatable :: FragAtoms
    type (FragInfo), dimension(:), intent(inout), allocatable :: FragInfo

    integer, intent(in) :: ELMLEN
    double precision, intent(in) :: RADII(ELMLEN)
    double precision, intent(in) :: FUDGE

    integer :: Iatom, Jatom, Matom, IIAN, JIAN, IatomN
    double precision :: DisSep, FUDGE_save

    do Jatom=1, Natoms
        if (CARTESIAN(Jatom)%Atomic_Number.le.0) cycle
        IatomN=FragInfo(Jatom)%Natoms

```







```

type (FragInfo), dimension(:), intent(out), allocatable :: FragInfo

integer :: Iatom
integer :: Jatom
integer :: ifound
integer :: ilevel
type (TwoDiArrInt), dimension(:), allocatable :: LevelConnect !
logical, dimension(:), allocatable :: found

allocate (FragAtoms(Natoms,Natoms))
allocate (FragInfo(Natoms))
allocate (LevelConnect(level))
allocate (found(Natoms))

CALL GetLevelConnect(level,LevelConnect)

do Jatom=1, Natoms
  if (Cartesian(JAtom)%Atomic_Number.eq.0) cycle
  FragInfo(Jatom)%NAtoms=1
  FragInfo(Jatom)%NumNeighbourAtoms=0

  ! Defined the main atom (Jatom) element_symbols
  FragAtoms(Jatom,1)%level=0
  FragAtoms(Jatom,1)%MUNIdx=Jatom ! the index of atom in MUNgauss
  FragAtoms(Jatom,1)%Atomic_Number=Cartesian(Jatom)%Atomic_Number
  FragAtoms(Jatom,1)%x=Cartesian(Jatom)%x
  FragAtoms(Jatom,1)%y=Cartesian(Jatom)%y
  FragAtoms(Jatom,1)%z=Cartesian(Jatom)%z
  FragAtoms(Jatom,1)%element=&
    element_symbols(Cartesian(Jatom)%Atomic_Number)
  FragAtoms(Jatom,1)%ctype=""
  FragAtoms(Jatom,1)%sorted= .true.

  ifound=1 ! index for each sub-atom found for Jatom

do Iatom=1, Natoms
  if (Cartesian(IAtom)%Atomic_Number.eq.0) cycle
  found=.false.
  do ilevel=1, level
    if ((LevelConnect(ilevel)%ijarray(Iatom,Jatom).ne.1).or.&
      (found(Iatom)).or.Iatom.eq.Jatom) cycle
    ifound=ifound+1
    found(Iatom)=.true.
    FragInfo(Jatom)%NAtoms=FragInfo(Jatom)%NAtoms+1
    if (ilevel.eq.1) FragInfo(Jatom)%NumNeighbourAtoms=&
      FragInfo(Jatom)%NumNeighbourAtoms+1
    FragAtoms(Jatom,ifound)%level=ilevel
    FragAtoms(Jatom,ifound)%MUNIdx=Iatom ! MUNgauss index
    FragAtoms(Jatom,ifound)%Atomic_Number=&
      Cartesian(IAtom)%Atomic_Number
    FragAtoms(Jatom,ifound)%x=Cartesian(IAtom)%x
    FragAtoms(Jatom,ifound)%y=Cartesian(IAtom)%y
    FragAtoms(Jatom,ifound)%z=Cartesian(IAtom)%z
    FragAtoms(Jatom,ifound)%element=&
      element_symbols(Cartesian(Iatom)%Atomic_Number)
    FragAtoms(Jatom,ifound)%ctype=""
    FragAtoms(Jatom,ifound)%sorted= .false.
  end do ! ilevel
end do ! Iatom
end do ! Jatom

CALL CalcFragConnect(FragAtoms,FragInfo)
CALL UpdateFragTypes(FragAtoms,FragInfo)

```



```

implicit none

integer, intent(in) :: level, Jatom
integer, dimension(:), allocatable, intent(out) :: atomAdj
integer :: Iatom, ilevel
type (TwoDiArrInt), dimension(:), allocatable :: LevelConnect !
logical, dimension(:), allocatable :: found

allocate (LevelConnect(level))
allocate (found(Natoms))
allocate (atomAdj(Natoms))

CALL GetLevelConnect(level,LevelConnect)
atomAdj=0
do Iatom=1, Natoms
  if (Cartesian(IAtom)%Atomic_Number.eq.0) cycle
  found=.false.
  do ilevel=1, level
    if ((LevelConnect(ilevel)%ijarray(Iatom,Jatom).ne.1).or.&
      (found(Iatom)).or.Iatom.eq.Jatom) cycle
    found(Iatom)=.true.
    atomAdj(Iatom)=ilevel
  end do ! ilevel
end do ! Iatom

deallocate(LevelConnect, found)
return
end SUBROUTINE FindAdj

SUBROUTINE GetLevelConnect(level,LevCon)
!*****
! Date last modified: *
! Author: Ibrahim Awad *
! Description: Calculate the connectivity matrix for the molecule at *
! different levels. *
!*****
!
USE GRAPH_objects
USE matrix_print

implicit none

integer, intent(in) :: level
type (TwoDiArrInt), dimension(:), intent(out), allocatable :: LevCon
integer :: ilevel

allocate(LevCon(level))
do ilevel=1, level
  allocate(LevCon(ilevel)%ijarray(Natoms,Natoms))
end do

CALL GET_object ('MOL', 'GRAPH', 'CONNECT')
LevCon(1)%ijarray=CONNECT ! adjacency matrices with level order
! call PRT_matrix(CONNECT,Natoms,Natoms)
do ilevel=2, level
  LevCon(ilevel)%ijarray=MATMUL(LevCon(ilevel-1)%ijarray,CONNECT)
end do
end SUBROUTINE GetLevelConnect

SUBROUTINE CalAtomicEle(grid_points,NApts_atom,rho_Atom,Bweights,ElemNum)
!*****
! Date last modified: *
! Author: Ibrahim Awad *
! Description: Calculate Number of electrons per Iatom *
!*****

```

```

! MODULEs:
  implicit none

  integer, INTENT(IN) :: NApts_atom
  type(type_grid_points),dimension(NApts_atom), INTENT(IN) :: grid_points
  double precision, dimension(NApts_atom), INTENT(IN) :: rho_Atom, Bweights
  double precision :: point_charge
  integer :: IApoint
  double precision, INTENT(OUT) :: EleNum

  EleNum=0.0d0
  do IApoint=1, NApts_atom
    point_charge=rho_Atom(IApoint)*grid_points(IApoint)%w*Bweights(IApoint)
    EleNum = EleNum+point_charge
  end do
  EleNum = FourPi*EleNum
  return
end SUBROUTINE CalAtomicEle
SUBROUTINE CalcVeeSelfA(grid_points,NApts_atom,rho_Atom,Bweights,Vee_A)
! *****
!   Date last modified: *
!   Author: Ibrahim Awad *
!   Description: Calculate Number of electrons per Iatom *
! *****
! MODULEs:
  implicit none

  integer, INTENT(IN) :: NApts_atom
  type(type_grid_points),dimension(NApts_atom), INTENT(IN) :: grid_points
  double precision, dimension(NApts_atom), INTENT(IN) :: rho_Atom, Bweights
  double precision :: pointI, pointJ, R_IJ
  integer :: IApoint, JApoint
  double precision, INTENT(OUT) :: Vee_A

  Vee_A=0.0d0
  do IApoint=1, NApts_atom
    do JApoint=IApoint+1, NApts_atom
      pointI=rho_Atom(IApoint)*grid_points(IApoint)%w*Bweights(IApoint)
      pointJ=rho_Atom(JApoint)*grid_points(JApoint)%w*Bweights(JApoint)
      R_IJ=DSQRT((grid_points(JApoint)%x-grid_points(IApoint)%x)**2+ &
        (grid_points(JApoint)%y-grid_points(IApoint)%y)**2+ &
        (grid_points(JApoint)%z-grid_points(IApoint)%z)**2)
      Vee_A = Vee_A + pointI*pointJ/R_IJ
    end do
  end do
  Vee_A = FourPi*FourPi*Vee_A
  return
end SUBROUTINE CalcVeeSelfA
subroutine get_Vne_Atom_Analytical(EV)
! *****
!   Date last modified: May 21, 2012 Version 1.0 *
!   Author: R.A. Poirier & Jessica Besaw *
!   Description: Calculate the atomic potential integrals (VAint) and *
!               the total potential integrals (VINT) in order to *
!               determine the atomic potential energies (EV) *
!               and the total potential energy (EVtotal) *
! *****
! Modules:
  USE program_constants
  USE type_molecule
  USE type_basis_set
  USE QM_defaults
  USE QM_objects
  USE INT_objects

```

```

USE matrix_print

implicit none

! Local scalars:
integer :: Ishell, Jshell, Ifirst, Jfirst, Ilast, Jlast
integer :: Istart, Jstart, Iend, Jend
integer :: Iatmshl, Jatmshl
integer :: LAMAX, LBMAX
integer :: Iatom
integer :: Irange, Jrange
integer :: Igauss, Jgauss
integer :: Iaos, JaoS
integer :: IGBGN, JGBGN, IGend, JGend
integer :: LPMAX, I, IA, INTC, IX, IY, IZ, IZERO, J, JX, JY, JZ, LIM1DS, NZERO
integer :: LENTQ
double precision ABX, ABY, ABZ, ARABSQ, ARG, AS, ASXA, ASYA, ASZA, &
BS, COEF, CUT1, EP, EPI, EPI02, PCX, PCY, PCZ, PEXP, &
PX, PY, PZ, RPCSQ, TWOASQ, TWOP, TWOPI, TWOPT2, XAP, XBP, &
YAP, YBP, ZAP, ZBP, ZCONST, ZT, ZTEMP
double precision :: XA, XB, XC, YA, YB, YC, ZA, ZB, ZC, RABSQ, TRACLO

! Local arrays:
double precision, dimension(:), allocatable :: VAint
double precision, INTENT(OUT) :: EV
integer INDIX(20), INDIY(20), INDIZ(20), INDJX(20), INDJY(20), INDJZ(20)
double precision :: TP(7), WP(7)
double precision A(45), CA(20), CB(20), CCX(192), CCY(192), CCZ(192), &
EEPB(100), EEPV(100), STWOCX(9), &
TWOCX(9), TWOCY(9), TWOCZ(9), XIP(80), YIP(80), ZIP(80)

parameter (TWOPI=TWO*PI_VAL, CUT1=-75.0D0)

! Data related to Rys Polynomials
DATA INDJX/1,2,1,1,3,1,1,2,2,1,4,1,1,2,3,3,2,1,1,2/, &
INDJY/1,1,2,1,1,3,1,2,1,2,1,4,1,3,2,1,1,2,3,2/, &
INDJZ/1,1,1,2,1,1,3,1,2,2,1,1,4,1,1,2,3,3,2,2/

! Obtaining the density matrix using HF theory
call GET_object ('QM', 'DENSITY_MATRIX', Wavefunction)

! Object:
if(.not.allocated(VINT))then
allocate (VAint((Basis%Nbasis*(Basis%Nbasis+1))/2))
allocate (VINT((Basis%Nbasis*(Basis%Nbasis+1))/2))
else
if(Basis%Nbasis.ne.size(VINT,1))then
deallocate (VINT)
allocate (VAint((Basis%Nbasis*(Basis%Nbasis+1))/2))
allocate (VINT((Basis%Nbasis*(Basis%Nbasis+1))/2))
end if
end if

! Initialize Sum to Zero
VINT=0.

call RYSSET

! Fill INDIX. (Related to Rys polynomials)
do I=1,20
INDIX(I)=4*(INDJX(I)-1)
INDIY(I)=4*(INDJY(I)-1)
INDIZ(I)=4*(INDJZ(I)-1)

```

```

        end do ! I

! Loop over atoms.
    Iatom=1

    IA=CARTESIAN(Iatom)%Atomic_number

! Perform calculatation on atoms, and not dummy atoms.
    IF(IA.GT.O)then
        XC=CARTESIAN(Iatom)%X
        YC=CARTESIAN(Iatom)%Y
        ZC=CARTESIAN(Iatom)%Z

! Loop over shells.
! Loop over Ishell.
        do Ishell=1,Basis%Nshells
            LAMAX=Basis%shell(Ishell)%Xtype+1
            Istart=Basis%shell(Ishell)%Xstart
            Iend=Basis%shell(Ishell)%Xend
            Irange=Iend-Istart+1
            IGBGN=Basis%shell(Ishell)%EXPBGN
            IGend=Basis%shell(Ishell)%EXPend
            Ifrst=Basis%shell(Ishell)%frstSHL

! Loop over Jshell.
            do Jshell=1,Ishell
                LBMAX=Basis%shell(Jshell)%Xtype+1
                Jstart=Basis%shell(Jshell)%Xstart
                Jend=Basis%shell(Jshell)%Xend
                Jrange=Jend-Jstart+1
                JGBGN=Basis%shell(Jshell)%EXPBGN
                JGend=Basis%shell(Jshell)%EXPend
                Jfrst=Basis%shell(Jshell)%frstSHL

                Ilast= Basis%shell(Ishell)%lastSHL
                Jlast= Basis%shell(Jshell)%lastSHL

! Loop over Iatmshl
                do Iatmshl=Ifrst,Ilast
                    XA=CARTESIAN(Basis%atmshl(Iatmshl)%ATMLST)%X
                    YA=CARTESIAN(Basis%atmshl(Iatmshl)%ATMLST)%Y
                    ZA=CARTESIAN(Basis%atmshl(Iatmshl)%ATMLST)%Z
                    Iaos=Basis%atmshl(Iatmshl)%frstAO-1

! Loop over Jatmshl
                    IF(Ishell.EQ.Jshell)Jlast=Iatmshl

                    do Jatmshl=Jfrst,Jlast
                        XB=CARTESIAN(Basis%atmshl(Jatmshl)%ATMLST)%X
                        YB=CARTESIAN(Basis%atmshl(Jatmshl)%ATMLST)%Y
                        ZB=CARTESIAN(Basis%atmshl(Jatmshl)%ATMLST)%Z
                        Jaos=Basis%atmshl(Jatmshl)%frstAO-1

                        LPMAX=LAMAX+LBMAX-1
                        LIM1DS=(LPMAX+3)/2
                        LENTQ=Irange*Jrange
                        NZERO=(LAMAX+LBMAX-2)/2+1
                        ABX=XB-XA
                        ABY=YB-YA
                        ABZ=ZB-ZA
                        RABSQ=ABX*ABX+ABY*ABY+ABZ*ABZ

! Zero accumulation area
                        do I=1,LENTQ

```

```

        EEPV(I)=ZERO
    end do ! I

! Loop over primitive Gaussians.

    do Igauss=IGBGN,IGend
        AS=Basis%gaussian(Igauss)%exp
        TWOASQ=TWO*AS*AS
        ASXA=AS*XA
        ASYA=AS*YA
        ASZA=AS*ZA
        ARABSQ=AS*RABSQ

    call FILLCC (LAMAX, Basis%gaussian(Igauss)%CONTRC, CA)

    do Jgauss=JGBGN,JGend
        BS=Basis%gaussian(Jgauss)%exp
        call FILLCC (LBMAX, Basis%gaussian(Jgauss)%CONTRC, CB)
        EP=AS+BS
        EPI=ONE/EP
        EPI02=PT5*EPI
        TWOP=EP+EP
        ARG=-BS*ARABSQ*EPI
        PEXP=ZERO
        IF(ARG.GT.CUT1)PEXP=DEXP(ARG)
        ZTEMP=TWOPI*EPI*PEXP
        PX=(ASXA+BS*XB)*EPI
        PY=(ASYA+BS*YB)*EPI
        PZ=(ASZA+BS*ZB)*EPI
        XAP=PX-XA
        XBP=PX-XB
        YAP=PY-YA
        YBP=PY-YB
        ZAP=PZ-ZA
        ZBP=PZ-ZB

    call GETCC1 (CCX, XAP, XBP, LAMAX+2, LBMAX+2)
    call GETCC1 (CCY, YAP, YBP, LAMAX+2, LBMAX+2)
    call GETCC1 (CCZ, ZAP, ZBP, LAMAX+2, LBMAX+2)

! Zero accumulation area.
! (I want to just say EEBV = 0., or is loop more cost effective?)
    do I=1,LENTQ
        EEPB(I)=ZERO
    end do ! I

! Calculate information about atom

    ZT=ZTEMP*DBLE(IA)
    PCX=XC-PX
    PCY=YC-PY
    PCZ=ZC-PZ
    RPCSQ=PCX*PCX+PCY*PCY+PCZ*PCZ
    ARG=EP*RPCSQ

    call RPOLX (NZERO, ARG, TP, WP)
    call GETA1 (A, EPI02, 0, LPMAX)

! Loop over zeroes of Rys polynomial.
    do IZERO=1,NZERO
        TWOPT2=TWOP*TP(IZERO)
        ZCONST=ZT*WP(IZERO)
        call GET2C (TWO CX, PCX, ONE, A, TWOPT2, 0, LPMAX)
        call GET2C (TWO CY, PCY, ONE, A, TWOPT2, 0, LPMAX)
    end do

```



```

      call GET2C (TWOCZ, PCZ, ZCONST, A, TWOPT2, 0, LPMAX)
      call GET3C (XIP, 0, TWOCX, CCX, LAMAX, LBMAX)
      call GET3C (YIP, 0, TWOCY, CCY, LAMAX, LBMAX)
      call GET3C (ZIP, 0, TWOCZ, CCZ, LAMAX, LBMAX)

! Loop over atomic orbitals for potential energy integrals,

      INTC=0
      do I=Istart,Iend
        IX=INDIX(I)
        IY=INDIY(I)
        IZ=INDIZ(I)

        do J=Jstart,Jend
          JX=INDJX(J)
          JY=INDJY(J)
          JZ=INDJZ(J)
          INTC=INTC+1
          EEPB(INTC)=EEPB(INTC)+XIP(IX+JX)*YIP(IY+JY)*ZIP(IZ+JZ)
        end do ! J
      end do ! I

! End of A0 loop.

      end do ! IZERO

! End of loop over Rys zeroes.

! Begin loop over atomic orbitals for potential energy integrals
! and apply the contraction coefficients.
! The potential integrals are already in EEPB.

      INTC=0
      do I=Istart,Iend
        do J=Jstart,Jend
          INTC=INTC+1
          COEF=CA(I)*CB(J)
          EEPV(INTC)=EEPV(INTC)-EEPB(INTC)*COEF
        end do ! J
      end do ! I

! End of loop over atomic orbitals

      end do ! Jgauss
      end do ! Igauss

! End of loop over Gaussians.

! FILMAT takes the atomic integrals in EEPB and stores them in
! their proper places in VAINT.

      call FILMAT (EEPV, VAINT, MATlen, Iend, Jend, &
                  Iatmshl, Jatmshl, Irange, Jrange, Iaos, JaoS)

      end do ! Jatmshl
      end do ! Iatmshl
      end do ! Jshell
      end do ! Ishell

! End of loop over shells.

! Calculate the atomic potential energies

```



```

T(j)=T(j)/n
end do
return
end SUBROUTINE GetCentroidCart
SUBROUTINE GetRotMatrix (NewCart,CurrCart,numberpoints,R,T)
!*****
!   Date last modified:
!   Author: Ibrahim Awad
!   Description: Return the rotation and translation matrices for a set of
!               points using least square method for fitting (SVD) .
!*****
!

implicit none

double precision, dimension(:,,:), intent(in), allocatable :: NewCart
double precision, dimension(:,,:), intent(in), allocatable :: CurrCart
integer, intent(in) :: numberpoints

double precision, dimension (3,3), intent(out) :: R
double precision, dimension (3), intent(out) :: T

double precision, dimension (:), allocatable :: Aver_A, Aver_B
double precision, dimension (:,:), allocatable :: H_a, H_b ! large arrays

double precision, dimension (1:3) :: temp_Ha, temp_Hb, SSS
double precision, dimension (1:3,1:3) :: H_matrix,HH,AAA,UUU,VVV,VT,UT

integer :: i,j,k,l

allocate (H_a(1:numberpoints,1:3))
allocate (H_b(1:numberpoints,1:3))
allocate (Aver_A(1:3))
allocate (Aver_B(1:3))

! calculate average of matrix A and matrix B
call GetCentroidCart (NewCart, numberpoints, Aver_A)
call GetCentroidCart (CurrCart, numberpoints, Aver_B)

! calculate H matrix
H_matrix=0.0d0; H_a=0.0d0 ; H_b=0.0d0;
do i=1, numberpoints
  do j=1,3
    H_a(i,j)=NewCart(i,j)-Aver_A(j) !H_a(i,j)=A(i,j)-A(1,j)
    H_b(i,j)=CurrCart(i,j)-Aver_B(j) !H_b(i,j)=B(i,j)-B(1,j)
  end do
end do
do i=1, numberpoints
  temp_Ha=0.0d0; temp_Hb=0.0d0
  do j=1,3
    temp_Ha(j)=H_a(i,j)
    temp_Hb(j)=H_b(i,j)
  end do
  HH=0.0d0
  do k=1,3
    do l=1,3
      HH(k,l)=temp_Ha(k)*temp_Hb(l)
    end do
  end do
  H_matrix=H_matrix+ HH
end do
CALL SVD(H_matrix,UUU,SSS,VVV,3,3) !Calculate SVD
VT=transpose(VVV)
UT=transpose(UUU)

```

```

R=matmul(VVV,UT) !Calculate Rotation matrix
T=matmul(-R,Aver_A)+Aver_B
deallocate(H_a, H_b, Aver_A, Aver_B)
return
end SUBROUTINE GetRotMatrix
SUBROUTINE DoRotation (OldCart,NewCart,numberpoints,R,T)
!*****
! Date last modified: *
! Author: Ibrahim Awad *
! Description: Get the new coordinate after applying the rotation and *
! the translation matrices. *
!*****
!
implicit none

integer, intent(in) :: numberpoints
double precision, dimension (:,:), intent(in), allocatable :: OldCart
double precision, dimension (3,3), intent(in) :: R
double precision, dimension (3), intent(in) :: T
double precision, dimension (:,:), intent(out), allocatable :: NewCart

double precision, dimension (1:3) :: NewPoint, point
integer::i

allocate(NewCart(1:numberpoints,3))

do i=1, numberpoints
point=(/OldCart(i,1),OldCart(i,2),OldCart(i,3)/)
NewPoint= Matmul(R,point) + T
NewCart(i,1)=NewPoint(1) ! X points
NewCart(i,2)=NewPoint(2) ! Y points
NewCart(i,3)=NewPoint(3) ! Z points
end do
return
end SUBROUTINE DoRotation
SUBROUTINE GetOldPoints (OldCart,NewCart,numberpoints,R,T)
!*****
! Date last modified: *
! Author: Ibrahim Awad *
! Description: Get the old coordinate before applying the rotation and *
! the translation matrices. *
!*****
!
USE mod_math

implicit none

double precision, dimension (:,:), intent(out), allocatable :: OldCart
double precision, dimension (3,3), intent(in) :: R
double precision, dimension (3), intent(in) :: T
double precision, dimension (:,:), intent(in), allocatable :: NewCart
integer, intent(in) :: numberpoints

double precision, dimension (:,:), allocatable :: RI
double precision, dimension (1:3) :: NewPoint, point
integer::i

allocate(OldCart(1:numberpoints,3))
allocate(RI(3,3))

call INV33(R,RI)
do i=1, numberpoints
point=(/NewCart(i,1),NewCart(i,2),NewCart(i,3)/)
NewPoint=Matmul(RI,(point-T))

```

```

        OldCart(i,1)=NewPoint(1)
        OldCart(i,2)=NewPoint(2)
        OldCart(i,3)=NewPoint(3)
    end do
    return
end SUBROUTINE GetOldPoints
SUBROUTINE GetStoredRotMatrix (OldCart,numberpoints,R,T)
! *****
! Date last modified: *
! Author: Ibrahim Awad *
! Description: Return the rotation and translation matrices that needed *
! to get the standard form. *
! *****
!
    implicit none

    double precision, dimension (:,:), intent(in), allocatable :: OldCart
    double precision, dimension (3,3), intent(out) :: R
    double precision, dimension (3), intent(out) :: T
    integer, intent(in) :: numberpoints

    double precision, dimension (:,:), allocatable :: Tempcart

    double precision :: AB_side,AC_side,BC_side
    double precision :: AD_side,BD_side,S_ABC,S_ABD
    double precision :: CD_side,CG_side,DG_side,Y_along4C
    double precision :: Y_along3,Z_along3,signang_ABC,signang_ABD
    double precision :: Y_alongC4,Z_alongC4,Y_along4,Z_along4,EG_side
    double precision :: ABS_ABC,BCS_ABC,ACS_ABC,ABS_ABD,BDS_ABD
    double precision :: ADS_ABD,ss1,ss2,ss3,dd,X4,X4_1,X4_2,DG_sideC,CD_sideC
    integer::I,J,signa_ABC,signa_ABD,numberw,signa_CDG,signang_CDG

    allocate (Tempcart(1:numberpoints,1:3))

    numberw=1
    TempCart=0.0d0
    SELECT CASE (numberpoints)
    CASE (1)
        TempCart(1,1:3)=0.0d0
        numberw=1
    CASE (2)
        numberw=2
        TempCart(1:2,1:3)=0.0d0
        TempCart(2,3)=DSQRT((OldCart(2,1)-OldCart(1,1))**2+(OldCart(2,2)-&
            OldCart(1,2))**2+(OldCart(2,3)-OldCart(1,3))**2)
        !TempCart(3,1)=0.0d0;TempCart(3,2)=0.0d0;TempCart(3,3)=0.0
    CASE (3:)
        numberw=3
        ! ABC triangle
        AB_side=DSQRT((OldCart(2,1)-OldCart(1,1))**2+(OldCart(2,2)-&
            OldCart(1,2))**2+(OldCart(2,3)-OldCart(1,3))**2)
        AC_side=DSQRT((OldCart(3,1)-OldCart(1,1))**2+(OldCart(3,2)-&
            OldCart(1,2))**2+(OldCart(3,3)-OldCart(1,3))**2)
        BC_side=DSQRT((OldCart(2,1)-OldCart(3,1))**2+(OldCart(2,2)-&
            OldCart(3,2))**2+(OldCart(2,3)-OldCart(3,3))**2)
        S_ABC = (AB_side+BC_side+AC_side)/2.0
        ABS_ABC=S_ABC-AB_side
        ACS_ABC=S_ABC-AC_side
        BCS_ABC=S_ABC-BC_side
        signang_ABC=AB_side**2+AC_side**2-BC_side**2
        signa_ABC=1
        if (signang_ABC.le.0) signa_ABC=-1
        Y_along3=DSQRT((4.0*S_ABC*ABS_ABC*ACS_ABC*BCS_ABC)/(AB_side**2))
        Z_along3=DSQRT(AC_side**2-Y_along3**2)*signa_ABC

```

```

TempCart=0.0d0
TempCart(2,3)=AB_side ! Z-axes for second point
TempCart(3,2)=Y_along3
TempCart(3,3)=Z_along3
if (numberpoints.ne.3) then
  numberw=4
  ! ABD triangle
  AD_side=DSQRT((OldCart(4,1)-OldCart(1,1))**2+(OldCart(4,2)-&
    OldCart(1,2))**2+(OldCart(4,3)-OldCart(1,3))**2)
  BD_side=DSQRT((OldCart(2,1)-OldCart(4,1))**2+(OldCart(2,2)-&
    OldCart(4,2))**2+(OldCart(2,3)-OldCart(4,3))**2)
  S_ABD = (AB_side+BD_side+AD_side)/2.0
  ABS_ABD=S_ABD-AB_side
  ADS_ABD=S_ABD-AD_side
  BDS_ABD=S_ABD-BD_side
  signang_ABD=AB_side**2+AD_side**2-BD_side**2
  signa_ABD=1
  if (signang_ABD.le.0) signa_ABD=-1
  ! perpendicular of triangle ABD
  Y_alongC4=DSQRT((4.0*S_ABD*ABS_ABD*ADS_ABD*BDS_ABD)/(AB_side**2))
  Z_along4=DSQRT(AD_side**2-Y_alongC4**2) *signa_ABD ! Z axes >>>OK<<<
  ! equation of plane ss1.X+ss2.Y+ss3.Z+dd=0 >>>
  ! >> Using Vectors to Describe a Plane
  ss1=(OldCart(2,2)-OldCart(1,2))*(OldCart(3,3)-OldCart(1,3))-&
    (OldCart(2,3)-OldCart(1,3))*(OldCart(3,2)-OldCart(1,2))
  ss2=(OldCart(2,3)-OldCart(1,3))*(OldCart(3,1)-OldCart(1,1))-&
    (OldCart(2,1)-OldCart(1,1))*(OldCart(3,3)-OldCart(1,3))
  ss3=(OldCart(2,1)-OldCart(1,1))*(OldCart(3,2)-OldCart(1,2))-&
    (OldCart(2,2)-OldCart(1,2))*(OldCart(3,1)-OldCart(1,1))
  dd=-(ss1*OldCart(1,1)+ss2*OldCart(1,2)+ss3*OldCart(1,3))
  ! Distance from a point to a plane
  X4_1=(ss1*OldCart(4,1)+ss2*OldCart(4,2)+ss3*OldCart(4,3)+dd)
  X4_2=DSQRT(ss1**2+ss2**2+ss3**2)
  if (abs(X4_2).lt.1.0d-06) then
    X4=0.0d0
  else
    X4=X4_1/X4_2
  end if
  !Sign Y4 ?????? CDG triangle
  CD_sideC=DSQRT((OldCart(3,1)-OldCart(4,1))**2+(OldCart(3,2)-&
    OldCart(4,2))**2+(OldCart(3,3)-OldCart(4,3))**2)
  CD_side=DSQRT(CD_sideC**2-X4**2)
  CG_side=Y_along3
  !>>>>>>>By USing the new coordinate Just Y and Z
  EG_side=Z_along3-Z_along4
  Y_along4C=DSQRT(Y_alongC4**2-X4**2) ! without sign
  DG_side=DSQRT(Y_along4C**2+EG_side**2)
  !DG_side=DSQRT((0-OldCart(4,1))**2+(0-OldCart(4,2))**2+(DG_sideC)**2)

  signang_CDG=CG_side**2+DG_side**2-CD_side**2
  signa_CDG=1
  if (signang_CDG.le.0) signa_CDG=-1
  Y_along4=Y_along4C*signa_CDG
  X4=abs(X4) ! we want X alwayes positive
  TempCart(4,1)=X4 ! Z-axes for second point
  TempCart(4,2)=Y_along4
  TempCart(4,3)=Z_along4
end if
end SELECT
call GetRotMatrix (OldCart,TempCart,numberw,R,T)
deallocate(TempCart)
return
end SUBROUTINE GetStoredRotMatrix
end MODULE mod_rotation

```



```

double precision, dimension (3), intent(out) :: TT

integer, intent(in) :: pointnumber

double precision, dimension (:,:), allocatable :: CurrCart, NewCart
integer :: ifound, pointnumber2

allocate (CurrCart(pointnumber,1:3))
allocate (NewCart(pointnumber,1:3))

do ifound=1, pointnumber !loop over cart atoms
  CurrCart(ifound,1)=OldCarts(ifound)%x
  CurrCart(ifound,2)=OldCarts(ifound)%y
  CurrCart(ifound,3)=OldCarts(ifound)%z

  NewCart(ifound,1)=NewCarts(ifound)%x
  NewCart(ifound,2)=NewCarts(ifound)%y
  NewCart(ifound,3)=NewCarts(ifound)%z
end do
pointnumber2=pointnumber
if (pointnumber.gt.3) pointnumber2=4
call GetRotMatrix (CurrCart,NewCart,pointnumber2,RR,TT)

deallocate(CurrCart, NewCart)
return
end SUBROUTINE GetCartRotMatrix
SUBROUTINE GetCartRotMatrixW(OldCarts,NewCarts,pointnumber,RR,TT)
!*****
! Date last modified: *
! Author: Ibrahim Awad *
! Description: iGet the the least square method rotation and translation *
! matrices after applying weights *
!*****
!

implicit none

type (FragAtomInfo), dimension(:), intent(in), allocatable :: OldCarts
type (FragAtomInfo), dimension(:), intent(in), allocatable :: NewCarts
double precision, dimension (3,3), intent(out) :: RR
double precision, dimension (3), intent(out) :: TT

integer, intent(in) :: pointnumber

double precision, dimension (:,:), allocatable :: CurrCart, NewCart
integer :: ifound,pointnumber2
integer, parameter :: weight0=2, weight1=1, weight2=0
integer :: Newpointnumber, Newifound, countfound

! calculate number of points
Newpointnumber=0
do ifound=1, pointnumber !loop over cart atoms
  if (OldCarts(ifound)%level.eq.0) Newpointnumber=2000
  if (OldCarts(ifound)%level.eq.1) Newpointnumber=Newpointnumber+&
    OldCarts(ifound)%Atomic_Number**(weight1+2)
  if (OldCarts(ifound)%level.eq.2) Newpointnumber=&
    Newpointnumber+OldCarts(ifound)%Atomic_Number**(weight2+2)
  ! if (OldCarts(ifound)%level.gt.2) Newpointnumber=&
  ! Newpointnumber+OldCarts(ifound)%Atomic_Number
end do
allocate (CurrCart(Newpointnumber,1:3))
allocate (NewCart(Newpointnumber,1:3))
countfound=0
do ifound=1, pointnumber !loop over cart atoms
  if (OldCarts(ifound)%level.eq.0) then

```



```

do Newifound=1, 2000
  countfound=countfound+1
  CurrCart(countfound,1)=OldCarts(ifound)%x
  CurrCart(countfound,2)=OldCarts(ifound)%y
  CurrCart(countfound,3)=OldCarts(ifound)%z

  NewCart(countfound,1)=NewCarts(ifound)%x
  NewCart(countfound,2)=NewCarts(ifound)%y
  NewCart(countfound,3)=NewCarts(ifound)%z
end do
end if
if (OldCarts(ifound)%level.eq.1) then
  do Newifound=1, OldCarts(ifound)%Atomic_Number**(weight1+2)
    countfound=countfound+1
    CurrCart(countfound,1)=OldCarts(ifound)%x
    CurrCart(countfound,2)=OldCarts(ifound)%y
    CurrCart(countfound,3)=OldCarts(ifound)%z

    NewCart(countfound,1)=NewCarts(ifound)%x
    NewCart(countfound,2)=NewCarts(ifound)%y
    NewCart(countfound,3)=NewCarts(ifound)%z
  end do
end if
if (OldCarts(ifound)%level.eq.2) then

  do Newifound=1, OldCarts(ifound)%Atomic_Number**(weight2+2)
    countfound=countfound+1
    CurrCart(countfound,1)=OldCarts(ifound)%x
    CurrCart(countfound,2)=OldCarts(ifound)%y
    CurrCart(countfound,3)=OldCarts(ifound)%z

    NewCart(countfound,1)=NewCarts(ifound)%x
    NewCart(countfound,2)=NewCarts(ifound)%y
    NewCart(countfound,3)=NewCarts(ifound)%z
  end do
end if
! if (OldCarts(ifound)%level.gt.2) then
!   do Newifound=1, OldCarts(ifound)%Atomic_Number**2
!     countfound=countfound+1
!     CurrCart(countfound,1)=OldCarts(ifound)%x
!     CurrCart(countfound,2)=OldCarts(ifound)%y
!     CurrCart(countfound,3)=OldCarts(ifound)%z
!
!     NewCart(countfound,1)=NewCarts(ifound)%x
!     NewCart(countfound,2)=NewCarts(ifound)%y
!     NewCart(countfound,3)=NewCarts(ifound)%z
!   end do
! end if
end do
call GetRotMatrix (NewCart,CurrCart,countfound,RR,TT)
deallocate(CurrCart, NewCart)
return
end SUBROUTINE GetCartRotMatrixW
SUBROUTINE GetDBCart(OldCarts,NewCarts,pointnumber,RR,TT)
! *****
!   Date last modified: *
!   Author: Ibrahim Awad *
!   Description: Get the database coordinates and the rotation and the *
!               translation matrices *
! *****
!
implicit none

type (FragAtomInfo), dimension(:), intent(in), allocatable :: OldCarts

```

```

type (FragAtomInfo), dimension(:), intent(out), allocatable :: NewCarts
double precision, dimension (3,3), intent(out) :: RR
double precision, dimension (3), intent(out) :: TT

integer, intent(in) :: pointnumber

double precision, dimension (:,:), allocatable :: CurrCart, NewCart
integer :: ifound

allocate (NewCarts(pointnumber))
allocate (CurrCart(pointnumber,1:3))
allocate (NewCart(pointnumber,1:3))

NewCarts=OldCarts

do ifound=1, pointnumber !loop over cart atoms
    CurrCart(ifound,1)=OldCarts(ifound)%x
    CurrCart(ifound,2)=OldCarts(ifound)%y
    CurrCart(ifound,3)=OldCarts(ifound)%z
end do
call GetStoredRotMatrix (CurrCart,pointnumber,RR,TT)
call DoRotation (CurrCart,NewCart,pointnumber,RR,TT)
do ifound=1, pointnumber !loop over cart atoms
    NewCarts(ifound)%x=NewCart(ifound,1)
    NewCarts(ifound)%y=NewCart(ifound,2)
    NewCarts(ifound)%z=NewCart(ifound,3)
end do
deallocate(CurrCart, NewCart)
return
end SUBROUTINE GetDBCart
SUBROUTINE GetDBFormFragments(FragAtoms,NewFragAtoms,FragInfo)
!*****
! Date last modified: *
! Author: Ibrahim Awad *
! Description: Get the database coordinates and the rotation and the *
! translation matrices for the fragments *
!*****
!

implicit none

type(FragAtomInfo),dimension(:,:),intent(in), allocatable :: FragAtoms
type(FragAtomInfo),dimension(:,:),intent(out), allocatable :: NewFragAtoms
type(FragInfo),dimension(:),intent(inout), allocatable :: FragInfo

double precision, dimension (3,3) :: RR
double precision, dimension (3) :: TT

double precision, dimension (:,:), allocatable :: CurrCart, NewCart
integer :: ifound,JAtom,pointnumbers

allocate (NewFragAtoms(Natoms,Natoms))

NewFragAtoms=FragAtoms

do JAtom=1, Natoms !loop over all atoms
    pointnumbers=FragInfo(JAtom)%NAtoms
    allocate (CurrCart(1:pointnumbers,1:3))
    allocate (NewCart(1:pointnumbers,1:3))
    do ifound=1, pointnumbers !loop over cart
        CurrCart(ifound,1)=FragAtoms(Jatom,ifound)%x
        CurrCart(ifound,2)=FragAtoms(Jatom,ifound)%y
        CurrCart(ifound,3)=FragAtoms(Jatom,ifound)%z
    end do
    call GetStoredRotMatrix (CurrCart,pointnumbers,RR,TT)

```

```

FragInfo(Jatom)%RMat=RR
FragInfo(Jatom)%TMat=TT
call DoRotation (CurrCart,NewCart,pointnumbers,RR,TT)
do ifound=1, pointnumbers !loop over cart
  NewFragAtoms(Jatom,ifound)%x=NewCart(ifound,1)
  NewFragAtoms(Jatom,ifound)%y=NewCart(ifound,2)
  NewFragAtoms(Jatom,ifound)%z=NewCart(ifound,3)
end do
deallocate(CurrCart, NewCart)
end do !loop over all atoms
return
end SUBROUTINE GetDBFormFragments
end MODULE rotation_Cart

```

```

SUBROUTINE CalcMoleProperties()
!*****
! Date last modified: *
! Author: Ibrahim Awad *
! Description: *
!*****
! MODULES:

USE AIMDFT_type
USE type_molecule

implicit none

interface
SUBROUTINE Build_Molecule_DB(DatabaseInform,FragProp,FRgrids,FragAtoms)
  use aimdft_type
  implicit none
  type (IndexFile), dimension (:), allocatable :: DatabaseInform
  type (PropFile), dimension (:), allocatable :: FragProp
  type (GridsDataBase), dimension (:,:), allocatable :: FRgrids
  type (FragAtomInfo), dimension (:,:), allocatable :: FragAtoms
end SUBROUTINE Build_Molecule_DB
SUBROUTINE Build_Molecule_Direct(DatabaseInform,FragProp,FRgrids)
  use aimdft_type
  USE type_molecule
  implicit none
  type (IndexFile), dimension (:), allocatable :: DatabaseInform
  type (PropFile), dimension (:), allocatable :: FragProp
  type (GridsDataBase), dimension (:,:), allocatable :: FRgrids
end SUBROUTINE Build_Molecule_Direct
SUBROUTINE CalcMolePropertiesDB(DatabaseInform,FragProp,FRgrids)
  use aimdft_type
  implicit none
  type (IndexFile), dimension (:), allocatable :: DatabaseInform
  type (PropFile), dimension (:), allocatable :: FragProp
  type (GridsDataBase), dimension (:,:), allocatable :: FRgrids
end SUBROUTINE CalcMolePropertiesDB
end interface

type (IndexFile), dimension (:), allocatable :: DatabaseInform
type (PropFile), dimension (:), allocatable :: FragProp
type (GridsDataBase), dimension (:,:), allocatable :: FRgrids
type (FragAtomInfo), dimension (:,:), allocatable :: FragAtoms

allocate (DatabaseInform(Natoms))
allocate (FragProp(Natoms))
allocate (FragAtoms(Natoms,Natoms))
allocate (FRgrids(Natoms,MaxGridPnt))

if(DirectMethod) then

```

```

        call Build_Molecule_Direct(DatabaseInform,FragProp,FRgrids)
    else
        call Build_Molecule_DB(DatabaseInform,FragProp,FRgrids,FragAtoms)
    end if

    call CalcMolePropertiesDB(DatabaseInform,FragProp,FRgrids)
    deallocate (DatabaseInform)
    deallocate (FragProp)
    deallocate (FragAtoms)
    deallocate (FRgrids)
    return
end SUBROUTINE CalcMoleProperties

```

```

SUBROUTINE FragmentsDisplay()
!*****
!   Date last modified:
!   Author: Ibrahim Awad
!   Description: Display the current and database cartesian coordinates
!               for the molecule fragments
!*****
!
    USE Sorted_AIMDFT
    USE mod_math
    USE symbol_AIMDFT

    implicit none

    character (SymMax), dimension (:), allocatable :: symbols
    type (FragInfo), dimension (:), allocatable :: FragInfo
    type (FragAtomInfo), dimension (:,:), allocatable :: FragAtoms, DBFragAtoms

    double precision, dimension (:,:), allocatable :: RR
    double precision, dimension (:), allocatable :: TT
    integer :: Iatom, JAtom, ifound

    !type (TwoDiArrInt), dimension (:), allocatable :: LevelConnect z

    allocate (FragInfo(Natoms))
    allocate (symbols(Natoms))
    allocate (FragAtoms(Natoms,Natoms))
    allocate (DBFragAtoms(Natoms,Natoms))
    allocate (RR(1:3,1:3))
    allocate (TT(1:3))
    !allocate (LevelConnect(Level_Number))

    CALL LOGO()

    !   CALL GetLevelConnect(Level_Number,LevelConnect)
    !   write(*,*) "*****"
    !   write(*,*) "           Adjacency matrix           "
    !   write(*,*) "*****"
    !   do i=1, Level_Number
    !       write(*,*) "for level # ", i
    !       write(*,*) "-----"
    !       call PRT_matrix(LevelConnect(i)%ijarray,Natoms,Natoms)
    !   end do

    CALL GetSortCartTerm (Level_Number,FragAtoms,FragInfo)

    write(*, '(A)') "*****"
    write(*, '(A)') "           Current cartesian coordinates           "
    write(*, '(A)') "*****"
    CALL PRNTCART(FragAtoms,FragInfo)

```

```

CALL GetDBFormFragments(FragAtoms,DBFragAtoms,FragInfo)
write(*,'(A)') "*****"
write(*,'(A)') "      Database cartesian coordinates  "
write(*,'(A)') "*****"
CALL PRNTCART(DBFragAtoms,FragInfo)

CALL GetFragSymbols(Level_Number,symbols)
write(*,'(A)') "*****"
write(*,'(A)') "      Molecule atoms symbols          "
write(*,'(A)') "*****"
do Iatom=1, Natoms
    if (Cartesian(IAtom)%Atomic_Number.eq.0) cycle
    write(*,'(I5,3X,A)') Iatom,trim(symbols(Iatom))
end do

deallocate (FragInfo)
deallocate (symbols)
deallocate (FragAtoms)
deallocate (DBFragAtoms)
deallocate (RR)
deallocate (TT)
return
end SUBROUTINE FragmentsDisplay

```

```

SUBROUTINE Build_FragFiles()
!*****e*****
!      Date last modified: *
!      Author: Ibrahim Awad *
!      Description: *
!*****e*****
! MODULES:

USE module_grid_points
USE NI_defaults
USE type_Weights
USE QM_defaults
USE Sorted_AIMDFT
USE symbol_AIMDFT

implicit none
!
! Local scalars:

type (IndexFile), dimension(:), allocatable :: MolIdxInfo
type (FragInfo), dimension(:), allocatable :: FragInfo
type (FragAtomInfo), dimension(:,:), allocatable :: SortedFragmentAtom
type (FragAtomInfo), dimension(:,:), allocatable :: DBFragmentAtom

character (SymMax), dimension (:), allocatable :: symbols
character (SymMax), dimension (:), allocatable :: UniSym
integer, dimension (:), allocatable :: MUNatom

integer :: katom
integer :: UniSymMum
integer :: IndexFoundNum
integer :: fileunit
integer :: ifound
integer :: newindex
integer :: maxindex
integer :: iSym

character (4) :: OPT_YN
character (60) :: cmd
character (60) :: radline

```

```

character (60) :: outfile
character (60) :: inputfile
logical :: Lerror

allocate (MolIdxInfo(Natoms))
allocate (FragInfo(Natoms))
allocate (SortedFragmentAtom(Natoms,Natoms))
allocate (DBFragmentAtom(Natoms,Natoms))
allocate (symbols(Natoms))
allocate (UniSym(Natoms))
allocate (MUNatom(Natoms))

CALL Check_DB_files()
CALL GetSortCartTerm(Level_Number,SortedFragmentAtom,FragInfo)
CALL GetDBFormFragments(SortedFragmentAtom,DBFragmentAtom,FragInfo)
CALL GetFragSymbols(Level_Number,symbols)
CALL GetUniSym (Level_Number,UniSym,MUNatom,UniSymMum)
CALL GetMoleculeAtomsIndex (MolIdxInfo,UniSym,UniSymMum,IndexFoundNum)
CALL GetMaxIndex (maxindex) ! Find Maxindex

write(*,'(A)') "*****"
write(*,'(A)') " The unique Symbols in the molecule "
write(*,'(A)') "*****"
do iSym=1, UniSymMum
  write (*,'(A,I4,A,A)') "SYM# ", iSym, " is ", trim(UniSym(iSym))
end do
  write(*,'(A)') "AIMDFT-AIMDFT-AIMDFT-AIMDFT-AIMDFT-AIMDFT-AIMDFT-AIMDFT"
!! WORKING WITH the nonavaliabile fragments within database
! add the new symbols
newindex=maxindex
do ifound=1, UniSymMum
  if (MolIdxInfo(ifound)%Availability) cycle
  newindex=newindex+1
  ! generate input files
  CALL INPUT_File_FRAG ('INPUT_','.dat',newindex,inputfile)
  CALL INPUT_File_FRAG ('INPUT_','.out',newindex,outfile)
  CALL GET_unit (inputfile, fileunit, Lerror)
  open(UNIT=fileunit, file=inputfile, status='REPLACE')
  write(fileunit,'(A)') &
  "MOLECULE MUltiplicity = 1 CHarge = 0 UNit = Angstrom"
  write(fileunit,'(A)')'TITLE = '//UniSym(ifound)//'
  write(fileunit,'(A)')'Cartesian'
  do katom=1,FragInfo(MUNatom(ifound))%NAtoms
    write(fileunit,'(A5,3f26.20)') &
    & DBFragmentAtom(MUNatom(ifound), &
    & katom)%element, &
    & DBFragmentAtom(MUNatom(ifound),katom)%x*Bohr_to_Angstrom, &
    & DBFragmentAtom(MUNatom(ifound),katom)%y*Bohr_to_Angstrom, &
    & DBFragmentAtom(MUNatom(ifound),katom)%z*Bohr_to_Angstrom
  end do
  write(fileunit,'(A)')'end ! Cartesian'
  write(fileunit,'(A/)')'end ! MOLECULE'
  write(fileunit,'(A12,A14,A4/)')'basis name= ',&
  trim(Basis_set_name),' end'

  write(fileunit,'(A25,I4.4,A9/)')'SET RUN NAME = "RUN_FRAG_',&
  newindex,'" end end'

! write(fileunit,'(A/)') &
! 'GRID MESH ORigin= ( 0.0 -5.0 -2.0 ) &
! MESH = ( 0.02 0.02 0.02 ) NX=0 NY=500 NZ=350 end end'
write(fileunit,'(A/)')'PLOT ATOM=false Molecule=false end'
OPT_YN=""
if (trim(OPT_status).eq.'OPT') then
  OPT_YN="OPT"

```



```

INQUIRE(FILE=trim(IndexFilePath), EXIST=index_exists)
INQUIRE(FILE=trim(CartFilePath), EXIST=cart_exists)
INQUIRE(FILE=trim(PropFilePath), EXIST=prop_exists)
if (.not.index_exists) then
CALL GET_unit (trim(IndexFilePath), File_unit, Lerror)
open(UNIT=File_unit,file=trim(IndexFilePath), &
      status='new',form='formatted')
write(File_unit,fmt_index_title) "Index#",          &
                                & "Symbol",        &
                                & "Atom#",          &
                                & "Grid#",          &
                                & "Grid",           &
                                & "Method",         &
                                & "Basisset",       &
                                & "Weight",         &
                                & "CoreSize",       &
                                & "Opt/NO-opt"
close(unit=File_unit)
end if

if (.not.cart_exists) then
CALL GET_unit (trim(CartFilePath), File_unit, Lerror)
open(UNIT=File_unit,file=trim(CartFilePath), &
      status='new',form='formatted')
write(File_unit,fmt_cart_title) "#","#", "Sym","X","Y","Z"
close(unit=File_unit)
end if

if (.not.prop_exists) then
CALL GET_unit (trim(PropFilePath), File_unit, Lerror)
open(UNIT=File_unit,file=trim(PropFilePath), &
      status='new',form='formatted')
write(File_unit,fmt_prop_title) &
                                & "#",              &
                                & "Electrons",       &
                                & "K",              &
                                & "K_HF",        &
                                & "T",           &
                                & "Vne_Ana",     &
                                & "Vne_Num",     &
                                & "Vee (Ana/Num)", &
                                & "J",           &
                                & "J_HF",       &
                                & "Jaa=Kaa",    &
                                & "Vee_DFT_AtomA"

close(unit=File_unit)
end if

end SUBROUTINE Check_DB_files
end SUBROUTINE Build_FragFiles

```

```

SUBROUTINE get_databaseform()
!*****
!   Date last modified:
!   Author: Ibrahim Awad
!   Description: Display the current and database cartesian coordinates
!               for the molecule fragments
!*****
!
USE AIMDFT_type
USE type_molecule
USE mod_rotation

```



```

implicit none

double precision, dimension (:,:), allocatable :: OldCart
double precision, dimension (:,:), allocatable :: ROTCart
double precision, dimension (:,:), allocatable :: NewCart
double precision, dimension (:,:), allocatable :: RR
double precision, dimension (:), allocatable :: TT
integer :: Iatom, JAtom, IROT

allocate (RR(1:3,1:3))
allocate (TT(1:3))
allocate (ROTCart(NAtoms,3))
allocate (OldCart(NAtoms,3))
allocate (NewCart(NAtoms,3))

if(NROTLlist.eq.0) then ! If no atoms specified, stop
write(*,*) "Error: No NROTLlist"
call exit()
end if

OldCart(1:Natoms,1)=Cartesian(1:Natoms)%x
OldCart(1:Natoms,2)=Cartesian(1:Natoms)%y
OldCart(1:Natoms,3)=Cartesian(1:Natoms)%z

do IROT=1,NROTLlist
Iatom=ROTLlist(IROT)
ROTCart(IROT,1:3) = OldCart(Iatom,1:3)
end do

call GetStoredRotMatrix (ROTCart,NROTLlist,RR,TT)
call DoRotation (OldCart,NewCart,Natoms,RR,TT)

if (RotUpdate) then
Cartesian(1:Natoms)%x=NewCart(1:Natoms,1)
Cartesian(1:Natoms)%y=NewCart(1:Natoms,2)
Cartesian(1:Natoms)%z=NewCart(1:Natoms,3)
write(*, '( /A,/A)') &
"RotUpdate is TRUE", "The Cartesian coordinates were updated"
write(*, '(A)') "The new coordinates are: (Angstrom unit)"
write(*, '(A4,3A14)') "Atom", "X", "Y", "Z"
do Iatom=1, Natoms
write(*, '(A4,3F14.8)') Cartesian(Iatom)%element, &
& NewCart(Iatom,1)*CartPrnFactor, &
& NewCart(Iatom,2)*CartPrnFactor, &
& NewCart(Iatom,3)*CartPrnFactor
end do
write(*,*) "-----"
end if

deallocate (ROTCart,OldCart,NewCart)
deallocate (RR)
deallocate (TT)
return
end SUBROUTINE get_databaseform

```

```

MODULE mod_math
! *****
! Date last modified: *
! Author: Ibrahim Awad *
! Description: *
! *****
!
implicit none

```



```

        + A(1,3)*A(2,1)*A(3,2) &
        - A(1,3)*A(2,2)*A(3,1)

RETURN
end FUNCTION DET33
double precision FUNCTION VectorsDistance(dim_num,v1,v2)
!*****
!   Date last modified:
!   Author: Ibrahim Awad
!   Description: Calculate the distance between two vectors
!*****
!
implicit none

integer, intent(in) :: dim_num
double precision, dimension(dim_num), intent(in) :: v1, v2

VectorsDistance=dsqrt(sum((v1(1:dim_num)-v2(1:dim_num))**2))

RETURN
end FUNCTION VectorsDistance
SUBROUTINE INV33(A,T)
!*****
!   Date last modified:
!   Author: Ibrahim Awad
!   Description: Calculate the inverse of a matrix (3x3), real values.
!*****
!
implicit none

double precision, dimension (3,3), intent(in) :: A
double precision, dimension (3,3), intent(out) :: T

T(1,1) = (A(2,2)*A(3,3)-A(2,3)*A(3,2))/DET33(A)
T(1,2) = -(A(1,2)*A(3,3)-A(1,3)*A(3,2))/DET33(A)
T(1,3) = (A(1,2)*A(2,3)-A(1,3)*A(2,2))/DET33(A)
T(2,1) = -(A(2,1)*A(3,3)-A(2,3)*A(3,1))/DET33(A)
T(2,2) = (A(1,1)*A(3,3)-A(1,3)*A(3,1))/DET33(A)
T(2,3) = -(A(1,1)*A(2,3)-A(1,3)*A(2,1))/DET33(A)
T(3,1) = (A(2,1)*A(3,2)-A(2,2)*A(3,1))/DET33(A)
T(3,2) = -(A(1,1)*A(3,2)-A(1,2)*A(3,1))/DET33(A)
T(3,3) = (A(1,1)*A(2,2)-A(1,2)*A(2,1))/DET33(A)

end SUBROUTINE INV33
SUBROUTINE RANDSEED(R)
!*****
!   Date last modified:
!   Author: Ibrahim Awad
!   Description: Return random number depend on the seed time.
!*****
!
implicit none

integer, INTENT(OUT) :: R
integer :: randnumI,seedsiz,values(1:8)
character(len=6) :: randnumS
integer, allocatable :: seed(:)
double precision :: randnum

call date_and_time(values=values)
call random_seed(size=seedsiz)
allocate(seed(seedsiz))
seed(:) = values(8)
call random_seed(put=seed)
call random_number(randnum)

```



```

!      Date last modified:
!      Author: Ibrahim Awad
!      Description:
! *****
!

implicit none

integer, intent(in) :: level
type(FragAtomInfo), dimension(:, :), intent(out), allocatable :: SortFragAtoms
type(FragInfo), dimension(:), intent(out), allocatable :: SortFragInfo

allocate (SortFragAtoms(Natoms, Natoms))
allocate (SortFragInfo(Natoms))

CALL GenerateFragments(level, SortFragAtoms, SortFragInfo)
CALL AddTerminalAtoms(level, SortFragAtoms, SortFragInfo)
! +1 >>> Also sort the new terminals
CALL SortFType(level+1, SortFragAtoms, SortFragInfo)

return
end SUBROUTINE GetSortCartTerm
SUBROUTINE SortLevel(level, SortFragAtoms, SortFragInfo)
! *****
!      Date last modified:
!      Author: Ibrahim Awad
!      Description:
! *****
!

implicit none

integer, intent(in) :: level
type(FragAtomInfo), dimension(:, :), intent(inout), allocatable :: SortFragAtoms
type(FragInfo), dimension(:), intent(inout), allocatable :: SortFragInfo
type(FragAtomInfo), dimension(:, :), allocatable :: TEMPFragAtoms
integer :: currvalue, nextvalue, inext
integer :: Jatom, Jatom, ilevel, NtempOrd, curr, next
integer, dimension(:, :), allocatable :: countLevel
integer, dimension(:), allocatable :: excludeLis
integer, dimension(10) :: tempOrd
character (SymMax) :: TempCopyChar
logical :: swap

allocate (TEMPFragAtoms(Natoms, Natoms))
allocate (countLevel(Natoms, level))
allocate (excludeLis(Natoms))

! FIRST :: Sort atoms by levels
! loop over all fragments for Jatoms
do Jatom=1, Natoms
! exclude dummy atoms
if (Cartesian(Jatom)%Atomic_Number.eq.0) cycle
NtempOrd=SortFragInfo(Jatom)%NAtoms
swap=.true.
do while(swap)
swap=.false.
do curr=1, NtempOrd-1
next=curr+1
currvalue=SortFragAtoms(Jatom, curr)%level
nextvalue=SortFragAtoms(Jatom, next)%level
if(currvalue.gt.nextvalue) then
TEMPFragAtoms(Jatom, curr)=SortFragAtoms(Jatom, curr)
SortFragAtoms(Jatom, curr)=SortFragAtoms(Jatom, next)
SortFragAtoms(Jatom, next)=TEMPFragAtoms(Jatom, curr)

```

```

        swap=.true.
    end if
end do ! do curr
end do ! do while(swap)
end do ! all atoms (atom by atom)

! Check each level if there is a UNIQUE level that has just one atom,
! then fix there positions
countLevel=0
do ilevel=1, level ! level loop
    do Jatom=1, NAtoms ! fragments loop
        do Iatom=1, SortFragInfo(Jatom)%NAtoms ! atoms within the fragments
            if (SortFragAtoms(Jatom,Iatom)%level.ne.ilevel) cycle
            if (SortFragAtoms(Jatom,Iatom)%sorted) cycle
            countLevel(Jatom,ilevel)=countLevel(Jatom,ilevel)+1
        end do ! subatom within fragment
    end do ! all atoms (atom by atom) (Fragments)
end do ! level by level
! Fixed the value for the UNIQUE atoms within ilevel
do ilevel=1, level ! level loop
    do Jatom=1, NAtoms ! fragments loop
        if (countLevel(Jatom, ilevel).eq.1) then
            do Iatom=1, SortFragInfo(Jatom)%NAtoms
                if (SortFragAtoms(Jatom,Iatom)%level.ne.ilevel) cycle
                if (SortFragAtoms(Jatom,Iatom)%sorted) cycle
                SortFragAtoms(Jatom,Iatom)%sorted=.true.
            end do ! subatom within fragment
        end if
    end do ! all atoms (atom by atom) (Fragments)
end do ! level by level
deallocate (TEMPFragAtoms)
deallocate (countLevel)
deallocate (excludeLis)
end SUBROUTINE SortLevel

SUBROUTINE SortZNUM(level,SortFragAtoms,SortFragInfo)
!*****
! Date last modified: *
! Author: Ibrahim Awad *
! Description: *
!*****
!
implicit none

integer, dimension(:), allocatable :: CountAtomicN, CountAtomicI
integer, dimension(:), allocatable :: CountAtomicT,excludeLis
integer :: NtempOrd, TempCopy, curr,next, iZ, Z_Vali, Z_Valj, excludeN
integer, intent(in) :: level
type(FragAtomInfo),dimension(:,:),intent(inout),allocatable::SortFragAtoms
type(FragsInfo),dimension(:),intent(inout),allocatable::SortFragInfo
type(FragAtomInfo), dimension(:,:), allocatable :: TEMPFragAtoms
integer :: currvalue, nextvalue, inext, Iatom, Jatom, ilevel
integer, dimension(:,:), allocatable :: countLevel
integer, dimension(10) :: tempOrd
character (SymMax) :: TempCopyChar
logical :: swap

allocate (CountAtomicN(Natoms))
allocate (CountAtomicI(Natoms))
allocate (CountAtomicT(Natoms))
allocate (excludeLis(Natoms))
allocate (TEMPFragAtoms(Natoms,Natoms))
allocate (countLevel(Natoms,level))

```

```

CALL SortLevel(level,SortFragAtoms,SortFragInfo)

! SECOND ::: Sort atoms by ATOMIC NUMBER and keep the level
! Now loop within each level for fragment Jatom (fixed the level)
do ilevel=1, level ! level loop
  do Jatom=1, NAtoms
    ! exclude dummy atoms
    if (Cartesian(Jatom)%Atomic_Number.eq.0) cycle

    NtempOrd=SortFragInfo(Jatom)%NAtoms
    swap=.true.
    do while(swap)
      !if (FragAtoms(Jatom,ifound)%level.ne.ilevel) cycle
      swap=.false.
      do curr=1,NtempOrd-1
        !Just ilevel
        if (SortFragAtoms(Jatom,curr)%level.ne.ilevel) cycle
        if (SortFragAtoms(Jatom,curr)%sorted) cycle
        ! set curr subatom
        currvalue=SortFragAtoms(Jatom,curr)%Atomic_Number

        ! search for the next subatom in the same level,
        ! then compare
        next=-1
        do inext=curr+1, NtempOrd
          if (SortFragAtoms(Jatom,inext)%sorted) cycle
          if (SortFragAtoms(Jatom,inext)%level.eq.ilevel) then
            nextvalue=SortFragAtoms(Jatom,inext)%Atomic_Number
            next=inext
            ! Found subatom in the same level with the curr atom
            exit
          end if
        end do
        ! There just one atom at that level (NO next atom)
        if (next.eq.-1) cycle

        ! compare
        if(currvalue.lt.nextvalue)then
          TEMPFragAtoms(Jatom,curr)=SortFragAtoms(Jatom,curr)
          SortFragAtoms(Jatom,curr)=SortFragAtoms(Jatom,next)
          SortFragAtoms(Jatom,next)=TEMPFragAtoms(Jatom,curr)
          swap=.true.
        endif
      end do ! do curr
      !write(*,*) currvalue, "<", nextvalue, v1,v2
    end do ! do while(swap)
  end do ! all atoms (atom by atom)
end do ! level by level

! NEXT step is to Check each level if there is a UNIQUE atom with ....
! ... (no other atoms have same Z), then fix its position
do Jatom=1, NAtoms ! fragments loop
  do ilevel=1, level ! level loop
    iZ=0
    CountAtomicN=0
    do Iatom=1, SortFragInfo(Jatom)%NAtoms !atoms within the fragments
      if (SortFragAtoms(Jatom,Iatom)%level.ne.ilevel) cycle
      if (SortFragAtoms(Jatom,Iatom)%sorted) cycle
      iZ=iZ+1
      CountAtomicN(iZ)=SortFragAtoms(Jatom,Iatom)%Atomic_Number
      CountAtomicI(iZ)=Iatom
    end do ! subatom within fragment
    ! now check and put the value within array
    excludeN=0
  end do
end do

```

```

        excludeLis=0
        do Z_Vali=1, iZ
            do Z_Valj=Z_Vali+1,iZ
                if (Z_Vali.eq.Z_Valj) cycle
                if (CountAtomicN(Z_Vali).eq.CountAtomicN(Z_Valj)) then
                    if (.not.ANY( excludeLis==CountAtomicI(Z_Vali))) then
                        excludeN=excludeN+1
                        excludeLis(excludeN)=CountAtomicI(Z_Vali)
                    end if
                    if (.not.ANY( excludeLis==CountAtomicI(Z_Valj))) then
                        excludeN=excludeN+1
                        excludeLis(excludeN)=CountAtomicI(Z_Valj)
                    end if
                end if
            end do
        end do
        do Iatom=1, SortFragInfo(Jatom)%NAtoms !atoms within the fragments
            if (SortFragAtoms(Jatom,Iatom)%level.ne.ilevel) cycle
            if (SortFragAtoms(Jatom,Iatom)%sorted) cycle
            if (.not.ANY( excludeLis==Iatom)) &
                SortFragAtoms(Jatom,Iatom)%sorted=.true.
        end do ! subatom within fragment
    end do ! level by level
end do ! all atoms (atom by atom) (Fragments)
deallocate (CountAtomicN)
deallocate (CountAtomicI)
deallocate (CountAtomicT)
deallocate (excludeLis)
deallocate (TEMPFragAtoms)
deallocate (countLevel)
return
end SUBROUTINE SortZNUM

SUBROUTINE SortFType(level,SortFragAtoms,SortFragInfo)
!*****
! Date last modified:
! Author: Ibrahim Awad
! Description:
!*****
implicit none

integer, dimension(:), allocatable :: CountAtomicN, CountAtomicI
integer, dimension(:), allocatable :: CountAtomicT,excludeLis
integer :: NtempOrd, TempCopy, curr,next, iZ, Z_Vali, Z_Valj, excludeN
integer, intent(in) :: level
type(FragAtomInfo),dimension(:,:),intent(inout),allocatable::SortFragAtoms
type(FragInfo),dimension(:),intent(inout),allocatable::SortFragInfo
type(FragAtomInfo),dimension(:,:),allocatable::TEMPFragAtoms
integer :: currvalue, nextvalue, inext, Iatom, Jatom, ilevel
integer, dimension(:,:), allocatable :: countLevel
integer, dimension(10) :: tempOrd
character (SymMax) :: TempCopyChar
logical :: swap

allocate (CountAtomicN(Natoms))
allocate (CountAtomicI(Natoms))
allocate (CountAtomicT(Natoms))
allocate (excludeLis(Natoms))
allocate (TEMPFragAtoms(Natoms,Natoms))
allocate (countLevel(Natoms,level))

CALL SortLevel(level,SortFragAtoms,SortFragInfo)
CALL SortZNUM(level,SortFragAtoms,SortFragInfo)

```



```

! Third :::: Sort atoms by ATOMIC TYPE for the fragment
! without the NEW terminal and keep the level
! Now loop within each level for fragment Jatom (fixed the level)
do ilevel=1, level ! level loop
  do Jatom=1, NAtoms
    ! exclude dummy atoms
    if (Cartesian(Jatom)%Atomic_Number.eq.0) cycle

    NtempOrd=SortFragInfo(Jatom)%NAtoms
    swap=.true.
    do while(swap)
      !if (FragAtoms(Jatom,ifound)%level.ne.ilevel) cycle !Just ilevel
      swap=.false.
      do curr=1,NtempOrd-1
        if (SortFragAtoms(Jatom,curr)%level.ne.ilevel) cycle
        if (SortFragAtoms(Jatom,curr)%sorted) cycle
        ! set curr subatom
        currvalue=SortFragAtoms(Jatom,curr)%type

        ! search for the next subatom in the same level,
        ! then compare
        next=-1
        do inext=curr+1, NtempOrd
          ! exclude T sorted
          if (SortFragAtoms(Jatom,inext)%sorted) cycle
          ! same atomic number
          if (SortFragAtoms(Jatom,curr)%Atomic_Number.ne.&
              SortFragAtoms(Jatom,inext)%Atomic_Number) cycle
          if (SortFragAtoms(Jatom,inext)%level.eq.ilevel) then
            nextvalue=SortFragAtoms(Jatom,inext)%type
            next=inext
            exit ! Found subatom in the same level and same
                  ! atomic number with the curr atom
          end if
        end do
        if (next.eq.-1) cycle ! There just one atom at that level

        ! compare
        if(currvalue.lt.nextvalue)then
          TEMPFragAtoms(Jatom,curr)=SortFragAtoms(Jatom,curr)
          SortFragAtoms(Jatom,curr)=SortFragAtoms(Jatom,next)
          SortFragAtoms(Jatom,next)=TEMPFragAtoms(Jatom,curr)
          swap=.true.
        endif
      end do ! do curr
      !write(*,*) currvalue, "<", nextvalue, v1,v2
    end do ! do while(swap)
  end do ! all atoms (atom by atom)
end do ! level by level

! NEXT step is to Check each level if there is a UNIQUE atom with ....
! ... (no other atoms have same Z and type), then fix its position
do Jatom=1, NAtoms ! fragments loop
  do ilevel=1, level ! level loop
    iZ=0
    CountAtomicN=0
    do Iatom=1, SortFragInfo(Jatom)%NAtoms !atoms within the fragments
      if (SortFragAtoms(Jatom,Iatom)%level.ne.ilevel) cycle
      if (SortFragAtoms(Jatom,Iatom)%sorted) cycle
      iZ=iZ+1
      CountAtomicT(iZ)=SortFragAtoms(Jatom,Iatom)%type
      CountAtomicN(iZ)=SortFragAtoms(Jatom,Iatom)%Atomic_Number
      CountAtomicI(iZ)=Iatom
    end do
  end do
end do

```

```

end do ! subatom within fragment
! now check and put the value within array
excludeN=0
excludeLis=0
do Z_Vali=1, iZ
  do Z_Valj=Z_Vali+1,iZ
    if (Z_Vali.eq.Z_Valj) cycle
    if (CountAtomicN(Z_Vali).eq.CountAtomicN(Z_ValJ).and.&
      CountAtomicT(Z_Vali).eq.CountAtomicT(Z_ValJ)) then
      if (.not.ANY( excludeLis==CountAtomicI(Z_Vali))) then
        excludeN=excludeN+1
        excludeLis(excludeN)=CountAtomicI(Z_Vali)
      end if
      if (.not.ANY( excludeLis==CountAtomicI(Z_ValJ))) then
        excludeN=excludeN+1
        excludeLis(excludeN)=CountAtomicI(Z_ValJ)
      end if
    end if
  end do
end do
do Iatom=1, SortFragInfo(Jatom)%NAtoms !atoms within the fragments
  if (SortFragAtoms(Jatom,Iatom)%level.ne.ilevel) cycle
  if (SortFragAtoms(Jatom,Iatom)%sorted) cycle
  if (.not.ANY( excludeLis==Iatom))&
    SortFragAtoms(Jatom,Iatom)%sorted=.true.
  end do ! subatom within fragment
end do ! level by level
end do ! all atoms (atom by atom) (Fragments)
deallocate (CountAtomicN)
deallocate (CountAtomicI)
deallocate (CountAtomicT)
deallocate (excludeLis)
deallocate (TEMPFragAtoms)
deallocate (countLevel)
return
end SUBROUTINE SortFType
SUBROUTINE GetSortCart(level,SortFragAtoms,SortFragInfo) ! not completed
!*****
! Date last modified: *
! Author: Ibrahim Awad *
! Description: *
!*****
!

implicit none

integer, intent(in) :: level
type(FragAtomInfo),dimension(:,,:),intent(out),allocatable :: SortFragAtoms
type(FragInfo),dimension(:,,:),intent(out),allocatable :: SortFragInfo
type(FragAtomInfo),dimension(:,,:),allocatable :: FragAtoms,TEMPFragAtoms
type(FragInfo),dimension(:,,:),allocatable :: FragInfo ! (main atom)
integer, dimension(:), allocatable:: sortindex,sorttype
integer :: Iatom,Jatom,ifound,ilevel,kfound,mfound,ifoundlevel
integer :: startilevel,nosimilar,nsim,munatom
integer :: currvalue, nextvalue, inext
integer, dimension(:,,:), allocatable :: countLevel
integer, dimension(:), allocatable :: CountAtomicT,excludeLis
integer, dimension(:), allocatable :: CountAtomicN, CountAtomicI
double precision, dimension(:), allocatable :: SumZ, SumZ_x
double precision, dimension(:), allocatable :: SumZ_y, SumZ_z
double precision, dimension(:,,:), allocatable :: Z_x,Z_y,Z_z
double precision, dimension(:,,:), allocatable :: Cent_x, Cent_y, Cent_z
double precision, dimension(:,,:), allocatable :: Cent_R
integer, dimension(10) :: tempOrd
integer :: NtempOrd, TempCopy, curr,next, iZ, Z_Vali, Z_Valj

```



```

      Cent_y(Jatom,Iatom)=(Z_y(Jatom,Iatom)-SumZ_y(Jatom))/SumZ(Jatom)
      Cent_z(Jatom,Iatom)=(Z_z(Jatom,Iatom)-SumZ_z(Jatom))/SumZ(Jatom)
      Cent_R(Jatom,Iatom)=dsqrt( Cent_x(Jatom,Iatom)**2 + &
                                & Cent_y(Jatom,Iatom)**2 + &
                                & Cent_z(Jatom,Iatom)**2)

      write(*,*) Jatom, Iatom, Cent_R(Jatom,Iatom),&
        SortFragAtoms(Jatom,Iatom)%element
    end do ! subatom within fragment
    !SumZ_x(Jatom)=SumZ_x(Jatom)/SumZ(Jatom)
    !SumZ_y(Jatom)=SumZ_y(Jatom)/SumZ(Jatom)
    !SumZ_z(Jatom)=SumZ_z(Jatom)/SumZ(Jatom)
    !write(*,'(I3,4F16.8)') Jatom, SumZ(Jatom), &
    !                               SumZ_x(Jatom), SumZ_y(Jatom), SumZ_z(Jatom)
  end do ! all atoms (atom by atom) (Fragments)
end if ! just comment

CALL CalcFragConnect(SortFragAtoms,SortFragInfo)
deallocate(FragAtoms,TEMPFragAtoms, FragInfo, countLevel)
deallocate(CountAtomicN,CountAtomicI,excludeLis,CountAtomicT)
deallocate(SumZ,SumZ_x,SumZ_y,SumZ_z,Z_x ,Z_y ,Z_z,)
deallocate(Cent_x,Cent_y,Cent_z,Cent_R)
end SUBROUTINE GetSortCart
end MODULE Sorted_AIMDFT

```

# **Bibliography**